

# Modern Physics Lab Guide

*The College of Wooster*

*Department of Physics*

*2017 December 3*

## Front Cover

A circular corral of radius 71.3 Å constructed by individually positioning 48 iron atoms with the tip of a 4-kelvin Scanning Tunneling Microscope. STM images reveal that the corral's density of states is dominated by the eigenstate density expected for an electron trapped in a round two-dimensional box.

M. F. Crommie, C. P. Lutz, D. M. Eigler, "Confinement of Electrons to Quantum Corrals on a Metal Surface", *Science*, **262** (5131), 218-220 (1993 October 8).



## Preface

*Last updated 31 August 2007*

Most of these “Modern Physics” experiments were first performed in or around the critical timeframe often called the thirty years that shook physics, 1896-1926. For reference, this was the time of Henry Ford’s Model T and Model A automobiles, and the map of Europe was still dominated by the German, Russian, Austro-Hungarian, and Ottoman Empires. These experiments were our first contact with the microworld. They forced upon us an unanticipated and nonintuitive worldview that physicists and philosophers still struggle with today.

Doing these experiments, you follow in the footsteps of that turbulent generation by observing firsthand the quantization of energy and wave-particle duality, and by determining four measures of our universe, the fundamental constants  $c$ ,  $e$ ,  $m$ ,  $h$ .



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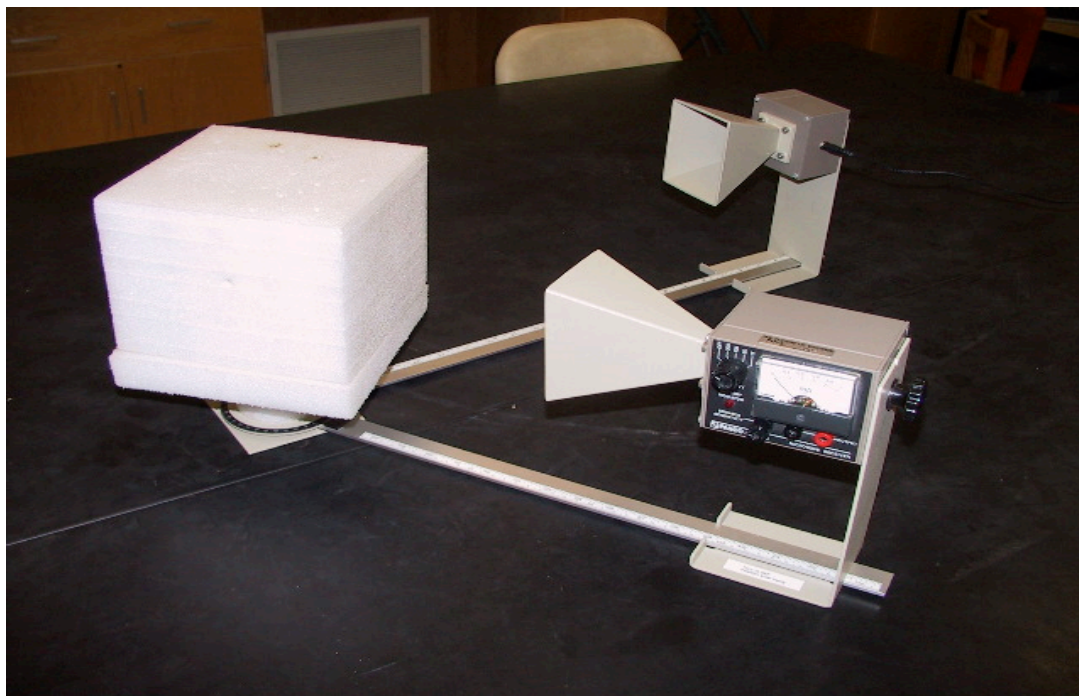
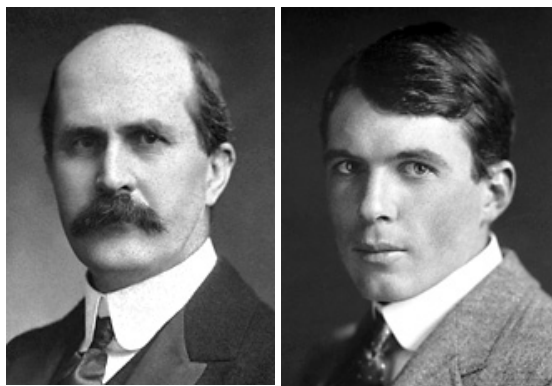


Fig. 1.1. Bragg microwave scattering analogue apparatus includes a microwave transmitter, receiver, and a cubic array of ball bearings suspended in Styrofoam.



Fig. 1.2. Bragg scattering x-ray apparatus includes a Mo x-ray source, a Geiger-Müller counter, and a NaCl monocrystal target.



## 1. Bragg Scattering

*Last updated 2017 October 29*

### Summary

Infer the invisible! Diffraction patterns reveal unseen structures over 8 orders of magnitude. Bragg reflection determines the lattice spacing of crystals, either with microwaves in the Fig. 1.1 Styrofoam and ball bearing macroscopic analogue or with x-rays in the Fig. 1.2 NaCl monocrystal.

### History

In 1895, Wilhelm Röntgen discovered x-rays, for which he was awarded a 1901 Nobel Prize. In 1912, Max von Laue used atomic crystals as (three-dimensional) *transmission* gratings for x-rays, for which he was awarded a 1914 Nobel Prize. Also in 1912, father and son **William Henry Bragg** and **William Lawrence Bragg** used atomic crystals as (three-dimensional) *reflection* gratings [1], for which they shared a 1915 Nobel Prize. In 1953, Francis Crick and James Watson inferred the double-helical structure of DNA from x-ray crystallographic data by Rosalind Franklin and Maurice Wilkins. Franklin died in 1958, while Crick, Watson, and Wilkins shared a 1962 Nobel Prize.

### Theory

This experiment illustrates the way in which radiation is diffracted by a regular three-dimensional array of scattering points when the spacing between the scatterers is of the same order as the wavelength of the incident radiation. The effect was first and most importantly observed in crystals, with the atoms at their regular lattice sites forming the 3-dimensional array. Since the spacing between atoms is  $\sim 1\text{\AA} = 10^{-8}\text{ cm}$ , the radiation needed lies in the x-ray region. X-ray diffraction today provides the most important way of finding crystal structures. Very complex structures including large biological molecules have been elucidated using these techniques. Furthermore, particles (first electrons and later neutrons) can also be diffracted by crystals when their energy is such that their de Broglie wavelength is in the appropriate range. The fact that such an interference effect can be observed for particles was the first experimental evidence that particles have a wave-like aspect.

The simplest analysis is due to the Braggs. Think of a series of parallel planes through the

atoms and assume the incident beam is partially reflected at each plane. Thus, the reflected beam contains components that have been reflected from several different planes. Assume that the usual law of reflection holds, so that the angles of incidence and reflection are equal. Label this common *grazing angle*  $\theta$ , as in Fig. 1.3. (Note that the angles are defined, by convention, relative to the surface, *not* the normal.)

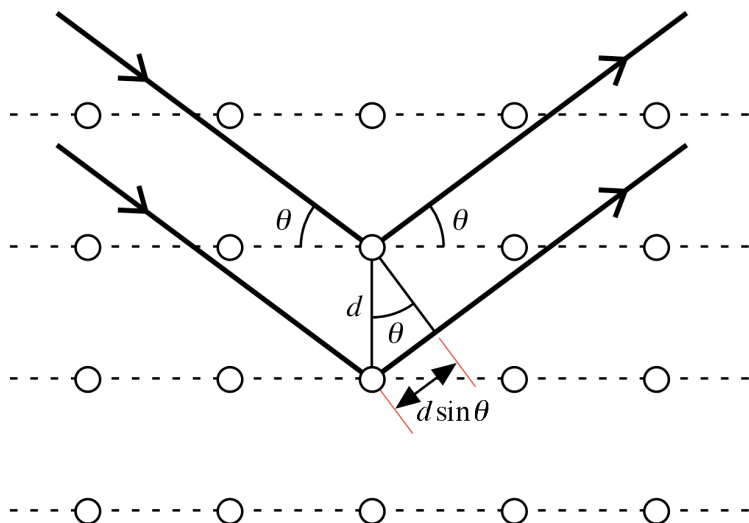


Fig. 1.3. Radiation Bragg-scatters from successive atomic planes.

For rays reflected from adjacent planes separated by a distance  $d$ , the path difference is  $2d \sin \theta$ . Constructive interference, a maximum in the reflected beam, occurs whenever the path difference is an integral number of wavelengths,

$$2d \sin \theta = n\lambda \quad (1.1)$$

where  $n$  is an integer. Thus, only certain discrete angles show constructive interference. Note that if the wavelength is longer than  $2d$  there is no interference. Thus, x-rays are needed to analyze real crystals.

## Procedure

### Microwaves

Align the Styrofoam “crystal” so that one cube edge is parallel to the direction of microwave travel. Note that the angle marker reads zero in this configuration. All waves incident on the crystal will be reflected back upon themselves so both the angle of incidence and reflection are zero. Now rotate the crystal  $5^\circ$ . The grazing angle reads  $5^\circ$  and you will need to set the microwave receiver at the  $10^\circ$  position so that the grazing angle equals the angle of reflection. In general, you will move the receiver  $2^\circ$  for every  $1^\circ$  the crystal moves.

Collect the Bragg scattering intensity as a function of grazing angle for the finest angle

resolution you can manage. Beware of interpreting the transmitted microwaves as scattering at high angles. You may wish to protect your apparatus from reflections off objects in the room by shielding it with wet paper towels; otherwise peaks may appear at additional angles not predicted by Eq. (1.1). **Better results may be obtained if the transmitter and receiver are rotated so that they are taller than they are wide.**

### X-rays

To install the target in the x-ray apparatus, loosen the target stage's knurled screw, and lay the NaCl crystal flat on the stage. Carefully raise the stage as far as it will go and tighten the knurled screw with care, pressing against the screw lightly to prevent it from stripping. Avoid mechanical stresses on the crystal; handle the crystal by the short faces only. Adjust the zero position of the goniometer measuring system as necessary.

If the counting rate is too low, reduce the distance between the target and the sensor somewhat. However, if this distance is too small, the angular resolution of the goniometer (angle measurer) will not be great enough to distinguish the characteristic  $K_\alpha$  and  $K_\beta$  lines of the Mo x-ray source.

Check that the x-ray apparatus is connected to a computer. Launch the x-ray Apparatus program and delete any existing measurement data. Set the x-ray high voltage  $U = 35.0$  kV, emission current  $I = 1.00$  mA, measuring time per angular step  $\Delta t = 10$  s, and angular step width  $\Delta\beta = 0.1^\circ$ . Press the COUPLED key on the device to enable  $2\nu$  coupling of the target and sensor; set the lower limit value of the target scan to  $2^\circ$  and the upper limit to  $25^\circ$ . Press the SCAN key to start the measurement and data transmission to the computer. When finished, save the measurement series to a file under a suitable name.

### Analysis

#### Microwaves

Make a graph of scattered intensity versus grazing angle and label any maxima present with the order of diffraction and planes giving rise to the diffraction. You will need to use Eq. (1) and an estimate of the spacing to predict what orders and planes give rise to what theoretical angles. Then you can use your measured angles, now appropriately identified in terms of planes and orders, to calculate the spacing. This iterative procedure is very realistic in terms of the ways scientists solve complex crystal structure problems with x-ray scattering. You should expect to see several maxima, and from the location of those maxima, arrive at several measurements for the spacing  $d$ . What is the relative (or percentage) difference between your best estimate of  $d$  from the scattering data and a direct measurement?

In Fig. 1.4, what is the spacing between planes for a “crystal” in a parallel configuration? What is the spacing between planes for a crystal in a diagonal configuration? Can you observe Bragg scattering for the diagonal configuration?

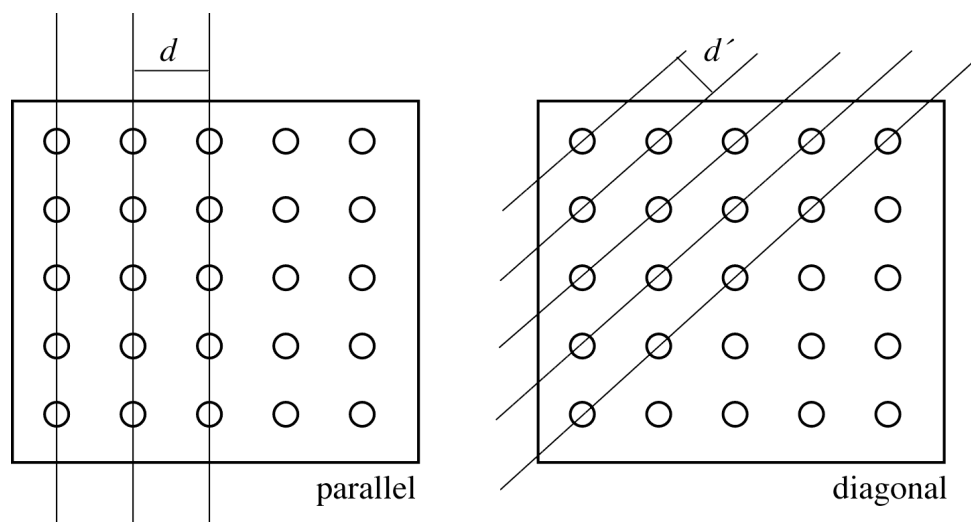


Fig. 1.4. The crystal has multiple sets of parallel Bragg planes.

### X-rays

The x-ray radiation consists of the bremsstrahlung continuum and several sharply defined lines that correspond to the characteristic x-ray radiation of the Mo anode, which originate in the  $K_\alpha$  and  $K_\beta$  transitions of the Mo atoms. Measure the angles of the peaks in the scattering intensity and use Eq. (1.1) to infer the NaCl lattice spacing. Compare with the  $d = 282$  pm expected value.

### Acknowledgment

These guidelines are based partly on Leybold Physics Leaflet P6.3.3.1.

### Reference

[1] William Lawrence Bragg, "The Diffraction of Short Electromagnetic Waves by a Crystal", *Proceedings of the Cambridge Philosophical Society*, **17**, 43–57(1913).







Fig. 2.1. Electron charge-to-mass ratio apparatus features a Helmholtz coil.





## 2. Thomson Electron Charge-to-Mass Ratio

*Last updated 2017 November 30*

### Summary

Measure the charge-to-mass ratio  $e/m$  of electrons by observing their deflection in a magnetic field of Fig. 2.1. This is the key that unlocked the subatomic world.

### History

In the late 1800s, the nature of “cathode rays” was mysterious. In 1883, Heinrich Hertz failed to observe deflection of cathode rays in an electric field, and confusion ensued. But in 1895, Jean Perrin discovered that such rays carried negative charge. Then in 1897, **Joseph John Thomson** measured a definite charge-to-mass ratio of  $176 \text{ C}/\mu\text{g}$  for these rays and thereby identified the first known subatomic particle, the electron [1]. Thomson subsequently was awarded a 1906 Nobel Prize.

### Theory

This experiment is designed to demonstrate the effect of electric and magnetic fields on charged particles, and to permit the measurement of the ratio of the charge of an electron to its mass. In the  $e/m$  tube, electrons are emitted from a directly heated cathode and accelerated by an electric voltage between the cathode and the anode. The  $e/m$  tube is filled with helium under a very low pressure of  $\sim 10^{-2}$  torr (where  $1 \text{ atm} \sim 760 \text{ torr} \sim 10^5 \text{ Pa}$ ). Due to the ionization of the helium by the electrons, the path they follow is visible as a fine blue luminous ring. Two circular Helmholtz coils produce a roughly homogeneous magnetic field. If a charged particle, such as an electron, moves perpendicularly to the magnetic field, a force whose magnitude is constant, but whose direction is always perpendicular to its velocity, forces the particle into a circular orbit.

The speed  $v$  of the electron depends on the electric potential difference (or voltage)  $V$  between the cathode and the anode. Assuming  $v \ll c$ ,

$$\frac{1}{2}mv^2 = eV, \tag{2.1}$$

where  $-e < 0$  is the charge and  $m$  is the mass of the electron. Referring to Fig. 2.2, the

Lorentz force law and Newton's second law give

$$evB = F = m \frac{v^2}{r}, \quad (2.2)$$

where  $r$  is the radius of its circular orbit. Furthermore, the Biot-Savart law implies that the magnetic field midway between the two Helmholtz coils is

$$B = \mu_0 NI \frac{R^2}{(d^2 + R^2)^{3/2}}, \quad (2.3)$$

where the coils are of radius  $R$  and separation  $2d$  and carry  $N = 130$  turns of current  $I$ . Together, these equations can be rearranged into the form

$$\frac{e}{m} = C \frac{2V}{I^2 r^2}, \quad (2.4)$$

where  $C$  is a constant that depends on the geometry  $(R, d, N)$ .

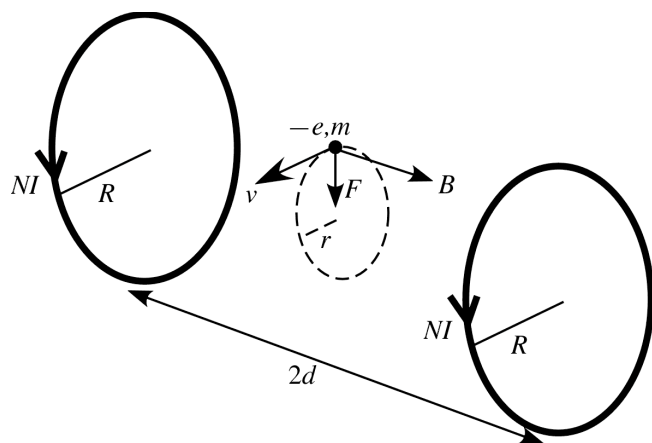


Fig. 2.2. Magnetic field of Helmholtz coils deflects the electron into a circular orbit.

### Procedure

Flip the toggle switch to the  $e/m$  MEASURE position; adjust the coil current knobs to OFF. Connect a 6.3 V AC fixed power supply between the terminals marked 1 and 2 on the base-plate. The cathode should glow a dark red when on. **Be careful not to overheat the cathode.** Connect a 300 V DC variable power supply between terminals marked 3 and 4 to supply an accelerating voltage. Connect a 10V DC power supply in series with a (milli)ammeter to the terminals 7 and 8 on the base-plate. Slowly turn the current adjust knob for the Helmholtz coils clockwise, but **don't let the current exceed 2 A.**

**Wait up to several minutes for the cathode to heat up.** When it does, the electron beam should emerge from the electron gun and be curved by the magnetic field from the Helmholtz coils. Check that the electron beam is parallel to the Helmholtz coils. If it is not, turn the tube until it is, but don't take it out of its socket: As you rotate the tube, the socket will turn.

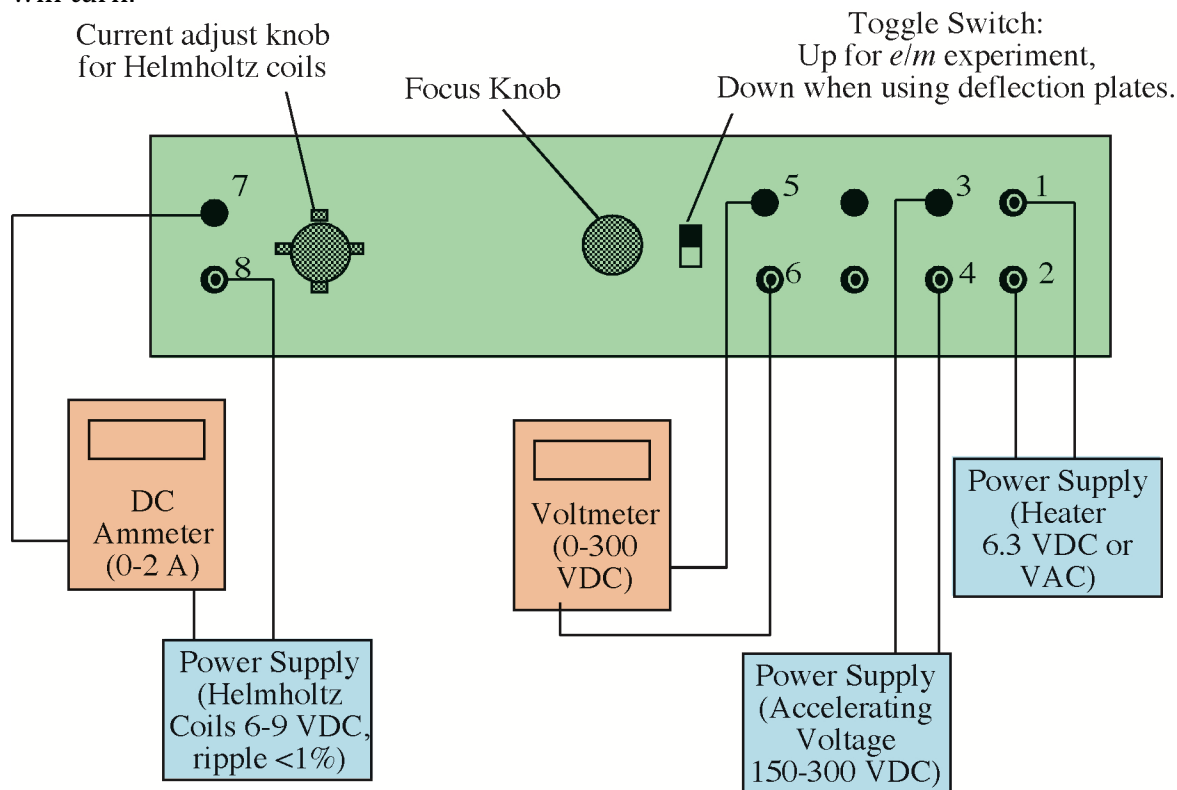


Fig. 2.3. Block diagram of the experimental apparatus.

