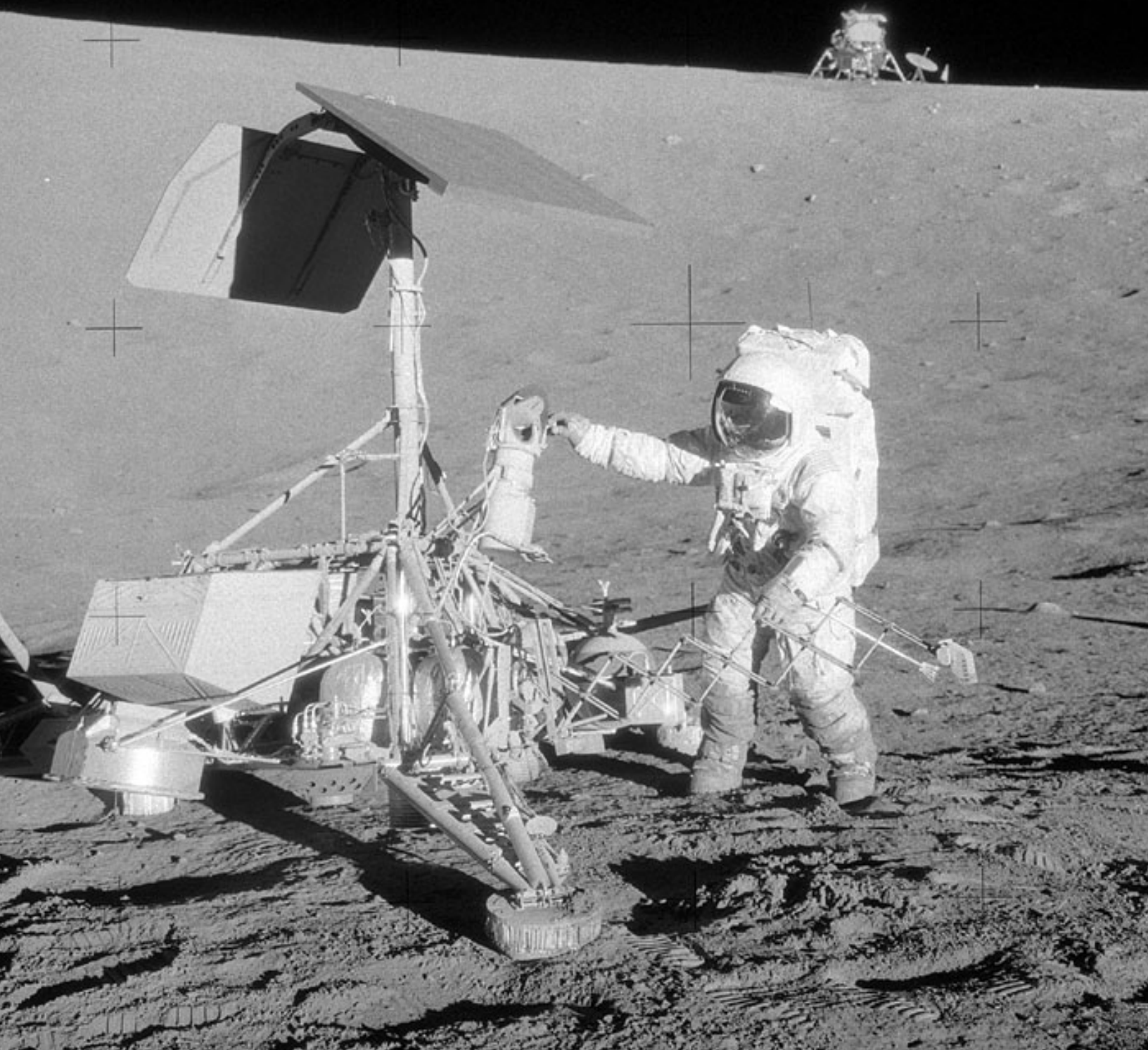


Physics Junior Independent Study
The College of Wooster
2012 May 6



Front cover

Two spacecraft on the moon: Surveyor 3 in the foreground and the Apollo 12 lunar module *Intrepid* in the background. Pete Conrad photographed Al Bean in November 1969.