Carefully read the current to the Helmholtz coils from your ammeter and the accelerating voltage from your voltmeter. Carefully measure the radius of the electron beam. Look through the tube at the electron beam. To avoid parallax errors, move your head to align the electron beam with the reflection of the beam that you can see on the mirrored scale. Measure the radius of the beam as you see it on both sides of the scale, and then average the results. Check by using the virtual millimeter scale projected into the plane of the beam.

### Analysis

Equation 2.4 suggests several different ways to best analyze the data. One strategy is to hold the accelerating voltage  $V$  fixed while varying the current  $I$  and measuring the orbital radius  $r$ . Then a plot of  $2CV/I^2$  versus  $r^2$  should be a straight line through the origin whose slope is the charge-to-mass ratio  $e/m$ . Propagate the uncertainties for each data point and use Igor Pro to do a weighted fit to the data (where the weight of each data point is the inverse of the area of its uncertainty footprint). Another strategy is to hold the current  $I$  fixed while varying the accelerating voltage  $V$  and recording the orbital radius  $r$ . A comparison of these two techniques can help identify systematic errors in the experiment. What is the relative (or percentage) difference between your measurement of  $e/m$  and the accepted value?

## Reference

[1] Joseph John Thomson, "Cathode Rays", *Philosophical Magazine*, **44**, 293 (1897).



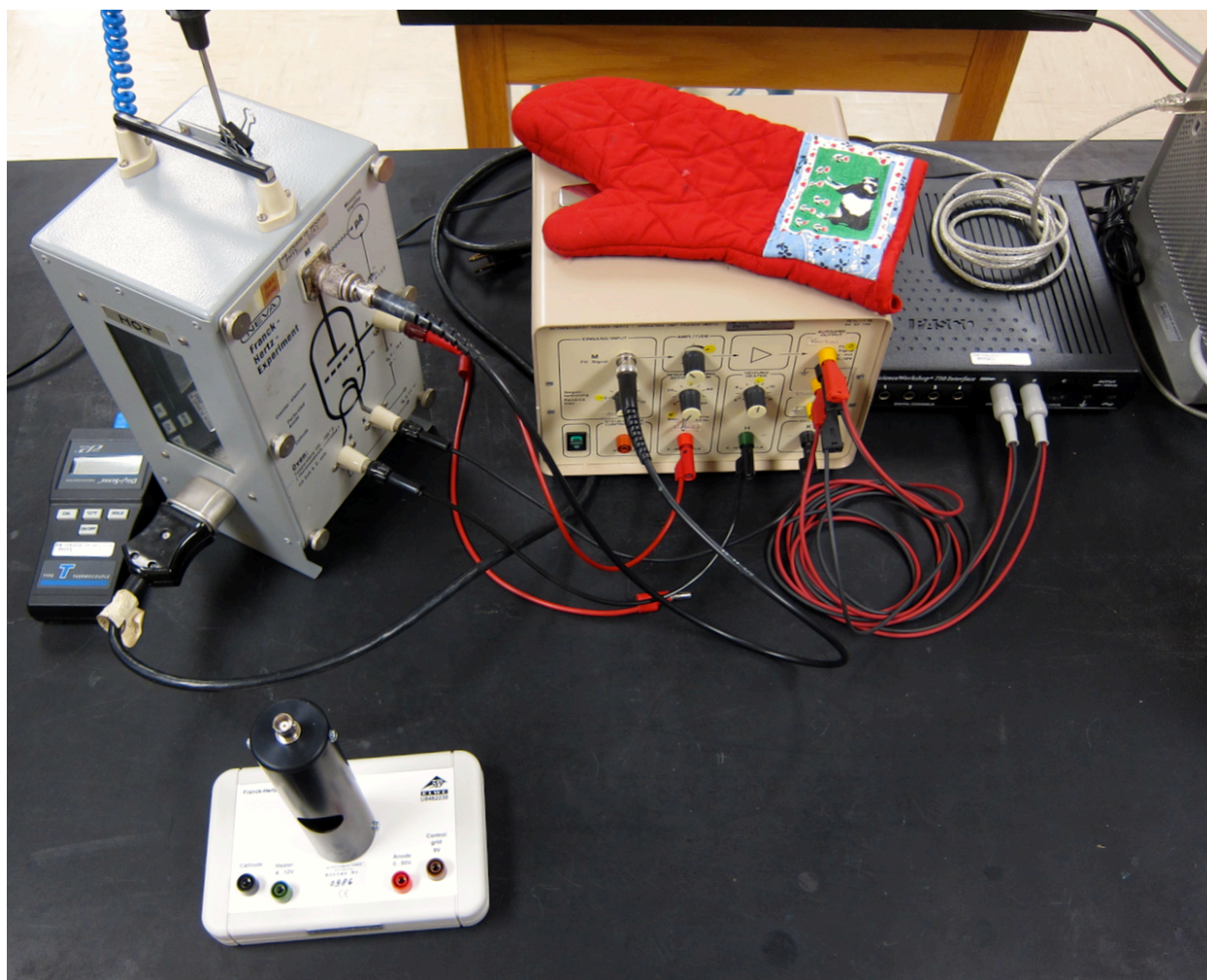


Fig. 3.1. Franck-Hertz apparatus features a controller as well as a tube of Ne and a thermostatically controlled oven surrounding a tube of Hg.



### 3. Franck-Hertz Energy Quantization

*Last updated 2017 December 3*

#### Summary

Atoms exist in discrete states of definite energy. Observe this quantization by measuring currents and voltages using *only* ammeters and voltmeters, as in Fig. 3.1.

#### History

In 1913, based upon insecure and contradictory foundation, Niels Bohr boldly proposed that atomic energy levels were quantized. The Bohr atom was a significant but incomplete step toward quantum mechanics. Soon after, in 1914, **James Franck** and **Gustav Hertz** (the nephew of Heinrich Hertz) directly observed this quantization by studying the absorption of energy by mercury atoms in collisions with electrons [1]. For this, they shared a 1925 Nobel Prize.

#### Theory

The design of the Klinger tube used in this experiment is like that originally used by Franck and Hertz. It provides for rigid mounting and stable positioning of the electrodes, which insures dependable results. To insure a high probability of collision, the distance between the grid (perforated anode) and the plate (counter electrode) is small whereas the distance between the cathode (source of electrons) and grid (perforated anode) is large in comparison to the free path of the electrons. Electrons are emitted thermionically from an indirectly heated cathode. A metal diaphragm connected to the cathode eliminates secondary and reflected electrons. Leakage current along the host glass wall of the tube is minimized by use of a ceramic feed-through on the plate (counter electrode).

The tube is highly evacuated and contains a measured quantity of metallic mercury. When raised to its proper operating temperature, the mercury within the tube is in vapor state, thus providing a suitable atmosphere for the measurement of excitation energies of mercury. To provide that temperature, a 300 W thermostatically controlled oven is used. The oven also contains a schematic diagram of the tube that clearly indicates and provides means for making all necessary electrical connections.

An accelerating voltage is applied between the cathode and grid (perforated anode). A



barrier electric potential is applied between the grid (perforated anode) and plate (counter electrode). As the accelerating voltage increases, more and more electrons are able to surpass the barrier potential and reach the plate (counter electrode). These electrons are recorded as an increasing current. When the accelerating voltage reaches the excitation energy of the mercury atom, an electron colliding with an atom of the mercury vapor will give up a quantum of energy to this atom. These electrons lose velocity to the extent that they cannot surpass the barrier potential energy and are recorded as a decreasing current [2].

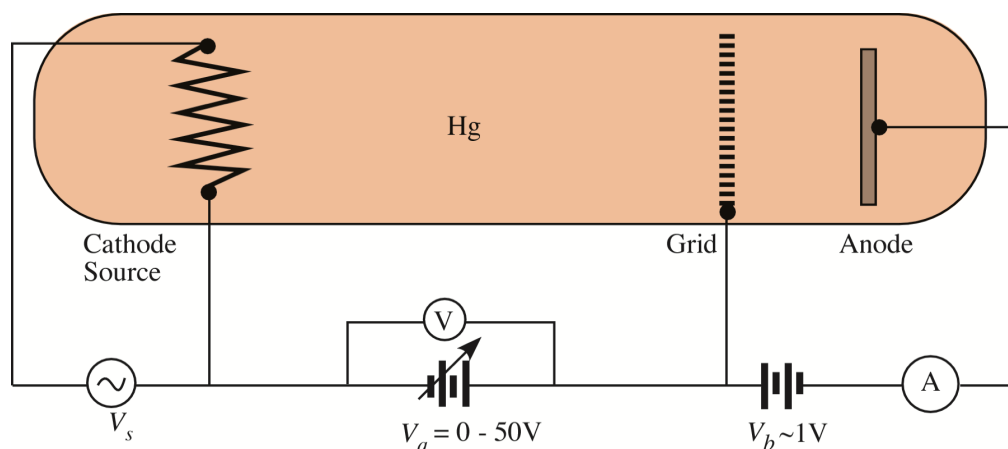


Fig. 3.2. The circuit diagram for the Franck-Hertz experiment shows one cathode and two anodes.

Compared to the distance between cathode and grid (perforated anode) the free path of the electrons is small. Electrons that have collided inelastically are able to regain more energy in the electric field. Some of these electrons are able to reach the plate (counter electrode) and are indicated by an increasing current. Plate current continues to increase as the accelerating voltage increases until the electrons again reach the mercury excitation level and lose their energy a second time.

The tendency of the curve to rise is related to the resistivity of the gas. If we knew the exact conversion factor used by the Klinger operating unit in converting anode current  $I_a$  to the electron voltage  $V_e$ , we could determine the resistivity. Since we do not know the conversion, we can only determine how the slope of the curve changes with temperature.

Energy transfers such as these take place several times as the accelerating voltage increases and are indicated by distinct current maxima and minima. In the voltage range from 0 V to  $\sim 60$  V as many as 13 minima can be observed. Measurement of these minima, which should occur in steps of  $\sim 4.9$  V with increasing accelerating voltage, give a direct measurement of the  $\sim 4.9$  eV excitation energy of mercury. The first peak, however, occurs near 7 V because of the additional energy of 2.1 eV required to remove an electron from the cathode.



## Procedure

### Hg Tube

Mercury's first excited states are at about 4.9 eV above its ground state. Transitions back to the ground state emit UV light.

Insert the thermometer in the top of the cabinet. (It should already be there.) Plug in the oven and adjust the thermostat so that the oven maintains a temperature of  $T \sim 180^\circ\text{C}$ . **Caution: This is hot!** Connect like labeled points on the tube to those on the Klinger operating unit. Turn on the operating unit and set the barrier voltage  $V_b \sim 0.5\text{ V}$  and the electron source voltage  $V_s \sim 10\text{ V}$ . Ramp the accelerating voltage  $V_a$  manually.

Use the Pasco Interface Box to record the electron voltage  $V_e$ , which is proportional to the electron current at the anode  $I_a \propto V_e$ , by connecting a Pasco voltage lead to the corresponding output of the Klinger operating unit. Connect a second Pasco voltage lead to the accelerating voltage  $V_a$  connection on the operating unit. Initially use Pasco Capstone to graph  $I_a$  versus  $V_a$ . Your graph should look something like Fig. 3.3. Later export to Igor Pro for final analysis.

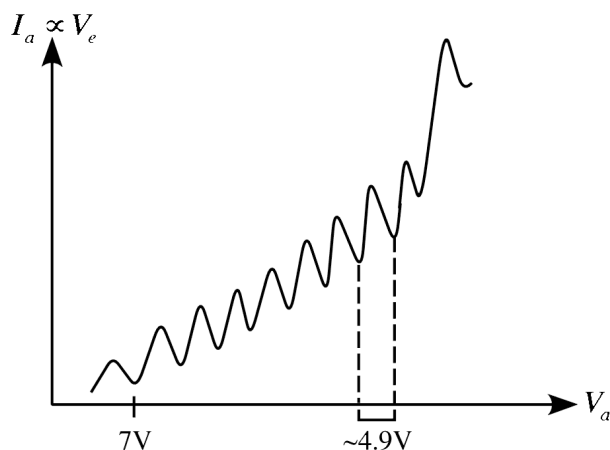


Fig. 3.3. Typical I-V plot for mercury vapor in the Franck-Hertz experiment.

### Ne Tube

Repeat the above procedure for neon, which will require very different settings for the Klinger operating unit. Unlike the mercury-filled Franck-Hertz tube, the neon filled tube operates at room temperature [3].

Neon's first excited states are in two bands at about 16.7 eV and 18.6 eV above its ground state. Transitions between these intermediate energy levels produce yellow-red light. Bands of light develop between the control grid and the accelerating grid where inelastic collisions of electrons and neon atoms occur. The plane-parallel geometry of the grids means that these bands can be viewed through a window. Observe and photograph the bands as you vary the accelerating voltage.

## Analysis

Note that the Klinger apparatus reports voltage in *dekavolts*, where  $1 \text{ daV} = 10 \text{ V}$ . Also, don't confuse electric potential (measured in volts V) and electric potential energy (measured in electron-volts eV or Joules J). What is your best estimate of the minimum excitation of mercury atoms? What is the relative (or percentage) difference between this estimate and the accepted value? Is the separation between the minima constant, or does it increase linearly [3]? Why does the mean slope of the mercury vapor  $I$ - $V$  curves (and hence the conductivity) increase as temperature increases?

Repeat your analysis for the excitation of neon atoms.

## References

[1] James Franck, Gustav Hertz, "Über Zusammenstöße zwischen Elektronen und Molekülen des Quecksilberdampfes und die Ionisierungsspannung desselben [On collisions between electrons and molecules of mercury vapor and the ionization potential of the same]," *Verhandlung Deutsche Physikalische Gesellschaft [Transactions of the German Physical Society]*, **16**, 457–467 (1914).

[3] A. C. Melissinos, J. Napolitano, *Experiments in Modern Physics, Second Edition* (Academic Press, 2003).

[3] Gerald Rapior, Klaus Sengstock, Valery Baev, "New features of the Franck-Hertz experiment", *American Journal of Physics*, **74**, 423-428 (2006).



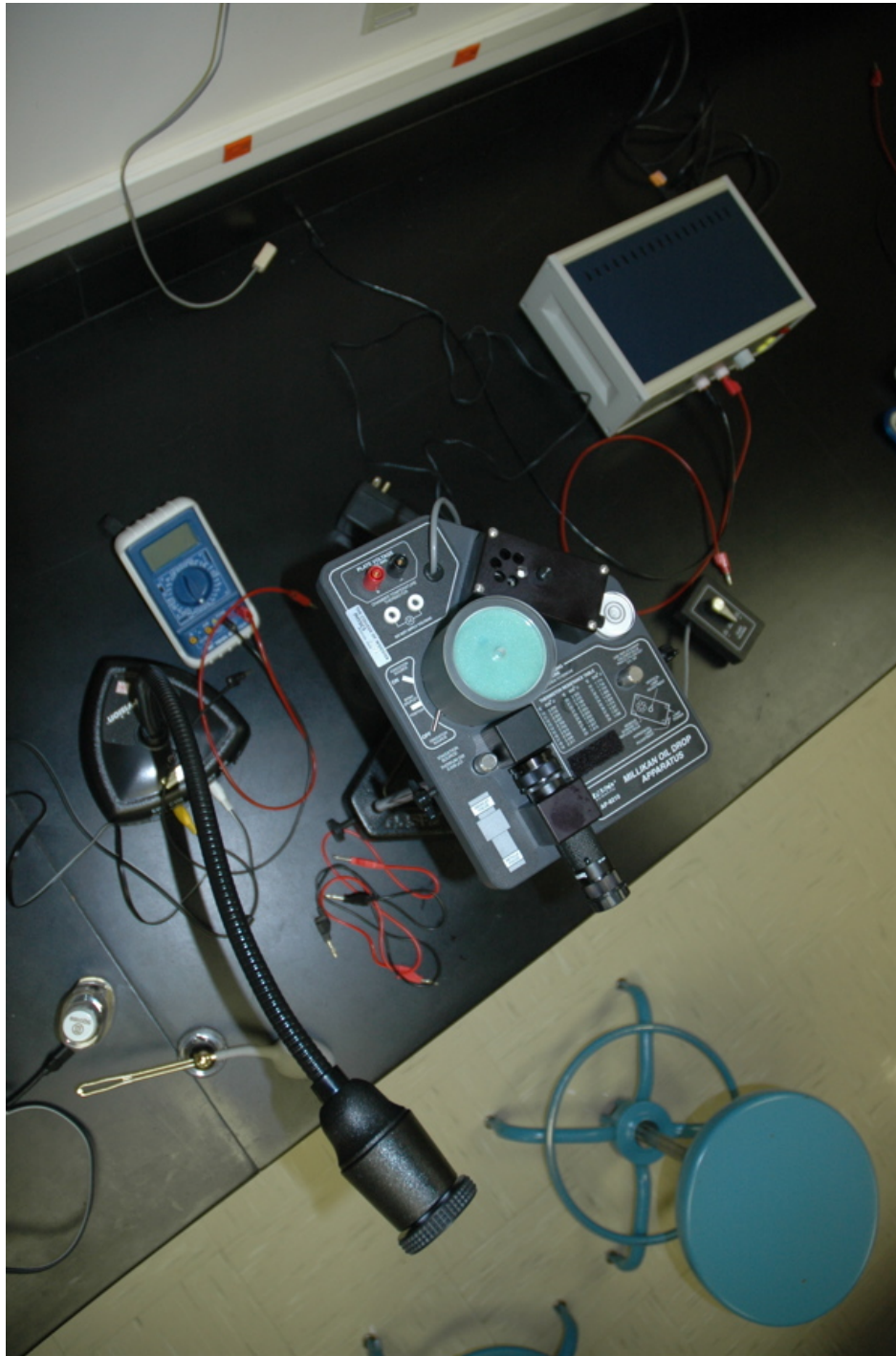


Fig. 4.1. Millikan oil-drop experiment includes a power supply and a video camera.



## 4. Millikan Electron Charge

*Last updated 2017 November 30*

### Summary

Discover that electrical charges always occur in multiples of a fundamental unit  $e$ . Measure the terminal velocities of a charged oil drop both falling and rising in combined gravitational and electrostatic fields, as in Fig. 4.1. The ratio of the rise and fall velocities is linear to the charge on the sphere.

### History

In 1897, Joseph John Thomson first measured the electron charge-to-mass ratio  $e/m$ . Subsequently, Thomson and his students attempted to measure the electron charge  $-e < 0$  by observing the motion of charged clouds. In 1909, **Robert Millikan** and **Harvey Fletcher** obtained a good estimate of the elemental charge  $e = 1.60 \times 10^{-19} \text{ C}$  by balancing the fall of individual charged droplets with an electrostatic field [1, 2]. For this measurement, and for his study of the photoelectric effect, Millikan was awarded a 1923 Nobel Prize.

### Theory

#### Basic Theory

Millikan wanted to determine whether electrical charges occurred in discrete units and, if they did, whether there was such a thing as an elementary charge. Like his original experiment, this apparatus uses oil drops, which move vertically between two metal plates. These drops are too small to be seen with the naked eye, and so a light projector and microscope are used to enable the eye to see the sphere as a small dot of light. A CCD array outputs the image to a computer monitor.

When there is no voltage applied to the plates, the drop falls slowly and steadily under the influence of gravity, quickly reaching its terminal velocity. When a voltage is applied to the plates, not only the force of gravity but also by the electric force affects the terminal velocity of the drop. Knowing the viscosity of the drops in air and the electric field created by the Millikan apparatus, a series of observations will produce a group of terminal velocities, which will be multiples of a lowest value. From this data, one can verify the quantization of charge and determine the elementary unit of charge  $e$  [3].

By convention,  $-e < 0$  is the charge on the electron while  $e > 0$  is the charge on the *positron* (antielectron).

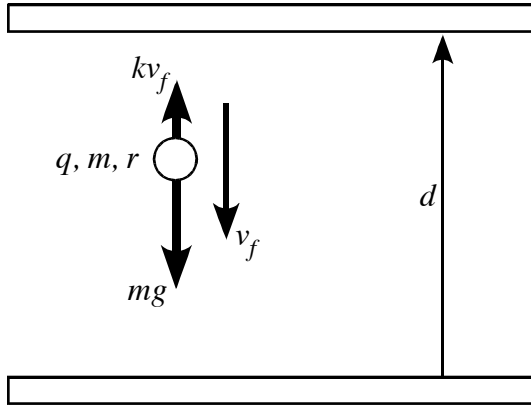


Fig. 4.2. Gravity and viscosity act on a drop as it falls.

Consider an oil drop of mass  $m$  and radius  $r$  subject to gravity and viscosity, as in Fig. 4.2. If the drop falls a distance  $d_f$  in a time  $t_f$ , then its terminal speed is

$$v_f = \frac{d_f}{t_f}. \quad (4.1)$$

Its effective mass, including the buoyancy of air, is

$$m = (\rho_o - \rho_a) \left( \frac{4}{3} \pi r^3 \right), \quad (4.2)$$

where the density of the oil is  $\rho_o \approx 886 \text{ kg/m}^3$  (check!) and the density of air  $\rho_a \approx 1.3 \text{ kg/m}^3$ . By Stokes' law, its drag coefficient is  $k = 6\pi\eta r$ , where the viscosity of air is  $\eta \approx 1.9 \times 10^{-5} \text{ Pa s}$ . Its equation of motion is

$$0 = F = mg - kv_f. \quad (4.3)$$

These equations can be solved to find the drop radius

$$r = \sqrt{\frac{9\eta d_f}{2t_f(\rho_o - \rho_a)g}}. \quad (4.4)$$

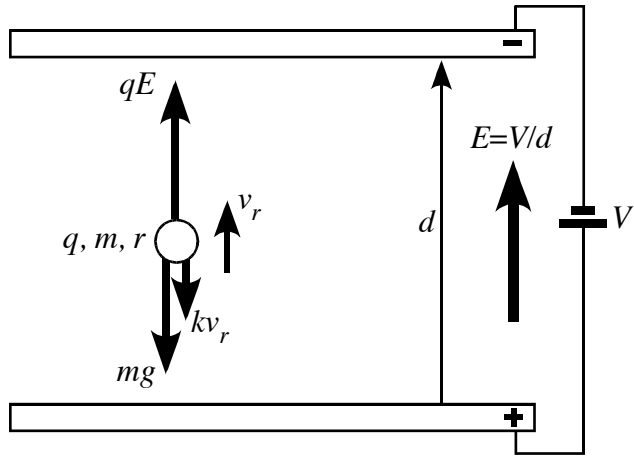


Fig. 4.3. Gravity, electricity, and viscosity act on a  $q > 0$  drop as it rises.

Next, consider an oil drop subject to gravity, viscosity, and electricity, as in Fig. 4.3. If the voltage between the plates is  $V$  and their separation is  $d$ , then the electric field is

$$E = \frac{V}{d}. \quad (4.5)$$

If the drop rises a distance  $d_r$  in a time  $t_r$ , then its terminal speed is

$$v_r = \frac{d_r}{t_r}. \quad (4.6)$$

Its equation of motion is

$$0 = F = mg + kv_r - qE. \quad (4.7)$$

Eliminating  $k$  from Eqs. 4.3 and 4.7 gives the charge

$$q = \frac{mg}{E} \left( 1 + \frac{v_r}{v_f} \right) = C \left( \frac{d_r}{t_r} + \frac{d_f}{t_f} \right) \sqrt{\frac{d_f}{t_f} \frac{d}{V}} \quad (4.8)$$

in terms of the rise and fall distances and times and the electric field, where  $C$  is a constant function of the densities, viscosity, and gravitational field.

### Advanced Theory

More precisely, because the small ( $\sim 2 \mu\text{m}$ ) radii of the drops is of the order of the mean free path of the air molecules, the air acts not like a perfect fluid but as a partial vacuum, and the viscosity in Stokes' law reduces to

$$\tilde{\eta} = \eta \left(1 + \frac{b}{pr}\right)^{-1} < \eta, \quad (4.9)$$

where  $p$  is the air pressure and the constant  $b \approx 8.2 \times 10^{-3}$  Pa m. Because  $r \propto \eta^{1/2}$  by Eq. (4.4), the effective drop radius reduces to

$$\tilde{r} = r \left(1 + \frac{b}{pr}\right)^{-1/2} < r. \quad (4.10)$$

Because  $q \propto m \propto r^3$  by Eq. (4.8), the effective charge reduces to

$$\tilde{q} = q \left(1 + \frac{b}{pr}\right)^{-3/2} < q. \quad (4.11)$$

## Procedure

Although this experiment allows one to measure the total charge on a drop, it is only through an analysis of the data obtained and a certain degree of experimental skill that the charge of a single electron can be determined. **By selecting drops that rise and fall slowly, one can be certain that the drop has a small number of excess electrons.** Several such drops should be observed and their respective charges calculated. If the charges on these drops are integral multiples of a certain smallest charge, then this is a good indication of the atomic nature of electricity. However, since a different drop has been used for measuring each charge, there remains the question as to the effect of the drop itself on the charge. Changing the charge on a single drop while the drop is under observation can eliminate this uncertainty. An ionization source placed near the drop will accomplish this. In fact, it is possible to change the charge on the same drop several times. If the results of measurements on the same drop yield charges that are integral multiples of some smallest charge, then this clinches the atomic nature of electricity.

To set up the experiment, remove the lid from the chamber. Remove the droplet hole cover from the capacitor plate and insert the knob with the focusing wire in the same place. Bring the wire into focus using the two focusing knobs on the telescope. Adjust the camera so that it is looking down the eyepiece as squarely as possible. Connect the camera to the computer, and monitor the camera's view using an appropriate application.

To begin the experiment, reassemble the droplet-viewing chamber by removing the focusing wire and placing the droplet hole cover on the top capacitor plate and then placing the lid on the housing. (The droplet hole cover prevents additional droplets from entering the chamber once the experiment has started.) Measure and record the plate voltage and the thermistor resistance (temperature).



Put non-volatile oil of known density into the atomizer. Prepare the atomizer by rapidly squeezing the bulb until oil is spraying out. Insure that the tip of the atomizer is pointed down ( $90^\circ$  to the shaft). Move the ionization source lever to the Spray Droplet Position to allow air to escape from the chamber during the introduction of droplets into the chamber. Place the nozzle of the atomizer into the hole on the lid of the droplet-viewing chamber. While observing through viewing scope, give the atomizer bulb one quick squeeze. Then squeeze it slowly to force the droplets through the hole in the droplet hole cover, through the droplet entry hole in the top capacitor plate, and into the space between the two capacitor plates. When a shower of drops appears in the viewing scope, move the ionization source lever to the OFF position.

**The exact technique of introducing drops must be developed by the experimenter.**

The object is to get a small number of drops, not a large, bright cloud from which a single drop can be chosen. It is important to remember that the drops are being forced into the viewing area by the pressure of the atomizer. Therefore, excessive use of the atomizer can cause too many drops to be forced into the viewing area and, more important, into the area between the chamber wall and the focal point of the viewing scope. Drops in this area prevent observation of drops at the focal point of the scope. If the entire viewing area becomes filled with drops, so that no one drop can be selected, either wait three or four minutes until the drops settle out of view, or disassemble the droplet viewing chamber (after turning off the DC power supply), thus removing the drops. When the amount of oil on the parts in the droplet-viewing chamber becomes excessive, carefully clean them. Remember: **the less oil that is sprayed into the chamber, the fewer times the chamber must be cleaned.**

From the drops in view, select a droplet that both **falls slowly** (about 0.02–0.05 mm/s) when the plate charging switch is in the “Plates Grounded” position and can be driven up and down by turning on the voltage. If too many droplets are in view, clear many of them by connecting power to the capacitor plates for several seconds. If too few droplets have net charges to permit the selection of an appropriately sized and charged drop, move the ionization lever to the ON position for about five seconds. After finding an appropriately sized and charged oil droplet, fine tune the focus of the viewing scope. The oil droplet is in best focus for accurate data collection when it appears as a pinpoint of bright light.

Because the drops move quickly, it may be helpful to film them through the microscope. To do this, monitor and record the rise and fall of the drops on the computer using iMovie. Later, use iMovie or VideoPoint to determine the terminal velocities. Save the movies in an appropriately labeled folder on the desktop or in the documents folder.

Measure the rise (plates charged) and fall (plates not charged) velocities of the selected droplet about 10–20 times. Maneuver the droplet as needed using the plate voltage switch. Achieve greatest accuracy by timing from the instant the bright point of light passes behind the first major reticule line to the instant the bright point of light passes behind the second major reticule line. (These lines are 0.5 mm apart.) Calculate the charge on the droplet. If the result of this first determination for the charge on the drop is greater than 5 excess electrons, use slower moving droplets in subsequent determinations. Introduce more oil

droplets into the chamber using the procedure previously described and select another droplet. Measure the rise and fall velocities of the selected droplet about 10–20 times or until the charge changes spontaneously or the droplet moves out of view.

Bring the droplet to the top of the field of view and move the ionization lever to the ON position for a few seconds as the droplet falls. If the rising velocity of the droplet changes, make as many measurements of the new rising velocity as possible. If the droplet is still in view, attempt to change the charge on the droplet by introducing more alpha particles, as described previously, and measure the new rising velocity 10–20 times, if possible. Repeat this procedure as many times as feasible. Record the plate potential, the oil density, the viscosity of air at the temperature of the droplet viewing chamber, and the barometric pressure, for each set of velocity measurements.

### Analysis

Analysis is based on Eq. (4.11). Create a table whose entries consist of the average charge for each droplet, over the time where the droplet is expected to have retained a constant charge. One way to analyze this data is to define the following **fitness function**

$$f[q] = \sum_{i=1}^N \left( \frac{\bar{q}_i}{q} - \text{int} \left[ \frac{\bar{q}_i}{q} \right] \right)^2, \quad (4.12)$$

where  $\bar{q}_i$  represents the average charge on the  $i^{\text{th}}$  droplet as discussed above, the notation  $\text{int}[\bar{q}_i/q]$  denotes the result of rounding the ratio  $\bar{q}_i/q$  to the nearest integer, and  $N$  is the number of droplets. To understand this equation, think of  $q$  as your “guess” for the potential value of the fundamental charge. If some average charge  $\bar{q}_i$  is an integer multiple of your guess  $q$ , then the quantity inside the square brackets will be zero for that  $i^{\text{th}}$  term in the sum; otherwise it will be some number between  $-0.5$  and  $0.5$ . The quantity in square brackets is squared to eliminate any negative signs and then is summed over each data point. The better your guess  $q$  fits your data set, then the smaller the sum  $f[q]$  will be; therefore, the values of  $q$  where  $f[q]$  is a minimum are candidates for the fundamental charge. Create a Mathematica program that plots  $f[q]$  versus  $q$  and locate any minima. Did you find more than one candidate for the value of the fundamental charge? Do you recognize limitations of this method?

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- [1] Robert Andrews Millikan, “On the Elementary Electric charge and the Avogadro Constant”, *Phys. Rev.* **2**, 109–143 (1913).
- [2] Harvey Fletcher, “My Work with Millikan on the Oil-Drop Experiment”, *Physics Today*, 43-47 (June 1982).
- [3] A. C. Melissinos, J. Napolitano, *Experiments in Modern Physics, Second Edition* (Academic Press, 2003).

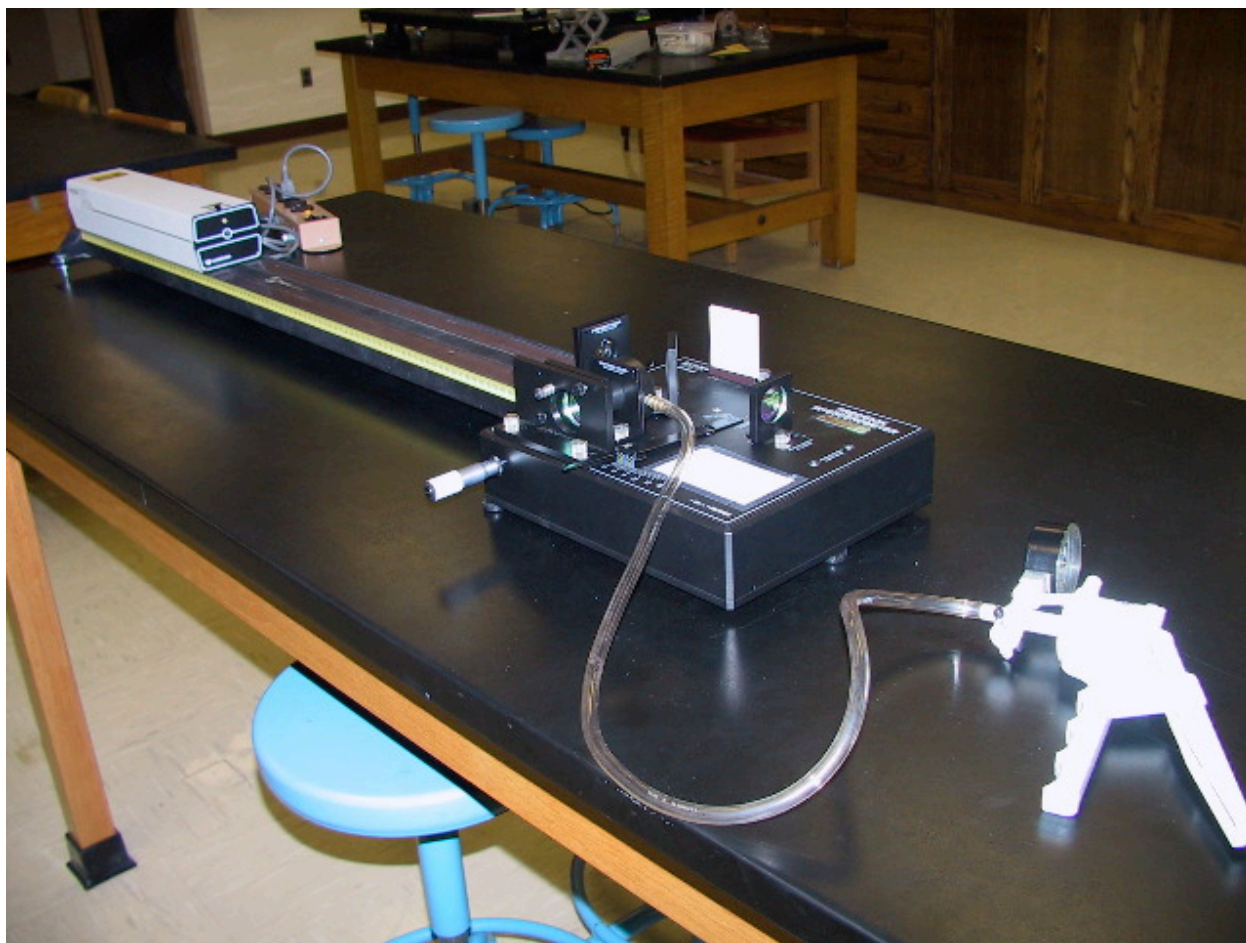
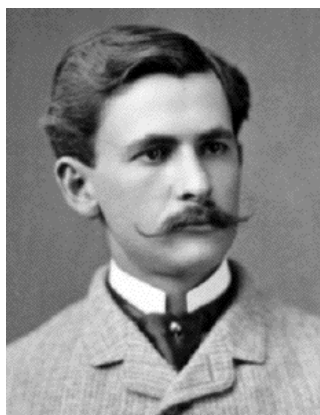


Fig. 5.1. Apparatus includes a Michelson interferometer, a laser, a vacuum cell, and hand-held pump.



## 5. Michelson Interferometry Air Refractive Index

*Last updated 2017 November 9*

### Summary

Use the precision Michelson interferometer of Fig. 5.1 to measure the index of refraction of air, a number that differs from unity by only a few parts in ten thousand!

### History

In 1881, **Albert Abraham Michelson** designed and built an interferometer to test for the existence of the ether, a hypothesized medium in which light propagated. He and Edward Williams Morley used the interferometer to show that light travels at a constant speed in all inertial reference frames, thereby invalidating the ether hypothesis [1]. Throughout his career, Michelson used interferometry to make ever more accurate measurement of the speed of light. Furthermore, his interferometer became widely used for measuring the wavelength of light and for using the wavelength of a known light source to measure extremely small distances. For these contributions, Michelson was awarded a 1907 Nobel Prize.

### Theory

In general, an interferometer can be used in two ways. If the characteristics of the light source are accurately known (wavelength, polarization, intensity), changes in the beam path can be introduced and the effects on the interference pattern can be analyzed. Alternately, by introducing specific changes in the beam path, information can be obtained about the light source that is being used. In this experiment, first use the interferometer to measure the wavelength of the light source and then use it to measure the index of refraction of air.

A Michelson interferometer splits incident monochromatic light into two paths, which are reflected by mirrors to be recombined and projected onto a viewing screen. If the paths differ by an odd number of half wavelengths, the light destructively interferes. If the paths differ by an even number of half wavelengths, the light constructively interferes. If a lens spreads the light rays, an interference pattern of dark and bright rings, or fringes, forms, as in Fig. 5.2.

Slowly displacing one mirror through a distance  $d$  (thereby changing one path length by  $2d$ ) and counting the number  $N$  of *fringe shifts* (inversions of the interference pattern), the wavelength of light  $\lambda$  can be calculated from

$$N\lambda = 2d. \quad (5.1)$$

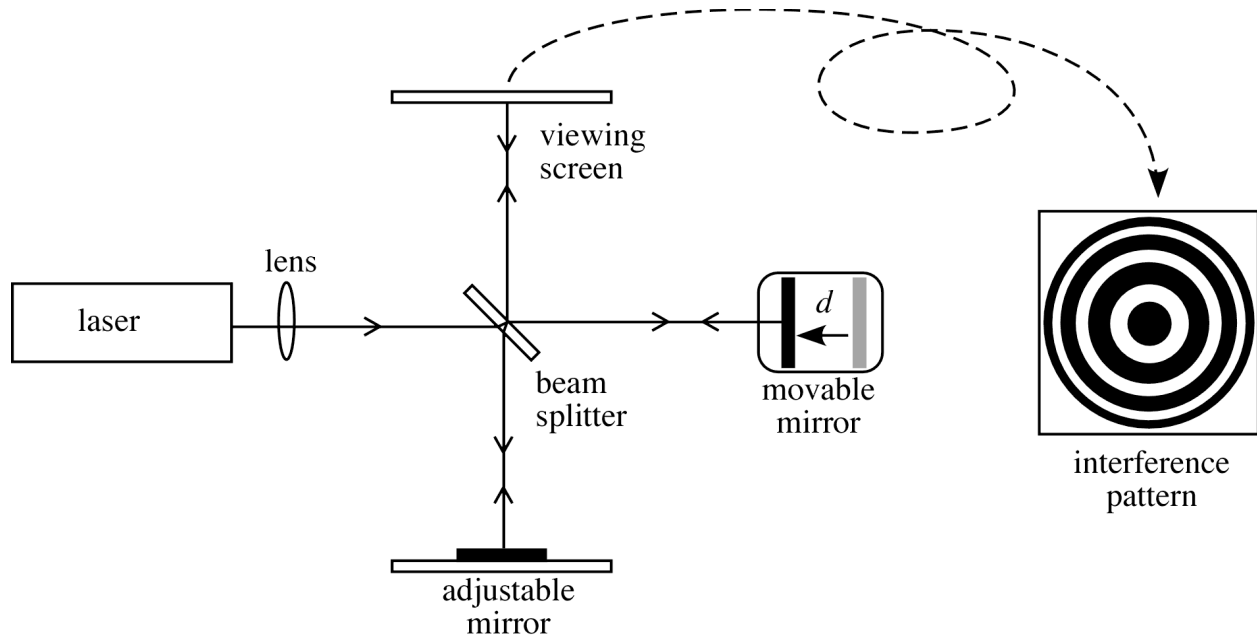


Fig. 5.2. Off axis rays in the interferometer produce a bull's-eye interference pattern.

Alternately, if the wavelength is known, a similar procedure can determine a displacement — or a refractive index. For example, suppose we place a vacuum cell of length  $d$  in one path and pump air out of it. If initially the cell contains  $N_i = 2d/\lambda_i$  wavelengths, then finally it contains  $N_f = 2d/\lambda_f$  wavelengths. If the wavelength of the light source in a vacuum is  $\lambda_0$ , then the *effective* wavelength of the light in air of refractive index  $n$  is  $\lambda = \lambda_0/n$ . Thus, as we pump air out, we should observe

$$N_i - N_f = \frac{2d}{\lambda_0}(n_i - n_f) \quad (5.2)$$

fringe shifts. Extrapolating to zero pressure,  $n_f = 1$  and

$$n - 1 = \frac{\lambda_0}{2d} \Delta N, \quad (5.3)$$

where  $\Delta N = N_i - N_f$ .

## Procedure

### Light Wavelength

Carefully align the laser and the interferometer in the Michelson mode, so an interference pattern is clearly visible on the viewing screen. The laser beam should go through the center of every optical element. Without the lens, light reflected to the laser should be a single spot coinciding with the laser's aperture. With the lens, a beautiful bull's-eye interference pattern, like that in Fig. 5.2, should appear.

Adjust the micrometer knob to a medium reading, approximately 50  $\mu\text{m}$ . (Each mark on the knob is 1  $\mu\text{m}$ . Each full revolution of the knob is 25  $\mu\text{m}$ , which corresponds with one notch on the axle. The  $\mu\text{m}$  knob adjusts the movable mirror.) In this position, the relationship between the micrometer reading and the mirror movement is most nearly linear. Turn the micrometer knob one full turn counterclockwise. Continue turning counterclockwise until the zero on the knob is aligned with the index mark. Record the micrometer reading, where each division represents 1  $\mu\text{m}$ .

When reversing the direction of turning the micrometer knob, there is a small amount of give before the mirror begins to move. This is called **mechanical backlash**, and is present in all mechanical systems involving reversals in direction of movement. To eliminate backlash errors, begin with a full counterclockwise turn, and then turn only counterclockwise when counting fringes.

One way to count fringe shifts is to keep an eye on the bull's-eye—note that as the interference pattern shifts through a full cycle, the bull's-eye changes from bright to dark and then bright again. An alternative way to count fringes is to adjust the position of the viewing screen so that one of the marks on the millimeter scale is aligned with one of the fringes in the interference pattern; one may then count the fringes that “shift” past this reference mark. When using this method, it is easier to count the fringes if the reference mark is one or two fringes out from the center of the pattern. Rotate the micrometer knob slowly counterclockwise. Count the number of fringe shifts using one of the above methods. Continue until some predetermined number of (at least 20) fringes has passed the mark. At the end of the count, the fringes should be in the same position with respect to your reference mark as they were when at the start of the count (or equivalently, the center of the bull's-eye should be dark at the beginning of the count, and dark once again at the end of the count). Record the final reading of the micrometer dial.

Record the distance  $d$  that the movable mirror moved toward the beam-splitter according to the readings of the micrometer knob. Remember, each small division on the micrometer knob corresponds to 1  $\mu\text{m} = 10^{-6}$  m of mirror movement. Also record the number  $N$  of fringe transitions. Repeat this procedure at several different distances, recording the results each time.

### Air Refractive Index

If necessary, realign the laser and interferometer in the Michelson mode. Place the component holder between the adjustable mirror and the beam-splitter. Attach the vacuum

cell to its magnetic backing and push the air hose of the vacuum pump over the air outlet hole of the cell. Adjust the alignment of the fixed mirror as needed so the center of the interference pattern is clearly visible on the viewing screen. (The fringe pattern will be somewhat distorted by irregularities in the glass end-plates of the vacuum cell. This is not a problem.)

For accurate measurements, the end plates of the vacuum cell must be perpendicular to the laser beam. Rotate the cell and observe the fringes. (How can observations assure that the vacuum cell is properly aligned?) Verify that the air in the vacuum cell is at atmospheric pressure. On the hand-held vacuum pump, this is accomplished by flipping the vacuum release toggle switch.

Record the initial reading of the vacuum pump gauge pressure  $P_i$ . Slowly pump out the air in the vacuum cell while counting the number  $N$  of fringe transitions. When done, record the final reading on the vacuum gauge pressure  $P_f$ . (Alternately, begin with the vacuum cell evacuated, then count fringes as air slowly pumps in.) Note that **most vacuum gauges measure pressure relative to atmospheric pressure**, and so the observed pressure  $P_o$  is the difference between the atmospheric pressure  $P_a$  and the gauge pressure  $P_g$ , namely  $P_o = P_a - P_g$ .

### Analysis

By Eq. (5.1), find the wavelength of the laser light source by plotting twice the mirror displacements  $2d$  versus the corresponding number of fringe shifts  $N$ . The result should be a straight line through the origin whose slope is the wavelength  $\lambda$ . What is the relative (or percentage) difference between the measurement and the accepted value for the laser?

The hand pump can't completely evacuate the cell. However, a plot of the number of fringe shifts versus the actual (not gauge) pressure can be extrapolated to zero pressure to find  $\Delta N$  in Eq. (5.3). Since  $\lambda_0 \approx \lambda$  and  $d \approx 3$  cm (check!), this determines  $n - 1$  (and  $n$ ). What is the relative (or percentage) difference between the measurement of  $n - 1$  and the accepted value? Note that the real challenge here is not finding  $n$  itself, which is very nearly unity, but finding the deviation of  $n$  from unity.

### Reference

[1] Albert Abraham Michelson, Edward Williams Morley, "On the Relative Motion of the Earth and the Luminiferous Ether," *American Journal of Science*, **34**, 333-345 (1887).





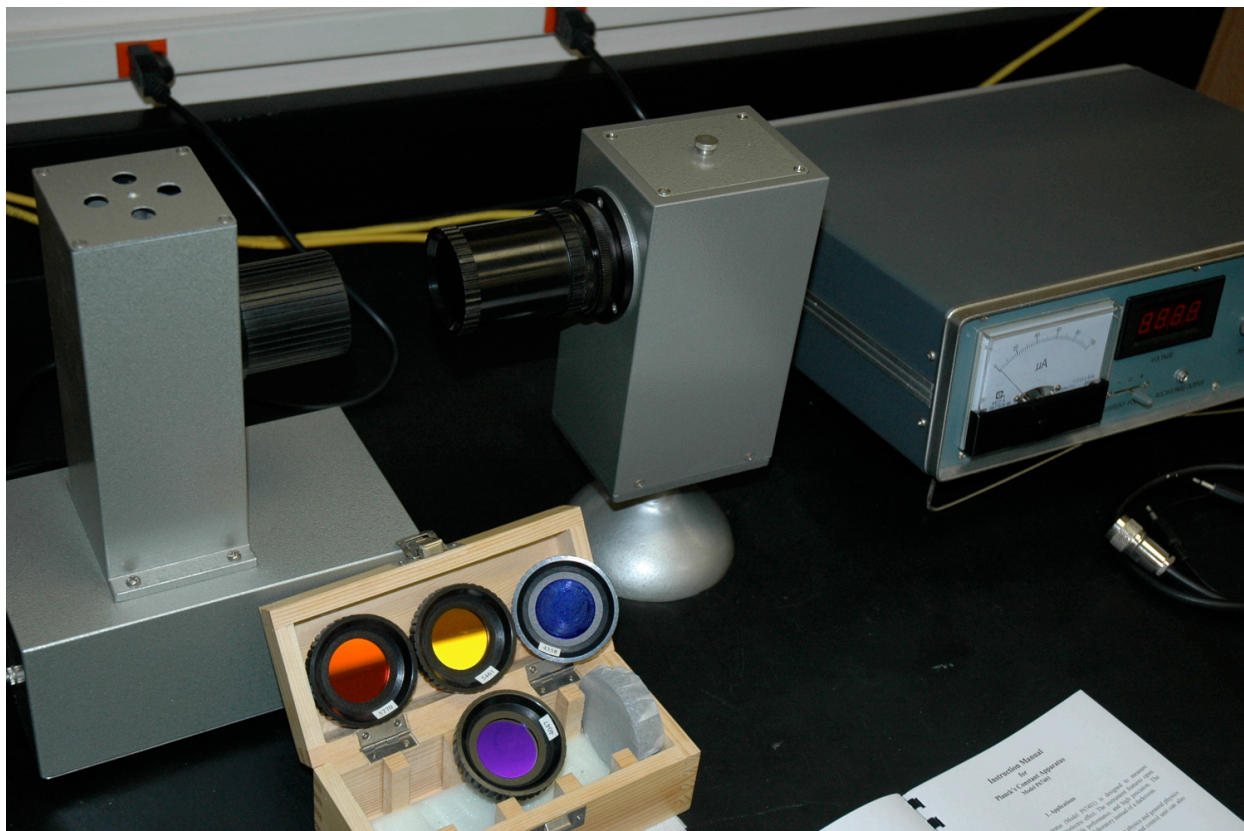
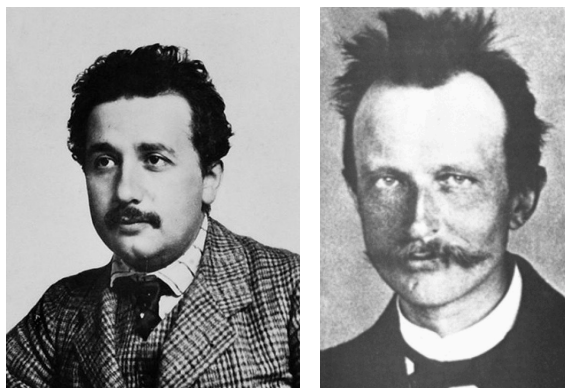


Fig. 6.1. The photoelectric apparatus includes a mercury light source, phototube with assorted colored filters and a sensitive ammeter.



## 6. Einstein Photoelectric Effect & Planck's Constant

*Last updated 2017 November 12*

### Summary

Shine light on metals and electrons may pop out. Understand the details by modeling light as particles called photons with energy  $E$  proportional to frequency  $\nu$ . Using the Fig. 6.1 apparatus, experimentally determine the proportionality constant, Planck's quantum of action  $h$ .

### History

In 1887, Heinrich Hertz discovered the photoelectric effect by noticing that ultraviolet radiation lowers the voltage for which two nearby electrodes will spark – ironically in a series of experiments confirming Maxwell's theory of electromagnetic *waves*. In 1902, Philipp Lenard extended Hertz's experimental work, showing that while the number of emitted electrons is proportional to the intensity of the incident light, the kinetic energy of the electrons is proportional to the *frequency* of the light. In 1905, **Albert Einstein** gave a simple interpretation of Lenard's results [1] by assuming light of frequency  $\nu$ , often denoted by the Greek letter nu, comes in quanta of energy  $h\nu$ . Previously introduced by **Max Planck** in his study of thermal or black-body radiation [2], the quantum of action  $h = 0.66 \text{ zJ/THz}$ , or about a zeptojoule per terahertz, is the rate at which a photon's energy increases with frequency. For this work and “for his services to Theoretical Physics”, Einstein received a 1921 Nobel Prize (while Lenard received a 1905 Nobel Prize for his work with cathode rays).

### Theory

When a photon of energy  $h\nu$  strikes a metal, the most energetic electrons emitted are those very close to the surface, and they will leave the cathode with maximum kinetic energy  $K_m$  such that

$$h\nu = K_m + W_e, \quad (6.1)$$

where  $W_e$  is the work required to remove the electron from the *emitter* (that is, the metal). To measure  $K_m$ , we bring a second metal surface nearby at an electric potential that is

lower than that of the emitter, as in Fig. 6.2, so that the (positive) potential difference  $V$  between this collector (at voltage  $V_c$ ) and the emitter (at voltage  $V_e$ ) may be expressed as  $V = V_e - V_c$ . Since the collector potential is lower than that of the emitter, the electrons are going “uphill” so they lose kinetic energy.

If we find the potential difference  $V$  between the collector and the emitter that **just barely stops** the photocurrent at the collector and call it  $V_s$ , then

$$K_{max} = eV_s + W_c - W_e, \quad (6.2)$$

where  $W_c$  is the work required to remove the electron from the *collector*, and  $e$  is the absolute value of the charge on the electron. Substituting Eq. (6.2) in Eq. (6.1), we get

$$h\nu = eV_s + W_c. \quad (6.3)$$

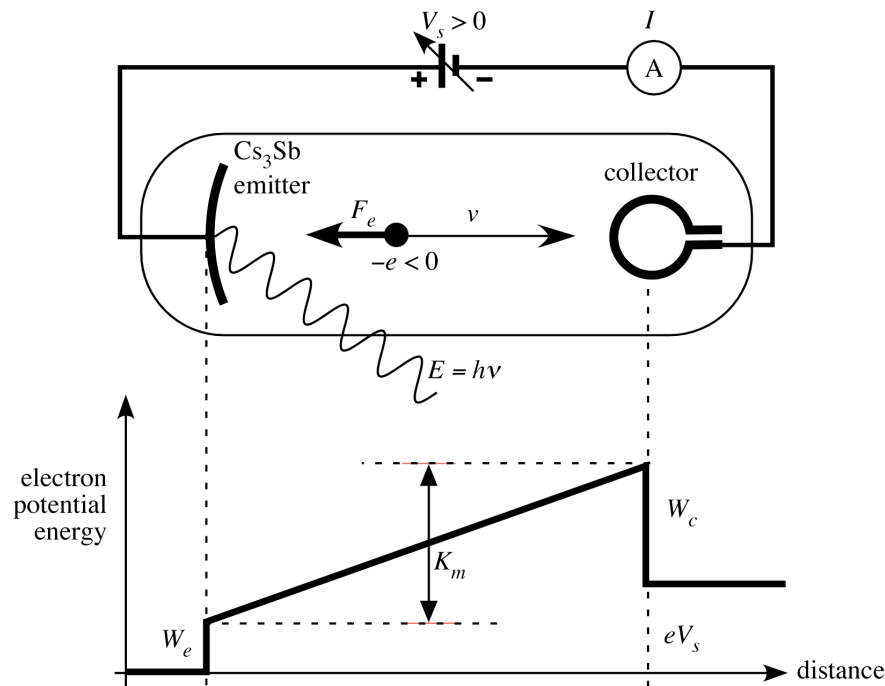


Fig. 6.2. Apparatus (top) and electron potential energy (bottom). Just below the stopping voltage  $V_s$ , electrons just make it from emitter to collector [3].

## Procedure

### Setup

Turn on the Hg lamp and let it warm up about 20 minutes. Turn on the Control Unit. Turn the range selector dial to “SHORT” and check that the analog meter reads 0. Turn the range selector dial to “FULL SCALE” and check that the meter reads 100.0.

To be sure that the current readings are due only to light hitting the cathode of the phototube, first measure the current when no light is present. With the objective cover on

the photocell unit and the range selector dial set to  $10^{-11}$  A, adjust the voltage slowly from  $-2$  to  $+2$  V. If the current reads more than  $10^{-11}$  A at any point, then record the current and voltage values.

### Photoelectric Data

Remove both the objective cover and the lid on the photocell unit to see inside. Adjust the Hg lamp and the photocell so that the light from the lamp is focused on the photocell. The distance between the lamp and photocell should be 30 to 50 cm. Replace the lid.

Place a colored filter on the input to the photocell unit. The colored filters (labeled with the wavelength in Ångstroms) allow only light of the labeled wavelength to pass through.

To acquire data, adjust the voltage and record the corresponding photocurrent. In this apparatus, negative voltages slow the electrons down; positive voltages accelerate the electrons from the cathode to the anode. This means that the *observed* apparatus voltage is opposite in sign to the potential difference defined above, so that  $-V = V_c - V_e$ .

For each wavelength  $\lambda$ , measure photocurrent  $I$  as a function of applied voltage  $V$  from saturation to beyond cutoff. At the smaller currents and longer wavelengths make sure that no extraneous light enters the phototube. Unfortunately, because of the photoelectron velocity distribution, the current does not fall abruptly to zero [4]. For more accurate data, record several measurements for the current as the voltage is both increased and decreased.

### Intensity

As the collector voltage  $V_c$  is increased relative to the emitter voltage  $V_e$ , it approaches a certain value relative to the emitter such that all electrons freed from the metal cathode should reach the anode and be measured as photocurrent. Increasing the collector voltage beyond this saturation point will not increase the current further [5].

The total number of electrons freed depends on the intensity of the light rather than its wavelength. For a given wavelength, try increasing and decreasing the light intensity by changing the distance between the lamp and detector. What happens to the saturation current?

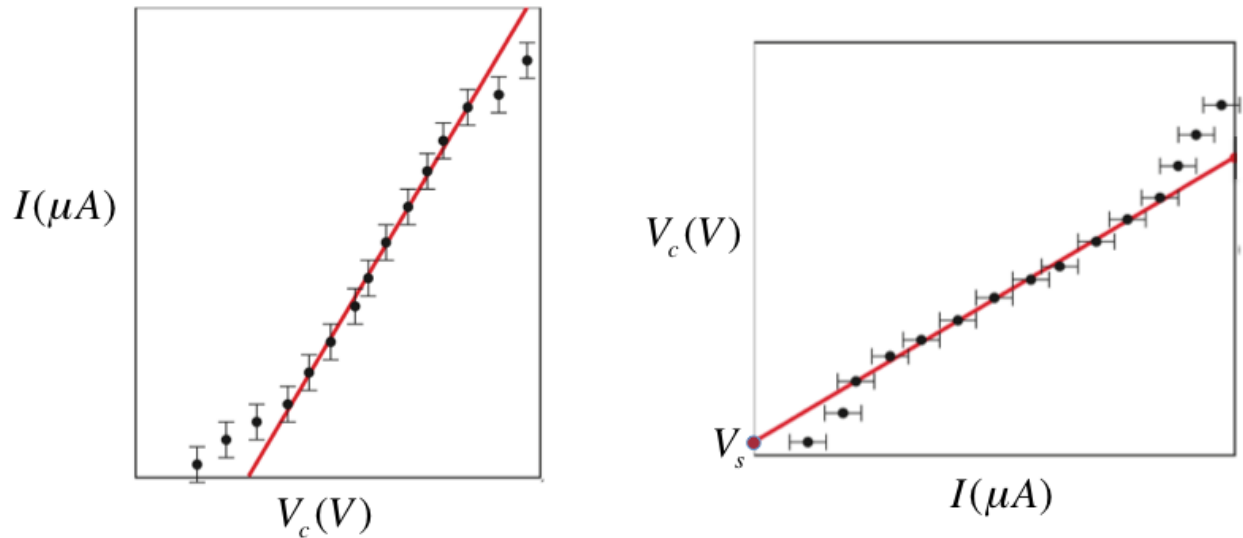


Fig. 6.3. Conceptually the data are current versus voltage (left), but practically reversing the axes (right) means a linear fit in Igor Pro can extrapolate the stopping voltage  $V_s$  and estimate its uncertainty.

### Analysis

For each wavelength  $\lambda$ , plot voltage  $V$  versus current  $I$ , as shown in Fig. 6.3, and estimate the (positive) stopping voltage  $V_s$  and its uncertainty. **You may need to exclude very low current data from the linear fits due to detector limitations.** Next, plot the stopping energy  $eV_s$  versus the light frequency  $\nu = c/\lambda$ , including error bars to indicate the uncertainty footprints. According to Eq. (6.3), the result should be a straight line whose slope is Planck's constant  $h$ . How good is a proportional fit, and what is the uncertainty in the measurement of  $h$ ? What is the relative (or percentage) difference between the measurement of  $h$  and the accepted value? What can be inferred from the intercept?

### References

- [1] Albert Einstein, "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt [On a Heuristic Viewpoint Concerning the Production and Transformation of Light]", *Annalen der Physik* **17** (6), 132–148 (1905).
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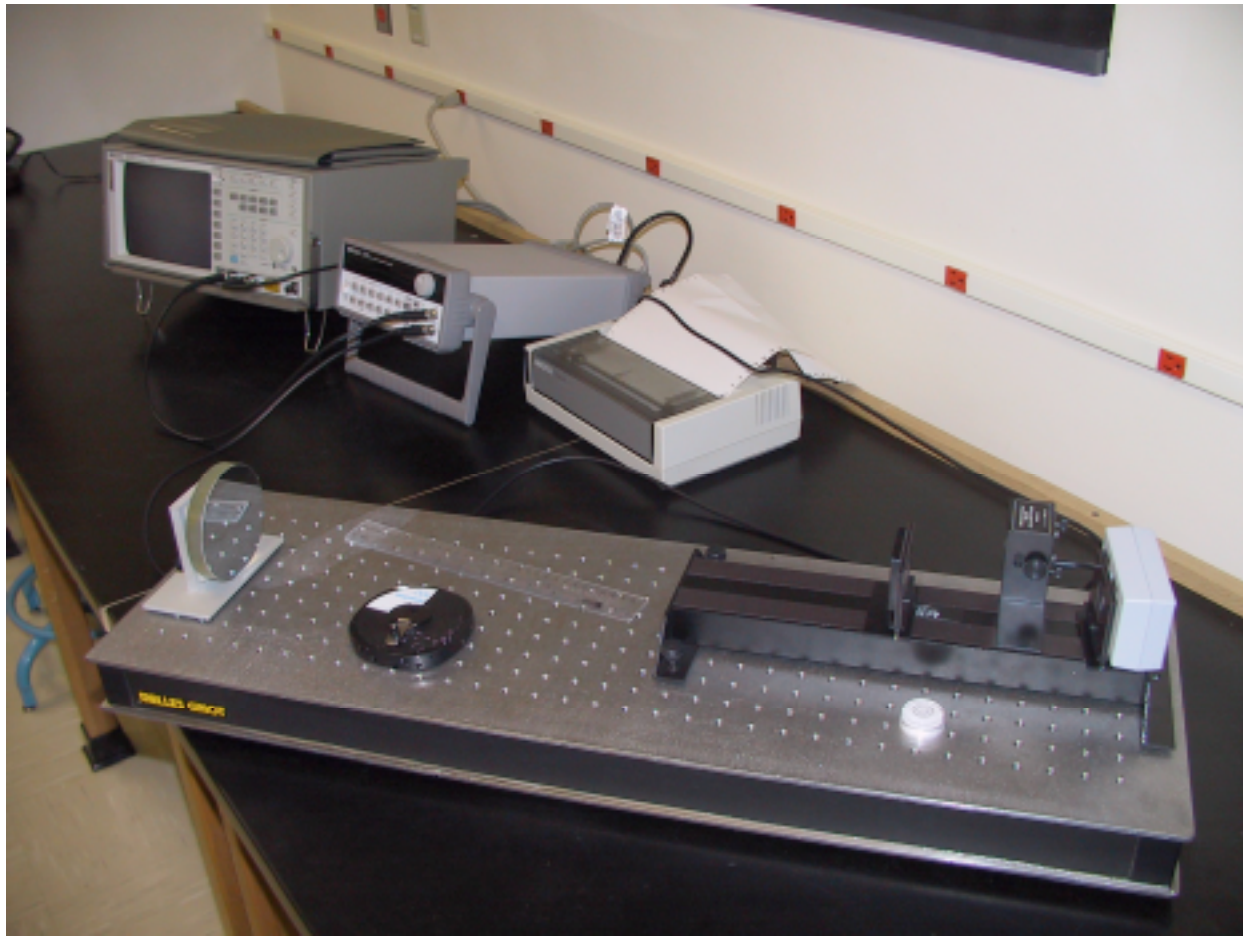
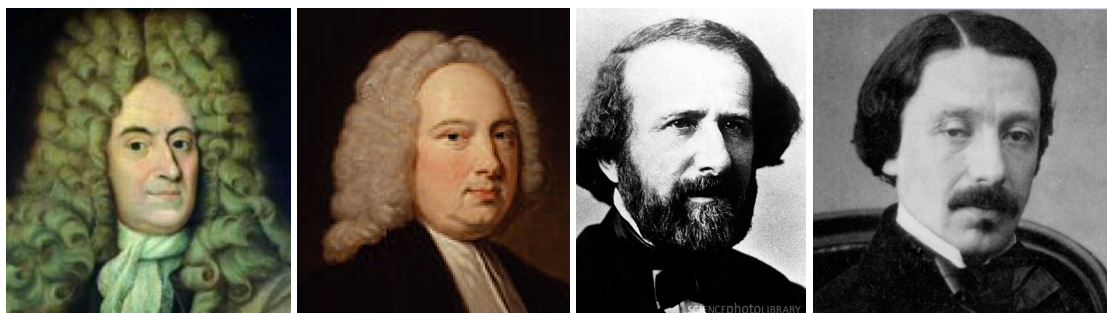


Fig. 7.1. Light speed apparatus includes diode laser, waveform generator, oscilloscope, and concave mirror.



## 7. The Speed of Light

*Last updated 2017 October 29*

### Summary

Measure the “speed limit of the universe”:  $c \sim$  billion k.p.h.  $\sim$  foot per nanosecond. Infer  $c$  by timing the emission, reflection, and return of pulsed laser light over increasingly large distances, using the apparatus of Fig. 7.1.

### History

In 1676, **Ole Rømer** first inferred the speed of light  $c \sim 214\,000\,000$  m/s from observation of Jupiter’s moons [1]. In 1728, **James Bradley** improved this estimate in discovering stellar aberration [2]. In 1849, in the first terrestrial measurement, **Armand Fizeau** measured  $c \sim 315\,300\,000$  m/s using a rotating toothed wheel [3]. Shortly thereafter, in the first laboratory measurement, **Jean Foucault** employed a rotating octagonal mirror [4]. Albert Michelson, in a lifetime (1852-1931) of increasingly ambitious experiments, significantly improved our knowledge of  $c$ . By the 1970s, experiments involving standing electromagnetic waves in cavities had determined  $c = \lambda\nu$  so accurately that, in 1983,  $c$  was *defined* to be exactly 299 792 458 m/s, which is about  $c = 0.3$  m/ns.

### Theory

In this experiment, measure the speed of light using a diode laser modulated on and off at a very high frequency and an oscilloscope. The total distance a laser pulses travels is

$$d' = 2d + \delta, \quad (7.1)$$

where  $d$  is the measured distance between the laser and the mirror and the constant  $\delta$  is the (unmeasured) distance between the laser and the photodiode.

The measured total time elapsed between the emission of a laser pulse and its reception at the photodiode is

$$t' = t + \frac{\delta}{c} + \tau, \quad (7.2)$$

Where  $t$  is the (unmeasured) time for the laser pulse to return to the laser and  $\tau$  is the



(unmeasured) time delay due to the electronics. Hence

$$\begin{aligned}
 2d &= ct \\
 &= ct' - \delta - c\tau \\
 &= ct' - \Delta,
 \end{aligned}
 \tag{7.3}$$

where the constant  $\Delta = \delta + c\tau$ .

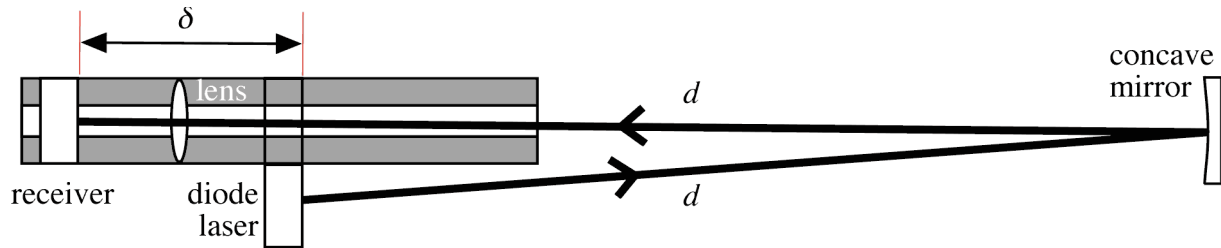


Fig. 7.2. Pulsed diode laser reflects off a concave mirror into the receiver.

### Procedure

Place the Pasco diode laser on one end of the laser bench facing outward. Connect the 9 V DC input of the diode laser to the output of the HP waveform generator. Set the waveform generator to a square wave with a frequency of 3.00 MHz, an amplitude of 5.00 V peak-to-peak, and an offset of 2.50 V. Connect the synchronized output of the waveform generator to channel 1 of the oscilloscope. Attach the Metrologic receiver to a magnetic support and place it on the opposite end of the laser bench with the photodiode facing inward. Because the video output is used to receive the incoming signal, verify that the photodiode directly beneath the video output is not covered by the magnetic support. Connect the video output of the receiver to channel 2 of the oscilloscope. Switch on the laser and receiver.

Place the concave mirror in the laser path and adjust it so that it reflects the laser back into the photodiode. The 127-mm convex lens can be placed on a magnetic support in the middle of the bench to aid in focusing the laser onto the photodiode. The laser path should go through the center of all optical components.

Adjust the oscilloscope so that it auto triggers on the edge of channel 1. Set the time base of the display to 50 ns/div, and display the signals from both channels. Set the vertical division to  $\sim 1$  V/div. At this point, the input square waveform should appear clearly on the oscilloscope. If the return waveform does not also appear, adjust the lens or the mirror to better reflect the laser on the photodiode. Adjust to maximize the amplitude of the return waveform.

What is important is how the time difference between the input and return waveforms *changes* as distance changes, and not the difference themselves. Consequently, it doesn't matter from which features of the waveforms the time difference is measured, as long as this is done consistently. For simplicity, use the times when each waveform crosses the horizontal axis. If the oscilloscope is properly configured to trigger off the square waveform,

this crossing point will always occur at 0 ns. Thus, only measure the time when the return waveform crosses the axis. In practice, the uncertainty in this measurement of time outweighs the uncertainties in the measurements of the distance to the mirror, and thus the smallest possible time division should be used when measuring the horizontal crossing of the return waveform. A convenient setting might be 20 ns/div.

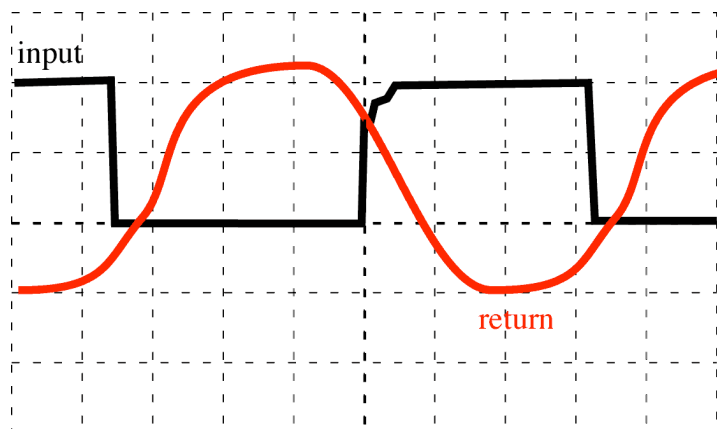


Fig. 7.3. Return waveform is not square because the diode laser does not respond instantaneously.

### Analysis

According to Eq. (7.3), a plot of the distances  $2d$  versus the times  $t'$  should be a straight line (*not* through the origin) whose slope is the speed of light  $c$ . How good is a linear fit, and what is the uncertainty in the measurement of  $c$ ? What is the relative (or percentage) difference between the measurement of  $c$  and the accepted value?

Since  $c$  is *defined* to be 299 792 458 m/s, what actually does this experiment measure?

### References

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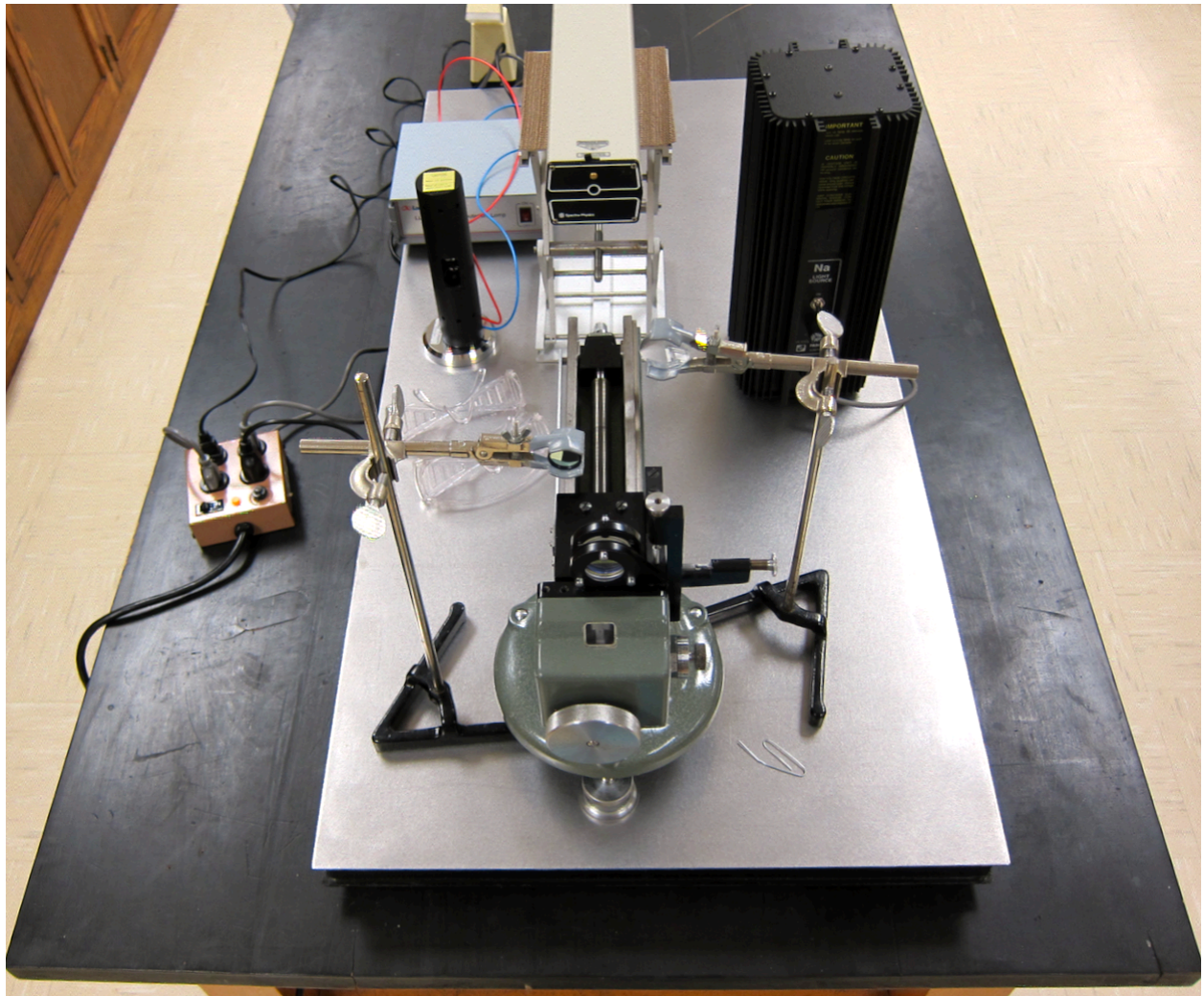


Fig. 8.1 Apparatus includes an H-D discharge tube, a practice Na-doublet discharge tube, a Fabry-Perot interferometer, and safety glasses.



## 8. Bohr Hydrogen-Deuterium Spectrum and Mass ratio

*Last updated 2012 November 10*

### Summary

Rise to the challenge of nineteenth century spectroscopy! Use the Fabry-Perot interferometer of Fig. 8.1 to measure the spectra of hydrogen and deuterium and thus infer the ratio of their masses.

### History

Nineteenth-century spectroscopy provided our first glimpse of the atomic structure of matter and our first contact with the quantum world. In 1913, **Niels Bohr** provided a revolutionary atomic model that successfully explained the spectra of one-electron atoms [1,2]. The “Bohr atom” was an important step toward a complete quantum theory of matter. For his contributions to the development of quantum theory, Bohr was awarded a 1922 Nobel Prize.

### Theory

The Bohr model of the hydrogen atom predicts energy levels

$$E_n = \frac{E_1}{n^2}, \quad (8.1)$$

where the ground state energy is

$$E_1 = -\frac{1}{2}m'c^2\alpha < 0, \quad (8.2)$$

and the inverse reduced mass of the electron and nucleus is

$$\frac{1}{m'} = \frac{1}{m_e} + \frac{1}{m_N}, \quad (8.3)$$

and the fine structure constant

$$\alpha = k_e \frac{e^2}{\hbar c} \approx \frac{1}{137}. \quad (8.4)$$

Energy transitions

$$\frac{hc}{\lambda} = h\nu = \hbar\omega = E_1 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right). \quad (8.5)$$

emit light of wavelength  $\lambda$ . Divide versions of Eq. (8.5) for Hydrogen and Deuterium to find

$$\frac{\lambda_H}{\lambda_D} = \frac{m_D}{m_H} \left( \frac{m_H + m_e}{m_D + m_e} \right) \quad (8.6)$$

or

$$\frac{m_H}{m_D} = 1 - \frac{\lambda_H - \lambda_D}{\lambda_H} \left( 1 + \frac{m_H}{m_e} \right). \quad (8.7)$$

Thus, we can determine the ratio of the mass of hydrogen to the mass of deuterium if we know the difference in spectral wavelengths  $\Delta\lambda = \lambda_H - \lambda_D$  for the two components and the wavelength of the spectral line  $\lambda_H$  in hydrogen [3]. Since the mass difference is very small, the wavelengths will be very close. How do we measure the difference in wavelength between two sources that may only differ by angstroms? Recall thin film effects from introductory physics—interference effects were great for determining film thickness on the order of angstroms! In a similar spirit, we can simulate the “film” with two separable mirrors and with a device known as a Fabry-Perot interferometer [4,5].

The Fabry-Perot interferometer is conceptually simple. Two partially reflecting (coefficient of reflection = 0.9) mirrors are mounted a distance  $d$  apart as shown below. Light passes through one mirror and is reflected multiple times at each mirror. Each time the beam strikes the mirror, some portion is reflected, but the rest transmitted. The various transmitted beams interfere with each other to produce a series of interference fringes. For example, in Fig. 8.2, beam B has traveled  $2d$  farther than beam A; the beams will constructively interfere if an integer number of wavelengths fits in the extra traverse. Varying the distance between the mirrors will change the position of the interference fringes.

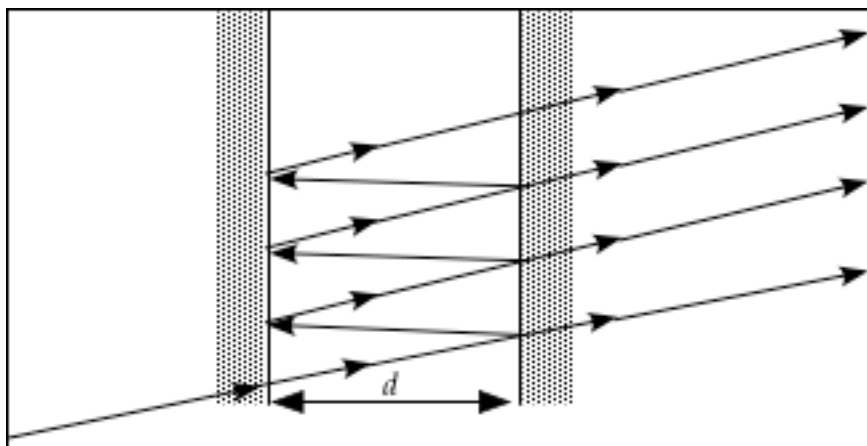


Fig. 8.2. Fabry-Perot mirrors and the pattern of light reflection. The set of mirrors is also called an etalon.

If two wavelengths are present, there will be two series of fringes superimposed, and a convex lens can focus them into overlapping bullseye patterns. If we scan along the radius of one of these circular double-fringe patterns, we will see a variation in intensity with position as shown in Fig. 8.3.

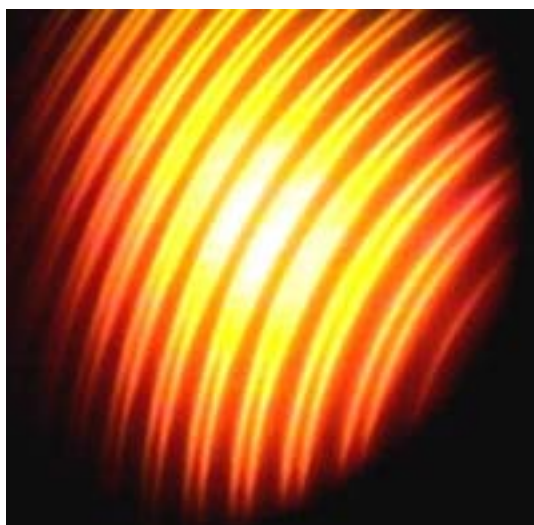


Fig. 8.3. Superimposed fringes for the bright Na doublet somewhere between coincidence and anti-coincidence [6].

Varying the spacing between the mirrors will cause one set of fringes to move relative to the other producing intensity patterns. Eventually, the fringes from one wavelength will exactly overlap the fringes of the second wavelength. This condition is called “coincidence”. Likewise, as the spacing continues to increase, the fringes of one wavelength will fall exactly between two fringes of the other wavelength, a condition known as “anti-coincidence”.



For normal incidence, constructive interference occurs when

$$2d = n\lambda \quad (8.8)$$

where  $d$  is the distance between the mirror and the integer  $n$  is the order of the maximum. If two wavelengths  $\lambda_1 > \lambda_2$  are present, successive coincidences occur when the shorter wavelength gets one order ahead of the longer wavelength as the mirror distance changes by

$$2\Delta d = \Delta n\lambda_1 = (\Delta n + 1)\lambda_2. \quad (8.9)$$

Hence

$$\Delta n = \frac{\lambda_2}{\lambda_1 - \lambda_2}, \quad (8.10)$$

and so

$$2\Delta d = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} \approx \frac{\lambda^2}{\Delta\lambda}, \quad (8.11)$$

if the wavelengths are close so that  $\lambda_1 \approx \lambda_2 \equiv \lambda$ . More generally, the mirror distance

$$d = n \frac{\lambda^2}{2\Delta\lambda} \equiv n s, \quad (8.12)$$

where the integer  $n$  is the order of the coincidence. Hence, a graph of mirror spacing  $d$  versus order of coincidence (or anti-coincidence)  $n$  will yield a line with slope  $s$ , which can be inverted to find the splitting  $\Delta\lambda$  once the slope  $s$  is known. By Eq. (8.7), the Hydrogen to Deuterium mass ratio

$$\frac{m_H}{m_D} = 1 - \frac{\Delta\lambda}{\lambda_H} \left(1 + \frac{m_H}{m_e}\right) = 1 - \frac{\lambda_H}{2s} \left(1 + \frac{m_H}{m_e}\right). \quad (8.13)$$

## Procedure

Because the H-D light spectrum contains ultraviolet light, **wear protective eyewear during the experiment.**

Configure the apparatus as shown in Fig. 8.4. Verify that the dial on the coarse adjustment knob is aligned with the millimeter mark on the main drive scale. Set the fine-tuning dial to zero (it will not be used during the experiment). Use the coarse adjust to bring the mirrors

about 2-3 mm apart.

Use a HeNe laser to align the mirrors. Use a paper clip to adjust the capstan screws on the two mirrors. Project the image on the wall. An image of several dots should appear; successively adjust the capstan screws on the mirrors so that the line of dots collapses to form a single dot. The image can be improved by placing a diverging lens in the beam. At this point, a faint bulls-eye interference pattern should be visible on the wall. View this laser pattern only on the wall! **Do NOT look through the etalon mirrors at the laser!**

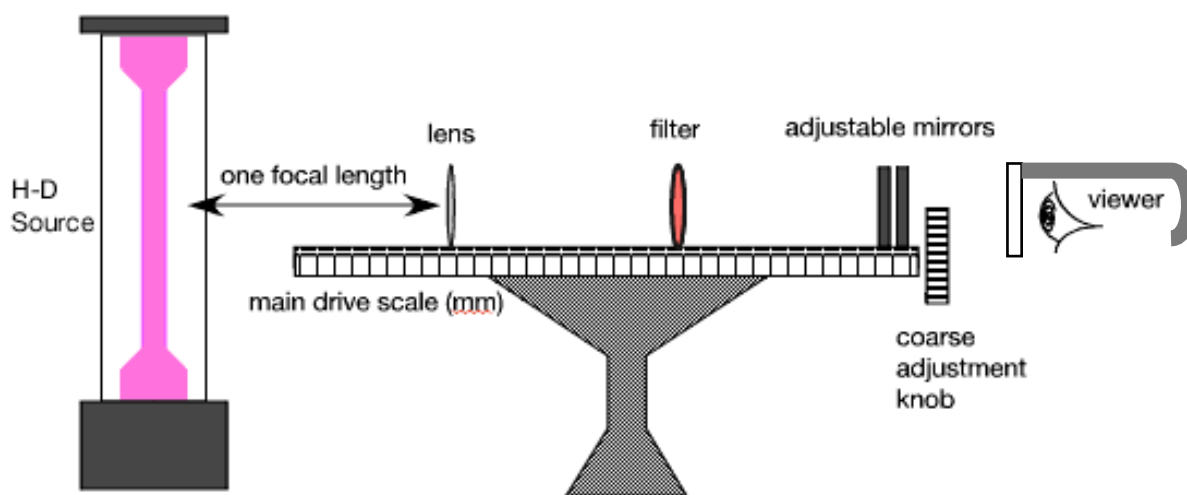


Fig. 8.4. Configuration of the H-D source and Fabry-Perot etalon.

Replace the laser with the H-D light source. Place the lens one focal length from the source and the filter near the mirrors as shown in the diagram.

Turn on the light source (this may take a few seconds) and view the image through the mirrors as in the diagram. If an interference pattern is not visible, added adjustment to the mirrors is needed.

When an interference pattern is apparent, it should not appear to contract or expand as the viewpoint changes up-down or left-right. If this occurs, use the fine adjust knobs until the effect disappears and a double set of concentric fringes appears. Turn the coarse adjustment knob back and forth until they overlap, then split, and then overlap again. This causes the fringes to pass through coincidence and anti-coincidence.

Begin taking data with the mirrors as close as possible and use the coarse-tuning knob to increase the spacing until the fringes pass through coincidence or anti-coincidence. While taking data, always end by turning the coarse-tuning knob in the same direction. Otherwise, the mechanical backlash in the knob will cause the measurements to be inconsistent. Record the mirror spacing from the micrometer value at coincidence or anti-coincidence.



Continue moving the mirrors and recording values. The side knob to the scale measures millimeters and the front measures fractions of millimeters.

**Tip:** Measuring the H-D splitting is hard. You may want to practice by first measuring the splitting of the bright Na doublet. If so, replace the H-D source with a Na source and remove the filter.

### Analysis

Graph the coincidence and anti-coincidence on the same graph, assigning integers to coincidence and half-integers to anti-coincidence. What is the wavelength difference between light emitted by deuterium and by hydrogen and its uncertainty? What is the ratio of the mass of hydrogen to the mass of deuterium and its uncertainty? Compare these results to their accepted values.

### References

- [1] Niels Bohr, "On the Constitution of Atoms and Molecules, Part I", *Philosophical Magazine*, **26**, 1-24 (1913).
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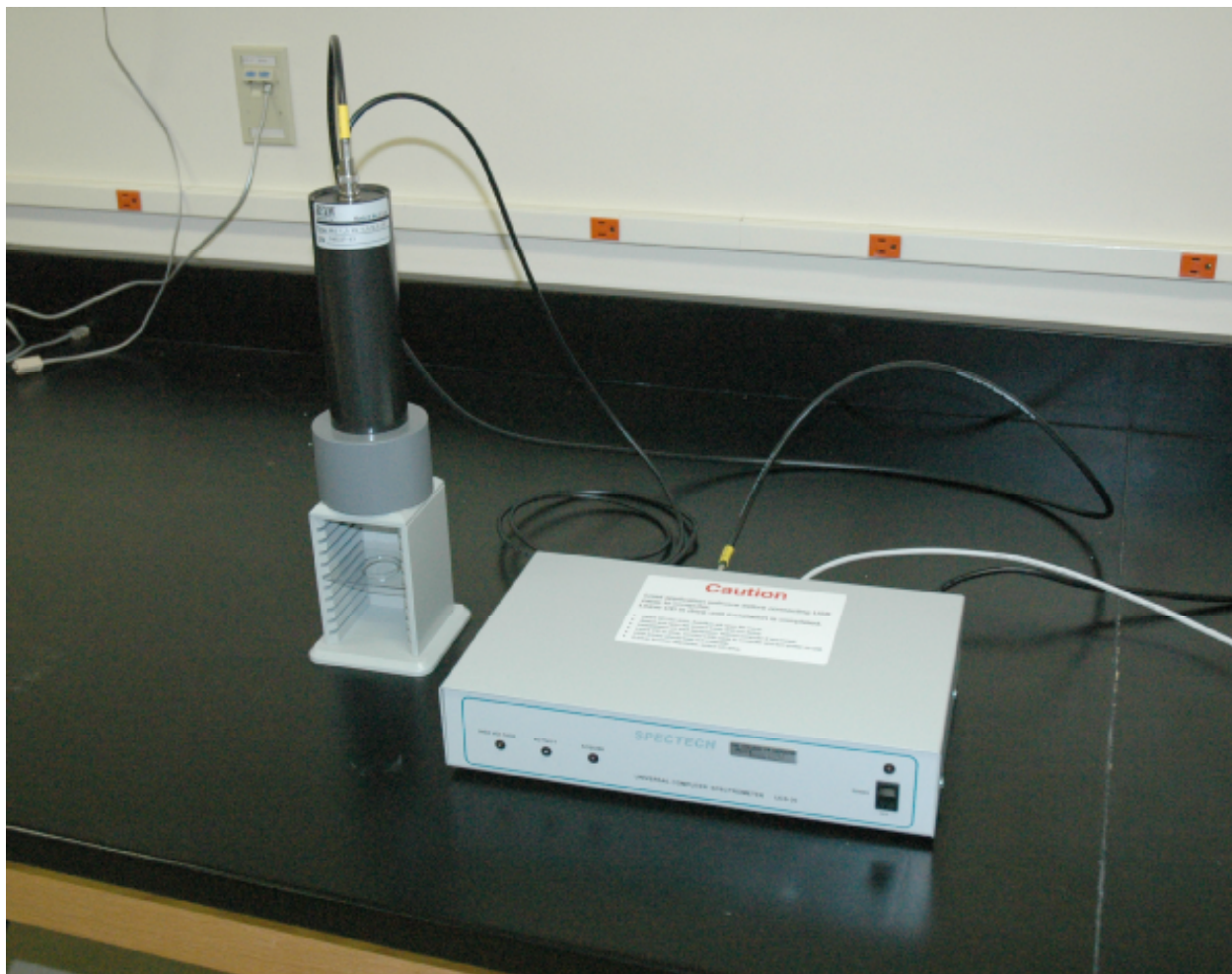
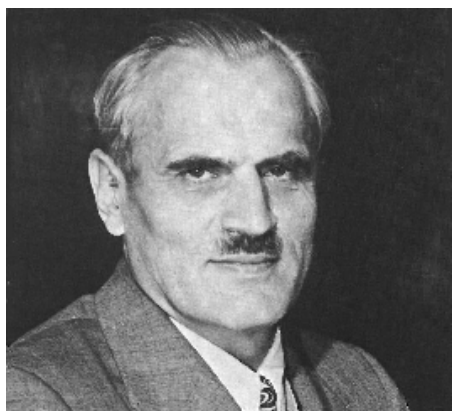


Fig. 9.1 Compton scattering apparatus includes a NaI scintillator and a pulse height analyzer interfaced to a computer.



## 9. Compton Scattering

*Last updated 2017 October 17*

### Summary

Scatter  $\gamma$ -ray photons from effectively free electrons with the Fig. 9.1 apparatus to demonstrate the particle nature of light — and follow in the footsteps of Wooster's own Arthur Compton '13.

### History

Newton developed a particle model of light in the 1600s. This was countered by Huygens wave model of light. Young's double slit interference experiments of 1800 demonstrated the wave nature of light. However, Einstein's photoelectric effect of 1905, suggested a particle model for light at low intensities. **Arthur Compton's** 1920s experiments scattering light from electrons as if it were particles clinched the idea of wave-particle duality, including the realization that both the particle and wave models were incomplete and inadequate descriptions of light [1]. For this work, Compton was awarded a 1927 Nobel Prize.

### Theory

A photon has energy  $E = \hbar\omega$  and momentum  $\vec{p} = \hbar\vec{k}$ , where the temporal and spatial frequencies of the corresponding classical light are related by  $\omega = kc$ . It scatters off an electron, as in Fig. 9.2.

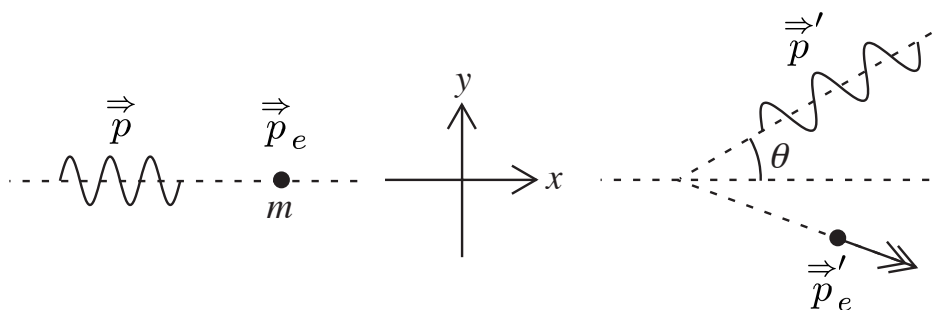


Fig. 9.2. Before (left) and after (right) the scattering of a photon and an electron.

Before the scattering, the photon has 4-momentum

$$\vec{p} = \begin{pmatrix} E/c \\ p \\ 0 \\ 0 \end{pmatrix} = \frac{h}{\lambda} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad (9.1)$$

and the stationary electron of mass  $m$  has 4-momentum

$$\vec{p}_e = \begin{pmatrix} mc \\ 0 \\ 0 \\ 0 \end{pmatrix} = mc \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (9.2)$$

After scattering, the photon has 4-momentum

$$\vec{p}' = \begin{pmatrix} E'/c \\ p' \cos \theta \\ p' \sin \theta \\ 0 \end{pmatrix} = \frac{h}{\lambda'} \begin{pmatrix} 1 \\ \cos \theta \\ \sin \theta \\ 0 \end{pmatrix}, \quad (9.3)$$

and the recoiling and undetected electron has 4-momentum  $\vec{p}'_e$ .

If the spacetime scalar product  $\vec{u} \circ \vec{v} = u_t v_t - u_x v_x - u_y v_y - u_z v_z$ , then the square of the 4-momenta  $\vec{p}'_e \circ \vec{p}'_e = \vec{p}_e \circ \vec{p}_e = (mc)^2$ . Relativistically conserve both energy and momentum by conserving 4-momentum

$$\vec{p} + \vec{p}_e = \vec{p}' + \vec{p}'_e. \quad (9.4)$$

Solve for  $\vec{p}'_e$ , square, and deduce

$$\frac{\Delta \lambda}{\lambda_c} = 1 - \cos \theta, \quad (9.5)$$

where  $\Delta \lambda = \lambda' - \lambda$  is the change in wavelength of the light,  $\theta$  is the scattering angle, and

$$\lambda_c = \frac{h}{mc} \sim \frac{1}{40} \text{\AA} \quad (9.6)$$

is the Compton wavelength of the electron [2, 3]. Alternately, rearrange to find the change in the reciprocal energies

$$\frac{1}{E'} - \frac{1}{E} = \frac{1 - \cos \theta}{mc^2}, \quad (9.7)$$

where the electron  $mc^2 = 511 \text{ keV}$ .

### Procedure

Measure the energies of photons from the radioactive decay of Cs-137 that scatter off effectively free electrons in an Al target, at angles of approximately  $180^\circ$  and  $90^\circ$ , and compare to the theoretical energies from the Compton equation. Cs-137 has a half-life of 30 years and undergoes  $\beta$  decay producing excited Ba-137\*, an electron, and an electron anti-neutrino. Ba-137\* is an unstable nucleus that decays by emitting photons of energy 0.662 MeV about 85% of the time, and it is these  $\gamma$ -ray photons that strike the Al target. (15% of the time the  $\gamma$ -ray doesn't leave the atom but ejects an inner electron, whose replacement by an outer electron results in an x-ray instead.)

Determine the energy of the deflected photons by using a photomultiplier tube (PMT) with a lead Pb collar and a sodium iodide NaI crystal scintillator connected to a pulse height analyzer (PHA), the UCS-20 Computer Spectrometer, which is controlled by *USX USB* software running under macOS. If the software doesn't recognize the hardware, disconnect and reconnect the USB cable or cycle the software and hardware on and off.

Within the software, you may use the automatic calibration function with the Cs-137 located within a few cm of the detector to properly calibrate the horizontal axis displayed on the computer screen.

When the  $\gamma$  rays hit the electrons in the Al target, they are scattered at different angles, in accordance with the Compton scattering formula. The source constantly emits radiation at all angles, which therefore hits the Al target at multiple angles. It is difficult to only view scattering from a specific angle with this experimental design, because photons scattering off the Al target can enter the PMT at angles with a range of values. (Can this problem be overcome?) For this reason, all measured Compton scattering angles should be considered approximate. The range of angles that can be detected is a function of the width of the Al target, the distance of the target from the PMT, the diameter of the PMT, and the height at which the incident  $\gamma$  rays hit the Al target. This requires a complicated 3-dimensional equation for the uncertainty of the angle.

During the experiment, the incident gamma rays should strike the middle of the Al target. Align the Al target's center with the center of the PMT's circular detector at a distance less than 3 cm. By centering the Al target, the radiation source, and the PMT as much as possible, the desired angle becomes the highest probable angle of detection and the peak pulse heights correspond to the energies of the centered angle.

Collect the intensity of radiation from the Cs-137 source with and without the Al target in

place at an angle of approximately  $180^\circ$  (you can—and should—also try other angles such as  $90^\circ$ ). It is important to keep the PHA preset time the same so the number of counts can be compared for equal times. It may be necessary to “integrate” (accumulate data) for many hours or days. Since the source is always radiating energy, the two energy spectra for each angle look identical except for the region representing the energy change of the scattered photons. By looking at the difference between the energy intensities with and without the Al target present, the scattered photon energy can be observed. This value can then be compared to the theoretical value for that scattering angle.

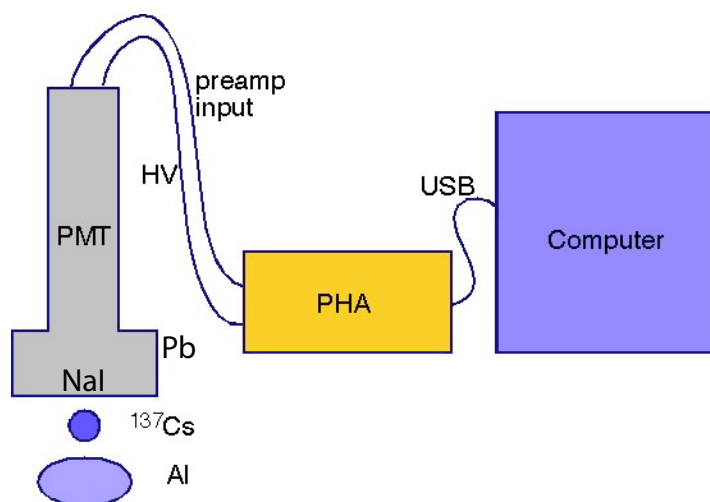


Fig. 9.2. Arrangement for observing  $180^\circ$  backscattering from the Al target into the NaI detector.

### Manual Calibration

The automatic calibration feature replaces the manual calibration required of the prior version of the software, which worked well when employing the following settings and procedure: set the High Voltage (HV) to 600 V, the coarse gain to 8, the fine gain to 1, the conversion gain to 1024. Set the lower level detector (LLD) to 2 and the upper level detector (ULD) to 100. These settings eliminate an x-ray peak due to interaction of electrons with the PMT's lead collar, at low energies, but extend well beyond the Cs-137 peak, at high energies. Set the live time, which is measured in seconds, to 600 s to obtain 10 minutes of data collection. Because the PMT needs to recover between detections, live time plus dead time equals actual time. Since the PHA reports a channel scale and not the needed energy scale, energy calibration of the channels is necessary. First collect intensity measurements for the three radioactive sources Co-60, Na-22, Cs-137. These sources have known radiation energies, which the PHA represents as peaks in the corresponding energy spectra. Plot the peak energies versus channel number and perform a line fit to form an expression that converts channel numbers to energies.

### Analysis

To export your collected data to Igor Pro for analysis, within the detector software, click save → file format → spectrum file. Then click save → file format → comma separated file

(CSV). Next open CSV file with text edit software, delete all metadata text at top of file, and save. Next open Igor Pro and click data → load waves → delimited text, and open your CSV file. Finally save your loaded wave with an appropriate name, click “new table”, and load waves as needed.

Identify and explain all the peaks in the spectrum, including the main x-ray and  $\gamma$ -ray peaks, the Pb collar peak, the Compton backscatter and edge peaks.  $\gamma$ -rays can directly transfer all their energy to a detector electron via the photoelectric effect to form a large Cs  $\gamma$ -ray peak, or they can directly Compton backscatter in the detector to form the Compton edge, or they can Compton backscatter externally *and then* directly transfer all their energy to a detector electron via the photoelectric effect to form the Compton backscatter peak.

Compare the measured and predicted energy shifts peaks. Carefully propagate the uncertainties. Are the measurements consistent with the theory? How can data be collected at other angles?

### Acknowledgment

We thank Andy Brinck for assistance with this entry.

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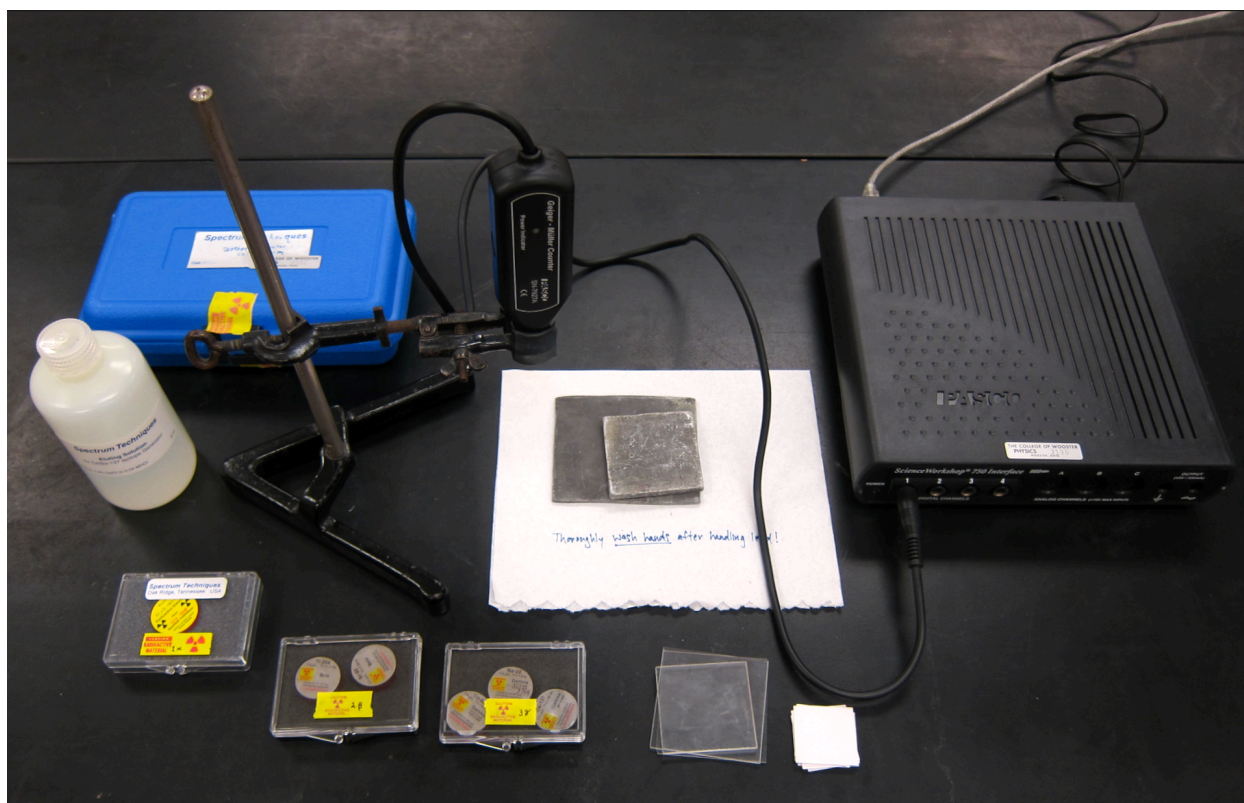


Fig. 10.1. Radioactive decay experiment includes a Geiger-Müller counter interfaced to a computer, commercial sources of alpha, beta, and gamma radiation, lead, plastic, and paper shielding, and a barium isotope generator kit.



## 10. Radioactive Decay

*Last updated 2017 October 29*

### Summary

Investigate the penetrating ability of three common types of nuclear radiation and the ability of different materials to stop them. Determine the half-life for Barium.

### History

In 1896, **Henri Becquerel** discovered that uranium salts spontaneously emit invisible rays like the recently discovered x-rays but not as penetrating [1,2]. Shortly thereafter, **Marie Skłodowska Curie** discovered that uranium is not the only natural emitter of such *radiation* [3]. By 1898, she and **Pierre Curie** isolated in pitchblende two new elements, now known as polonium and radium, even more radioactive than uranium [4]. For this work, Becquerel and the Curies shared a 1903 Nobel Prize. Note that 1 Bq = 27pCi (one becquerel equals 27 pico-curies) is one decay per second, and 1 Ci is approximately the activity of 1 g of radium-226.

### Theory

Today we understand that three common types of nuclear radiation, originally named by Ernest Rutherford after the first three letters of the Greek alphabet,  $\alpha$ ,  $\beta$ ,  $\gamma$ : alpha emissions are helium nuclei, beta emissions are electrons, and gamma emissions are highly energetic photons. Typically, large unstable nuclei decay quantum mechanically with certain probabilities to more stable nuclei by the emission of one or more of these radiations.

For example, polonium-210 alpha decays to lead-206 mainly by



where the superscripts indicate the number of protons and neutrons, and the subscripts indicate the number of positive charges. As another example, strontium-90 beta decays to yttrium-90 via



where internally a neutron becomes a proton via

$${}_0^1\text{n} \rightarrow {}_1^1\text{p} + {}_{-1}^0\text{e}, \quad (10.3)$$

or in the language of elementary particle physics, a down quark becomes an up quark via

$$d^{-1/3} \rightarrow u^{+2/3} + e^{-1} + \bar{\nu}_e, \quad (10.4)$$

where now the superscripts indicate electrical charge (in multiples of the anti-electron charge), and the electron anti-neutrino  $\bar{\nu}_e$  is often neglected in the nuclear physics equations.

Nuclear radiation counts typically decay exponentially in space

$$N = N_0 e^{-x/x_0} \quad (10.5)$$

with an  $e$ -folding depth  $x_0$  dependent on the kind of radiation and the type of shielding. Nuclear radiation activities, or counts per second, typically decay exponentially in time

$$A = A_0 e^{-\lambda t} = A_0 2^{-t/t_h} \quad (10.6)$$

with a decay rate  $\lambda$  and half-life  $t_h$ .

## Procedure

### Shielding

The radioactive sources are mounted in 2.5 cm diameter sealed plastic disks. They consist of polonium-210 (an alpha emitter with a half-life of 138 days), strontium-90 (a beta emitter with a half-life of 28 years), and cobalt-60 (a gamma emitter with a half-life of 5.3 years). First measure the background radiation, and then subtract this from subsequent measurements. Measure the activity as a function of layers of shielding. Be sure to choose the counting (or integration) time of the Geiger-Müller (GM) counter long enough to obtain counts for multiple layers of paper or plastic (and possibly lead). Use vinyl gloves while handling the lead or wash your hands thoroughly afterward.

### Half-life

Activate your barium sample and measure its activity as a function of time. The SpecTech Cs/Ba-137m isotope generator contains a small quantity ( $< 10 \mu\text{Ci}$ ) of radioactive Cs-137 bound to a special ion exchange medium. The Cs-137 parent isotope beta decays with a 30.17-year half-life to produce metastable Ba-137m, which in turn decays with a 2.55-minute half-life, generating a 661.6 keV  $\gamma$ -ray emission, as in Fig. 10.2. This  $\gamma$ -ray may be readily detected using a GM or scintillation radiation detector. An eluting solution is used to

selectively extract the Ba-137 isotope from the exchange medium leaving the parent Cs-137 isotope in place to regenerate more Ba-137. Equilibrium is reestablished in less than 1 hour.

Approximately 30 minutes after elution, the residual activity of the Ba-137 solution has decayed to less than 1/1000 of its original activity making it safe for normal disposal. When used with the eluting solution supplied, bleed through of the Cs-137 parent isotope is less 50 Bq/ml. To maintain correct chemical stability, use only the provided eluting solution. (Additional solution may be ordered or prepared as 0.9% NaCl in 0.04M HCl. When making solution, use distilled or de-ionized water to avoid unwanted mineral contamination.)

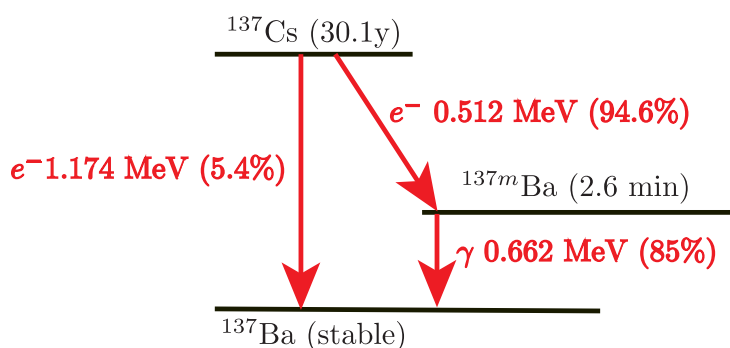


Fig. 10.2. Cs-137 decays to Ba-137 mainly via a short-lived metastable state that emits a  $\gamma$ -ray.

To operate the isotope generator, place the planchet on the sample tray being used. Draw the eluting solution into the syringe from the bottle or other suitable container. Remove both stoppers from the generator column. Insert the syringe firmly into the hole on the top of the generator and, while holding the generator vertically, force approximately seven drops of solution through the generator onto the planchet.

**Under no circumstances attempt to draw solution back through the generator as this may rupture the internal filters. The capsule is marked with a flow direction indicator.**

Immediately place the sample in the planchet into the counter and begin counting for the predetermined counting time. After 10 minutes the sample will have decayed to more than three half-lives and may be discarded. After use, remove the syringe from the column, replace both stoppers and empty any unused solution back into the bottle. Wash and dry the planchets before reusing them.

### Analysis

Which of three common types of nuclear radiation is most penetrating? Plot counts versus the thickness  $x$  of shielding for plastic (and possibly paper and lead). Assuming an Eq. (10.5) exponential decay, determine the  $e$ -folding depth  $x_0$ . Is the activity of radioactive barium exponential in time  $t$ ? Assuming an Eq. (10.6) exponential activity, determine the

decay rate  $\lambda$  and half-life  $t_h$  of Ba-137. Remember to subtract the background counts.

### Acknowledgment

These guidelines are based in part on the Spectech isotope generator operating instructions.

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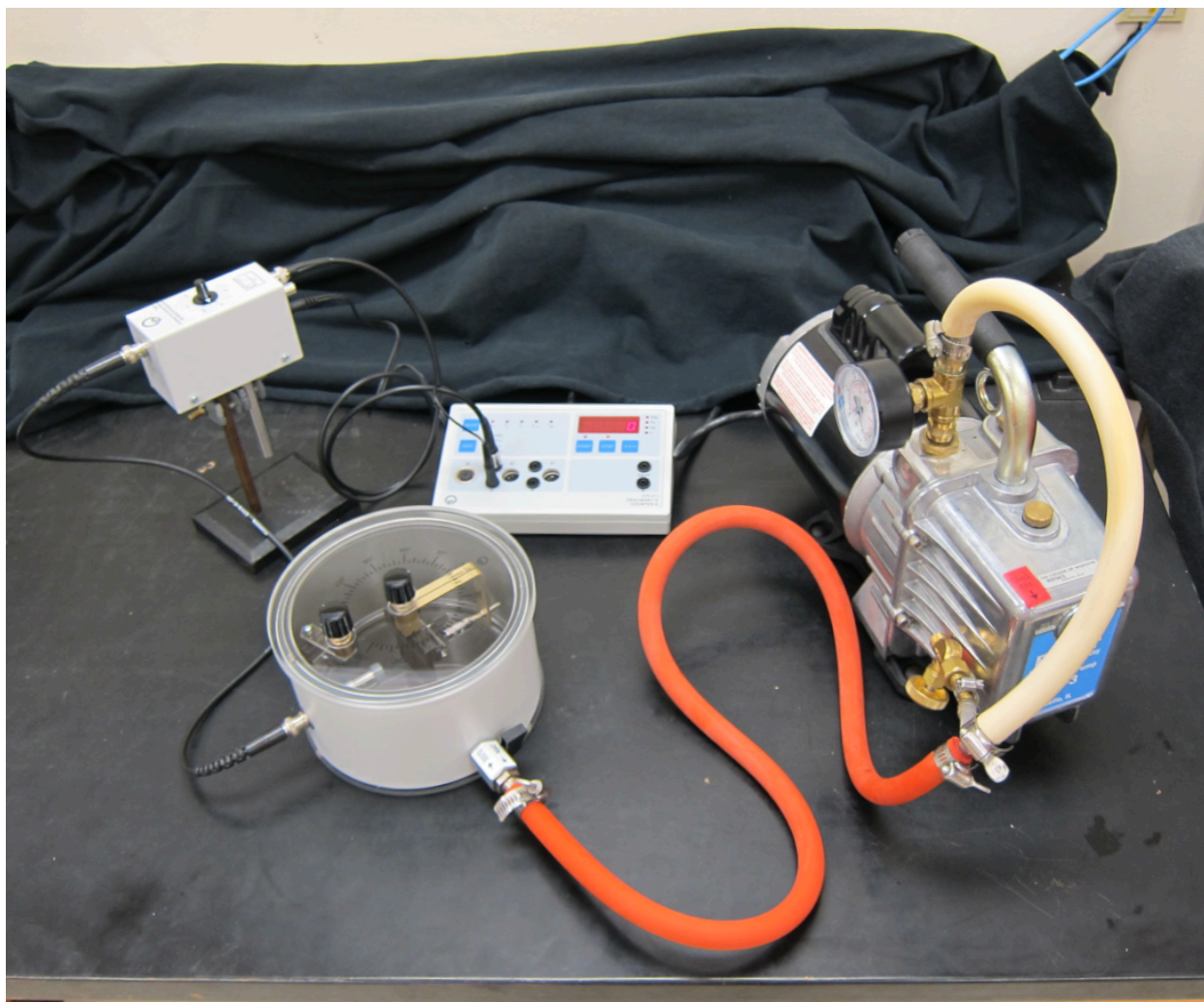
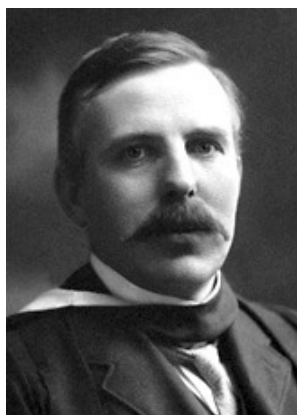


Fig. 11.1. Rutherford apparatus features (counterclockwise from right) a vacuum pump, vacuum chamber with Am-241 radioactive source and gold foil and detector, discriminator preamplifier, and counter.



## 11. Rutherford Scattering from a Gold Foil

*Last updated 2017 October 17*

### Summary

Determine unseen structure by the scatter pattern of deflected projectiles. Rutherford thus exposed the nuclear atom. Here, we will confirm the Rutherford scattering formula by bombarding a thin gold foil with  $\alpha$  particles, as in Fig. 11.1.

### History

In 1911, **Ernst Rutherford** proposed the first nuclear model of the atom [1], inspired by and confirmed in a beautiful series of experiments by his students, Hans Geiger and Ernst Marsden in the years 1909-1914, who were scattering  $\alpha$  particles off thin metal foils [2]. The surprise was that some  $\alpha$  particles were deflected through large angles, which was “almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you”. (For earlier work on radioactivity, Rutherford earned a 1908 Nobel Prize.)

### Theory

Scattering experiments like this one are extremely useful in many areas of physics. Bombarding nuclei, atoms, and molecules with photons, electrons, protons, neutrons and  $\alpha$  particles and observing what scatters at different angles is very revealing. Definite quantitative conclusions about the scatterers can be obtained even though they cannot be seen. Many particles bombard the target and the desired information is encoded in the relative number scattered at different angles.

In the Rutherford scattering experiment,  $\alpha$  particles pass through a gold film. Interactions with gold nuclei deflect the  $\alpha$  particles, which are doubly charged helium nuclei, from their initial paths. The particles can be detected and counted after they hit the film target at the end of an evacuated cylinder.



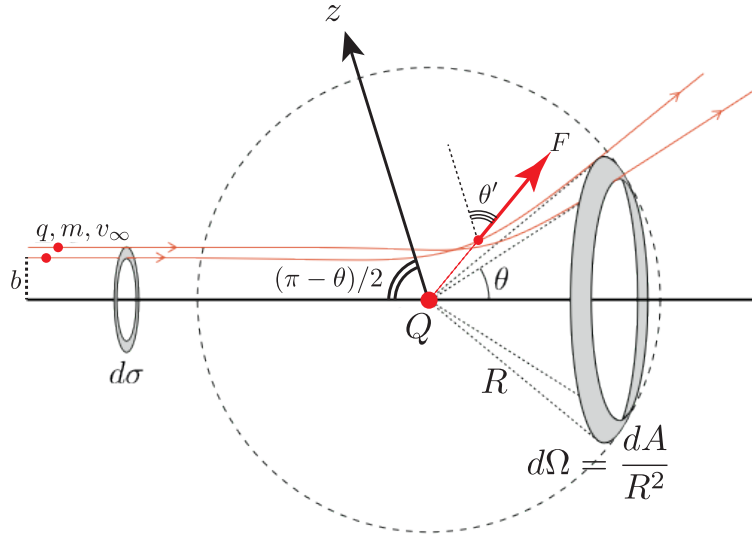


Fig. 11.2. Charges entering via the annulus  $d\sigma$  scatter through an angle  $\theta$  and exit through the solid angle  $d\Omega$ .

Assume a charge  $q$  of mass  $m$ , faraway speed  $v_\infty$ , and impact parameter offset  $b$  scatters from a fixed charge  $Q$  through an angle  $\theta$ , as in Fig. 11.2. Its momentum change

$$\Delta p_z = m\Delta v_z = m 2v_\infty \cos \left[ \frac{\pi - \theta}{2} \right] = 2mv_\infty \sin \left[ \frac{\theta}{2} \right]. \quad (11.1)$$

Angular momentum conservation (or Kepler's second law) relates infinitesimal changes in angle  $d\theta$  and time  $dt$  by

$$mr^2 \frac{d\theta}{dt} = mr^2 \omega = L_\perp = mrv = mbv_\infty, \quad (11.2)$$

so the total impulse

$$\mathcal{I} = \Delta p_z = \int_{-\infty}^{+\infty} F_z dt = \int_{-(\pi-\theta)/2}^{+(\pi-\theta)/2} \left( k \frac{qQ}{r^2} \cos \theta' \right) \left( \frac{r^2 d\theta'}{bv_\infty} \right) = 2 \frac{kqQ}{bv_\infty} \cos \left[ \frac{\theta}{2} \right], \quad (11.3)$$

where  $k = 1/4\pi\epsilon_0$  is Coulomb's constant. Combine the Eq.(11.1) and Eq.(11.3) momentum changes so solve for the impact parameter

$$b = \frac{kqQ}{mv_\infty^2} \frac{\cos[\theta/2]}{\sin[\theta/2]} = \frac{kqQ}{2E_\infty} \cot \left[ \frac{\theta}{2} \right]. \quad (11.4)$$

If  $\sigma = \pi b^2$  is the cross-sectional area of a target circle of radius  $b$ , then the differential area

$$\begin{aligned} d\sigma &= |2\pi b \, db| \\ &= \left| 2\pi \left( \frac{kqQ}{2E_\infty} \cot \left[ \frac{\theta}{2} \right] \right) \left( -\frac{kqQ}{2E_\infty} \csc^2 \left[ \frac{\theta}{2} \right] \frac{d\theta}{2} \right) \right| = 4\pi \left( \frac{kqQ}{2E_\infty} \right)^2 \frac{\cos[\theta/2]}{\sin^3[\theta/2]} d\theta \end{aligned} \quad (11.5)$$

subtends the solid angle

$$d\Omega = \frac{dA}{R^2} = \int_{\phi=0}^{2\pi} \sin \theta \, d\theta d\phi = 2\pi \sin \theta \, d\theta = 4\pi \sin \left[ \frac{\theta}{2} \right] \cos \left[ \frac{\theta}{2} \right] d\theta, \quad (11.6)$$

as in Fig. 11.2. The differential scattering cross section is the Eq. (11.5) to Eq. (11.6) ratio

$$R^2 \frac{d\sigma}{dA} = \frac{d\sigma}{d\Omega} = \left( \frac{kqQ}{4E_\infty} \right)^2 \frac{1}{\sin^4[\theta/2]}, \quad (11.7)$$

which is proportional to the number of detector counts per unit time at an angle  $\theta$  from the beam. Checking the famous inverse-sine-to-the-fourth dependence of the scattering rate on the scattering angle is the major goal of this experiment.

### Procedure

Record the scattering rate as a function of the angle.

In test trials, the chamber seems to hold the vacuum with the valve closed long enough to take all data (about 1.5 hours). But you could leave the chamber open and the vacuum pumping the entire time (see below), if you notice the counts decreasing for a given angle (indicating the vacuum is not as good).

Although there does not seem to be any difference between covering and uncovering the chamber with a black cloth, fluorescent lights may affect the sensor.

Setting the discriminator to  $V \geq 0.2V$  seems to give reasonable results. However, you should try a few trials at different angles to decide on a reasonable setting, and then leave the **discriminator voltage constant** throughout the entire data collection.

**To draw a vacuum** on the chamber: Ensure the chamber valve is open. Ensure the brass valve at the T in the vacuum tube is closed (**right = tight**). Turn on the pump. Press on lid to make sure the seal is good and let the vacuum run until it reaches about  $-100$  kPa. Close the chamber valve.

**To turn off the pump** (so oil is not sucked out of the vacuum!): Check that the chamber is closed. Slowly and completely open the brass valve at the T in the vacuum tube (**left = loose**). Turn off the vacuum pump.

**To open the chamber:** Orient the delicate gold foil so that the inflow of air will be parallel to its surface; i.e. turn the swivel arm to  $0^\circ$ . Slowly and only partially open the chamber valve, just until you begin to hear air hissing in at the brass valve in the vacuum tubing. Wait for the sound of airflow to stop. Pull the lid off the chamber, but be extremely careful not to damage the gold foil in the black plastic slit case.

**To adjust the gate time:** Press Gate to select 100 s. Then press and *hold* Gate (selects “manual”) and press Mode at the same time. Hold both for a second until the manual time set is displayed (defaults to 0 s) and then release both buttons. Adjust the gate time using the Mode button to increase time and the Gate button to decrease the time. Note: the last used manual gate time is saved as long as the power remains on. Click the start button to count.

### Analysis

Plot the scattering rate as a function of angle. Try to plot some function of  $\theta$  to get a straight line, such as inverse counts versus sine-tesseracted half-angle,  $1/N_\theta$  versus  $\sin^4[\theta/2]$ . Alternately, take the logarithm of Eq. (11.7) and check for a slope of  $-4$ . Compare results for positive and negative angles.

### References

[1] Ernest Rutherford, “The Scattering of  $\alpha$  and  $\beta$  Particles by Matter and the Structure of the Atom”, *Philosophical Magazine*, Series 6, **21**, 669-688 (1911).

[2] Hans Geiger, Ernst Marsden, “On a Diffuse Reflection of the  $\alpha$ -Particles”, *Proceedings of the Royal Society*, Series A **82**, 495-500 (1909).



## Scientific Writing

Last updated 2017 September 13

### Overview

Your experimental reports should be modeled on professional articles in the *American Journal of Physics* and the *Physical Review*, many examples of which are available online and at the Science Library. A scientific report should be well organized and readable. Your goal is to present the essential ideas and important details concisely so that more people will read and understand your work.

A scientific report should be well written. Good grammar, complete sentences, well-formed paragraphs, accurate spelling, and proper punctuation are essential. Avoid sentence fragments and numbered outlines. For reference, the American Institute of Physics (AIP) Author Resource Center is available online [2].

A useful book on scientific writing is *The Technical Writer's Handbook* by Matt Young [1], which is available in Timken Science Library and by the printer in the Jr. I.S. lab. For clarity in writing, start with the entries on *gobbledygook*, *it*, *passive voice* and *wordiness*.

Reports do not all have to follow the same format, but most good scientific reports include several basic elements, which are discussed below. Scientific reports do usually use headings (such as **Theory**, **Procedure**, **Results**, and so on) to give context to the reader.

### Title

The title should be descriptive, with sufficient key words to attract a desired reader who might be searching a computer database. For example, the titles “Lab #1” and “Photoelectric Lab” are unprofessional and unhelpful, while the title “Using the Photoelectric Effect to Measure Planck’s Constant” is much better. Beneath the title, write your name first and your partner’s name second, followed by the name and address of the College, and the date of the current draft.

### Abstract

The abstract should state very concisely (usually in one paragraph) the scope and nature of the subject discussed, the basic method or approach, and a summary of the major results. It is more valuable for the reader to learn, for example, “we measured light speed to be  $(2.97 \pm 0.03) \times 10^8 \text{ m/s} = 297 \pm 3 \text{ m}/\mu\text{s}$ , which is within 0.67% of the accepted value” than to read that “the speed of light was measured”. The abstract should stand alone without reference to the rest of the paper, since many people will read only your abstract and not the full paper. See the entry on abstract in Young’s book.

### Introduction

The introduction should outline for the reader exactly what is to be discussed in the paper, the purpose of the work, and a brief history of previous work relevant to the investigation. Here the authors of an original research paper can describe what is new in their work and how it contributes to the field. See the entry on *getting started* in Young’s book.

## Theory

Experiments are designed and performed within a theoretical context. Describe this context here and summarize, motivate, or derive the relevant equations. **Equations should be treated like words in sentences.** They should not stand outside sentences, but should be punctuated as words inside the sentences, even when they are indented and numbered. Good examples are contained in this lab manual. Use superscripts for exponents in scientific notation. Be sure to explain what each symbol means. (Do not assume the reader knows what  $F$  or  $i$  is.) When referring to variables in a sentence, they are traditionally italicized (as in the previous parentheses). See the entries on *equations*, *the* and *symbols* in Young's book.

## Procedure

Briefly describe what you did in full sentences and complete paragraphs. **Don't write this section as a numbered list of steps, and don't imitate a lab manual! Use the past tense.** Figures of the apparatus, including either schematics or digital photographs, are often valuable here, but all figures should be numbered and captioned. Serial numbers of each piece of equipment are not necessary, but you should identify the major equipment used. This is also the place to describe experimental difficulties and how (or whether!) they were overcome and any corrections or calibrations which were used.

## Results and Analysis

Concisely summarize your results and discuss them. This section will probably include plots and tables to display your results. **Plots and tables do not stand alone; each one must be discussed in the text.** Talk the reader through the important things to notice when looking at the figure.

All graphs, figures and tables must be labeled **and captioned**. Graphs must be titled, tables should have headings, and both should be labeled with units. Graphs and figures should be large enough to be clear; include error bars in your graphs. Graphs and tables should be on the page they are first mentioned or on the next page, not appended to the end of the report. **Verify that your results (including slopes of graphs) have proper units, the appropriate number of significant figures, and uncertainty estimates.** For example, "We estimate Planck's constant  $h$  to be  $(7.0 \pm 1.9) \times 10^{-34} \text{ J s} = 0.70 \pm 0.19 \text{ zJ/THz}$ , which is within one standard deviation of the accepted value." Typically, you will *not* include in your report all the raw data in your lab notebook. See the entries on *naked decimal point*, *SI units*, *kelvin*, *table* and *graph* in Young's book.

## Conclusions

In some cases, the purpose of the experiment is the measurement of specific quantities (for example, Millikan Electron Charge or The Speed of Light). Your result should be compared, whenever possible, with previously measured values, handbook or textbook values, or a theoretically calculated result. You should indicate the *extent* of agreement or disagreement and discuss any discrepancies.

In other experiments (for example, Franck-Hertz Energy Quantization) an important aspect of the investigation is to illustrate an effect or to distinguish between rival models or

theories. Here is where you should draw conclusions based on your data. This may also involve a discussion of uncertainties. In some cases, you may have to say that no conclusions can be drawn from your data. You might also suggest possible improvements in the experiment.

### Acknowledgments

If you borrowed apparatus from another laboratory, obtained ideas in discussions with others, or had financial support for your experiment, you should acknowledge such assistance here.

### References

If you refer to previously published work, your textbook, or your class notes, you should list these in a section at the end of the paper. References are numbered in order of use in the text. Be sure you cite the author's name and the name, volume, page, and date of the journal; the AIP reference style is generally the best choice. You can also insert parenthetical arguments or comments here that would interrupt the flow of thought in the main body of the paper. **Any quoted or paraphrased material used in your report must be referenced.**

### Appendixes

Supplementary material that would not be appropriate to put in the main body of the paper can be given in an appendix. Examples are lengthy derivations, details of uncertainty propagation, or extensive tables of data.

### References

[1] Matt Young, *The Technical Writer's Handbook: Writing with Style*, Revised Edition (University Science Books, 2002).

[2] American Institute of Physics (AIP) Author Resource Center,  
[www.aip.org/pubservs/style/4thed/toc.html](http://www.aip.org/pubservs/style/4thed/toc.html), last accessed 2017 September 13.



## Report Checklist

Last updated 2017 November 19

- ◇ RevTeX two-column format PDF.
- ◇ Descriptive title like “Scattering Photons from Electrons” (but not “Lab #2” or “Compton Experiment”).
- ◇ Your Name, Partner’s Name.
- ◇ If you measured something, report uncertainty and compare to accepted value like “ $c = (3.00 \pm 0.01) \times 10^8$  m/s, which is within 1% of the accepted value”.
- ◇ Include at least one
  - `\cite{}` reference to the **original literature** in the bibliography (which the *Modern Physics Lab Guide* helpfully references).
  - `\ref{}` to a figure in the report.
  - indented and numbered equation properly punctuated inside a sentence.
- ◇ Figure
  - (and especially graph) text is comparable to size of paragraph text.
  - captions end in periods.
  - captions often start with a noun phrase like “FIG. 1: **[shows]** The apparatus.”, where **[shows]** is implicit to save journal space. Subsequent sentences in the caption are normal.
- ◇ Unitalicize **units** everywhere.
- ◇ Italicize **variables** everywhere.
- ◇ Do not use lists (except in unusual circumstances).
- ◇ Use “experiment” instead of “lab”, as the latter rarely appears in professional scientific articles.
- ◇ Cite the lab guide as
  - Physics Department, *Modern Physics Lab Guide* (The College of Wooster, 2017).where “Physics Department” functions as author and “The College of Wooster” functions as publisher.

## Uncertainty Propagation

Last updated 28 August 2007

### Quantifying Uncertainties

There is some uncertainty associated with every measurement. Consider a *population* of a very large number of measurements  $x_n$ . The *population mean* is

$$\mu = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N x_n. \quad (\text{U.1})$$

The *deviations* from the mean are  $\Delta x_n = x_n - \mu$ . The average of the deviations is clearly zero, with just as many measurements larger than the mean as smaller, but the average of the *square* of the deviations does not vanish; the *population variance*

$$\sigma^2 = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N (\Delta x_n)^2 = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N (x_n - \mu)^2 \quad (\text{U.2})$$

is a nonzero measure of the spread of the measurements, but the square root of the variance or *standard deviation*  $\sigma$  is an even better measure, as it has the same units as  $x_n$ .

Often you can't measure the entire population but must be content to measure a small *sample*. The *sample mean*

$$\bar{x} = \frac{1}{N} \sum_{n=1}^N x_n \quad (\text{U.3})$$

is a good, unbiased estimate of the population mean  $\mu$ . The *sample variance*

$$s^2 = \frac{1}{N-1} \sum_{n=1}^N (x_n - \bar{x})^2, \quad (\text{U.4})$$

is a good, unbiased estimate of the population variance  $\sigma^2$ . The  $N-1$  in the denominator reflects the fact that, effectively, the number of degrees of freedom in the sample has been reduced by one in the calculation of the sample mean. As  $N \rightarrow \infty$ , the population and sample variances converge, and in practice they differ significantly only for small  $N$ . In physics, the same symbol  $\sigma^2$  is typically (and confusingly) used for both. By default, does your calculator compute the population variance Eq. (U.2) or the sample variance Eq. (U.4)?

### Propagating Uncertainties

Often you will measure quantities  $x$  and  $y$  and use a theoretical formula to compute a quantity  $z = f[x, y]$ . From multidimensional calculus, if  $x$  and  $y$  undergo infinitesimal changes  $dx$  and  $dy$ , then  $z$  undergoes an infinitesimal change

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy. \quad (\text{U.5})$$

Similarly, if  $x$  and  $y$  are uncertain by small amounts  $\Delta x$  and  $\Delta y$ , then  $z$  is uncertain by a small amount

$$\Delta z \approx \frac{\partial z}{\partial x} \Delta x + \frac{\partial z}{\partial y} \Delta y. \quad (\text{U.6})$$

Hence the variance in  $z$  can be written

$$\begin{aligned} \sigma_z^2 &= \frac{1}{N} \sum_{n=1}^N (\Delta z_n)^2 \\ &\approx \frac{1}{N} \sum_{n=1}^N \left( \frac{\partial z}{\partial x} \Delta x_n + \frac{\partial z}{\partial y} \Delta y_n \right)^2 \\ &= \frac{1}{N} \sum_{n=1}^N \left( \frac{\partial z}{\partial x} \Delta x_n \right)^2 + \frac{1}{N} \sum_{n=1}^N \left( \frac{\partial z}{\partial y} \Delta y_n \right)^2 + \frac{1}{N} \sum_{n=1}^N 2 \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \Delta x_n \Delta y_n \\ &\approx \left( \frac{\partial z}{\partial x} \right)^2 \frac{1}{N} \sum_{n=1}^N (\Delta x_n)^2 + \left( \frac{\partial z}{\partial y} \right)^2 \frac{1}{N} \sum_{n=1}^N (\Delta y_n)^2 + 0 \\ &= \left( \frac{\partial z}{\partial x} \right)^2 \sigma_x^2 + \left( \frac{\partial z}{\partial y} \right)^2 \sigma_y^2, \end{aligned} \quad (\text{U.7})$$

where the cross terms sum to nearly zero as half are likely to be positive and half negative. Thus,

$$\sigma_z = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial y}\right)^2 \sigma_y^2} = \sqrt{\left(\frac{\partial z}{\partial x} \sigma_x\right)^2 + \left(\frac{\partial z}{\partial y} \sigma_y\right)^2}, \quad (\text{U.8})$$

with similar expressions for formulas with more than two dependent variables.

### Example 1

Suppose  $z = x \pm y$ . Then from Eq. (U.8),

$$\sigma_z = \sqrt{\left(\frac{\partial z}{\partial x} \sigma_x\right)^2 + \left(\frac{\partial z}{\partial y} \sigma_y\right)^2} = \sqrt{\sigma_x^2 + \sigma_y^2}. \quad (\text{U.9})$$

Uncertainties are said to *add in quadrature*.

### Example 2

Suppose we perform  $N$  measurements  $x_n$ , each with most likely uncertainty  $\sigma$ . What is the uncertainty in the mean  $\bar{x}$  of these measurements? Our formula is Eq. (U.3), and so by Eq. (U.8),

$$\sigma_{\bar{x}} = \sqrt{\sum_{n=1}^N \left(\frac{\partial \bar{x}}{\partial x_n} \sigma_{x_n}\right)^2} = \sqrt{\sum_{n=1}^N \left(\frac{1}{N} \sigma\right)^2} = \sqrt{N \left(\frac{1}{N} \sigma\right)^2} = \frac{\sigma}{\sqrt{N}}. \quad (\text{U.10})$$

This is a famous result: averaging  $N$  results each with uncertainty  $\sigma$  reduces the uncertainty of the average by  $1/\sqrt{N}$ .

### Example 3

Suppose

$$z = f[x, y] = x^2 y^3. \quad (\text{U.11})$$

Then

$$\frac{\partial z}{\partial x} = 2xy^3 = 2 \frac{z}{x}, \quad (\text{U.12})$$

and

$$\frac{\partial z}{\partial y} = 3x^2y^2 = 3\frac{z}{y}, \quad (\text{U.13})$$

and so, dividing Eq. (U.8) by  $z$ ,

$$\frac{\sigma_z}{z} = \sqrt{\left(2\frac{\sigma_x}{x}\right)^2 + \left(3\frac{\sigma_y}{y}\right)^2}. \quad (\text{U.14})$$

Note that the coefficients inside the parentheses of Eq. (U.14) are the exponents in Eq. (U.11).

#### Example 4

In the Thomson electron charge-to-mass ratio experiment, we encounter the formula

$$\rho = k \frac{V}{I^2 r^2}, \quad (\text{U.15})$$

where  $k$  is a constant. Suppose the most likely relative uncertainties in  $V$ ,  $r$ , and  $I$  are 2%, 3%, and 4%, respectively. From the obvious generalization of Eq. (U.12), the most likely relative uncertainty in  $\rho$  is

$$\frac{\sigma_\rho}{\rho} = \sqrt{\left(\frac{\sigma_V}{V}\right)^2 + \left(2\frac{\sigma_r}{r}\right)^2 + \left(2\frac{\sigma_I}{I}\right)^2}, \quad (\text{U.16})$$

where we have omitted the negative signs because the terms are squared. Hence,

$$\frac{\sigma_\rho}{\rho} = \sqrt{2^2 + 6^2 + 8^2} \% = \sqrt{104} \% \approx 10\%. \quad (\text{U.17})$$

#### Bibliography

P. R. Bevington, D. K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences, Third Edition* (McGraw-Hill, 2002).

A. C. Melissinos, J. Napolitano, *Experiments in Modern Physics, Second Edition* (Academic Press, 2003).

J. R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements, Second Edition* (University Science Books, 1997).

## LaTeX Example

*Last updated 2017 October 29*

```
\documentclass[aps, twocolumn,amscd,amsmath,amssymb,verbatim]{revtex4}
% replace "twocolumn" by "preprint" for first drafts

\usepackage{graphicx} % for figures

\begin{document}

% %%%%%%%%%%%%%%%
\title{This is the Title of the Article}
\author{Jane Doe}
\affiliation{Physics Department, The College of Wooster, Wooster, Ohio 44691, USA}

\date{\today}

\begin{abstract}
The abstract is a short summary of the article. This is filler text. This is filler text. This is filler text.
This is filler text. This is filler text. This is filler text. This is filler text. This is filler text.
\end{abstract}

\maketitle % generate title, including abstract

% %%%%%%%%%%%%%%%
\section{Introduction}
This is text. This is \textbf{bold} text. This is text with \emph{emphasis}. This is ``double quotes".

Paragraphs are separated by one or more blank lines.

% %%%%%%%%%%%%%%%
\section{Math & Citations} % "&" is a special character
Examples of inline math are  $\alpha = \sqrt{\gamma^2 + \Gamma^2}$  and  $\vec{v} = 7 \hat{x} - 5 \hat{y}$  and  $\vec{u} \times \vec{v}$  and  $c = (2.99 \pm 0.01) \times 10^8 \text{ m/s}$ . One example of
block (display) math is
%
\begin{equation}
\int_0^1 x^2 dx = \frac{1}{3},
\label{myIntegral}
\end{equation}
%
and a second example is
%
\begin{equation}
\xi = \alpha \left( \frac{1}{\omega_0^2 + \omega^2} \right).
\label{signal}
\end{equation}
%
```

Note how block math is punctuated like words in a sentence! The block math equations are automatically numbered. We can reference Eq.~\ref{myIntegral} or Eq.~\ref{signal} by inserting labels in the block, but then we must compile `\LaTeX` twice.

We can readily cite both articles `\cite{Duke2003}` and books `\cite{Loecher2002}` in our bibliography, but again we must compile `\LaTeX` twice.

`% %%%%%%%%%%`

`\section{Figures & Tables}`

We can also include figures but need to use package `graphicx` under document class. We can reference Fig.~\ref{SchematicDiagram} like equations. All figures should have captions.

```
\begin{figure}[ht] % "ht" = here or top
  \includegraphics[width=0.8\linewidth]{CoWFigure} % PDFs or PNGs
  \caption{Figure captions go on bottom.}
  \label{SchematicDiagram}
\end{figure}
```

Finally, we can also include tables, such as Table~\ref{demoTable}. Like figures, we can also `\emph{attempt}` to force their positions.

In the document class line, we can easily convert from `preprint` one-column, double-spacing for rough drafts to `twocolumn` single spacing for final drafts!

```
\begin{table}[h] % indenting is optional
  \caption{Table captions go on top.}
  \label{demoTable}
  \begin{ruledtabular}
    \begin{tabular}{cc} % "cc" = center each column
      absicssa & ordinate\\
      \hline
      1.0 s & 5.6 m\\
      3.0 s & 9.9 m
    \end{tabular}
  \end{ruledtabular}
\end{table}
```

`% %%%%%%%%%%`

`\begin{thebibliography}{99} % place widest label in final brackets`

```
\bibitem{Duke2003} T. Duke and F. J. \textit{ulicher},
\textit{Phys. Rev. Lett.}, \textbf{90}, 158101 (2003).
```

```
\bibitem{Loecher2002} M. Loecher,
\textit{Noise Sustained Patterns} (World Scientific Publishing Company, 2002).
```

`\end{thebibliography}`

`\end{document}`



## Physical Constants

Last updated 2003 December 29

$c$	Light speed	299 792 458 m/s (exact)
$h$	Planck	$6.626\,0693(11) \times 10^{-34}$ J s $4.135\,667\,43(35) \times 10^{-15}$ eV s
$\hbar = h/2\pi$	Planck reduced	$1.054\,571\,68(18) \times 10^{-34}$ J s $6.582\,119\,15(56) \times 10^{-16}$ eV s
$e$	Elementary charge	$1.602\,176\,53(14) \times 10^{-19}$ C
$m$	Electron mass	$9.109\,3826(16) \times 10^{-31}$ kg $0.510\,998\,918(44)$ MeV/ $c^2$
$m_p$	Proton mass	$1.672\,621\,71(29) \times 10^{-27}$ kg $938.272\,029(80)$ MeV/ $c^2$
$G$	Newton gravity	$6.6742(10) \times 10^{-11}$ m <sup>3</sup> kg <sup>-1</sup> s <sup>-2</sup>
$k$	Boltzmann	$1.380\,6505(24) \times 10^{-23}$ J K <sup>-1</sup>
$\sigma$	Stephan-Boltzmann	$5.670\,400(40) \times 10^{-8}$ W m <sup>-2</sup> K <sup>-4</sup>
$N_A$	Avogadro	$6.022\,1415(10) \times 10^{23}$ mol <sup>-1</sup>
$\mu_0$	SI Magnetic	$12.566\,370\,614... \times 10^{-7}$ N A <sup>-2</sup> (exact) $4\pi \times 10^{-7}$ N A <sup>-2</sup> (exact)
$\epsilon_0 = 1/\mu_0 c^2$	SI Electric	$8.854\,187\,817... \times 10^{-12}$ F m <sup>-1</sup> (exact)
$\hbar c$		197.326 968(17) MeV fm 1973.26 968(17) eV Å
$\alpha = e^2/4\pi\epsilon_0\hbar c$	Fine-structure	$7.297\,352\,568(24) \times 10^{-3}$
$\alpha^{-1} = 4\pi\epsilon_0\hbar c/e^2$	Inverse fine structure	137.035 999 11(46)
$m_p = \sqrt{\hbar c/G}$	Planck mass	$2.176\,45(16) \times 10^{-8}$ kg
$l_p = \hbar/m_p c = \sqrt{\hbar G/c^3}$	Planck length	$1.616\,24(12) \times 10^{-35}$ m
$t_p = l_p/c = \sqrt{\hbar G/c^5}$	Planck time	$5.391\,21(40) \times 10^{-44}$ s

Source: 2002 CODATA, <http://physics.nist.gov/cuu/Constants/index.html>. Parentheses indicate uncertainties in the last couple of decimal places.