Back Cover

The Lunar Reconnaissance Orbiter photographed the Apollo 12 landing site 40 years later in November 2009.

All of this required some physics.



Coulomb



Ampère



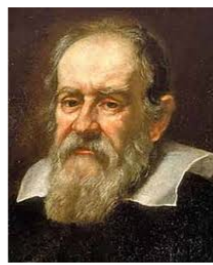
Hall



Feigenbaum



Stokes



Galileo



Noether



Duffing



Cavendish



Kelvin



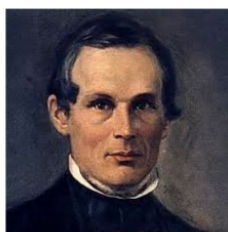
Gabor



Feynman



Ashkin



Ångström



Kamerlingh
Onnes



Stefan



Boltzmann



Brown

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Electromagnetism & Condensed Matter

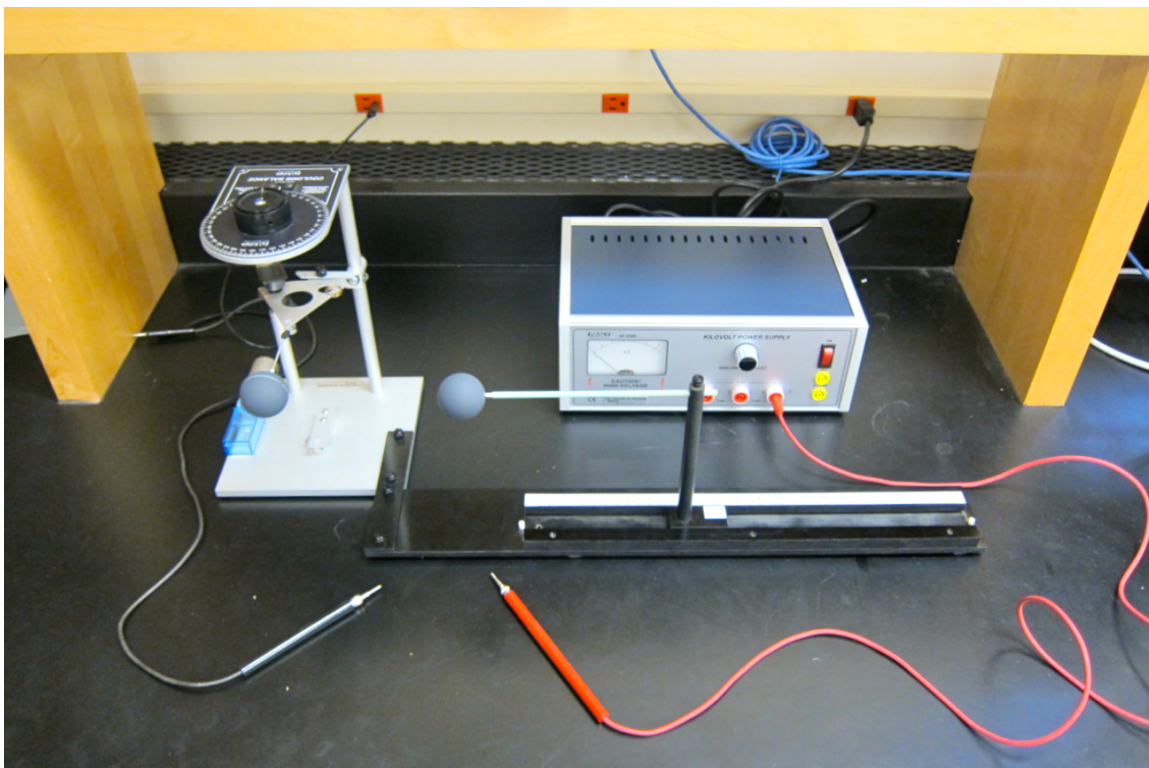


FIG. 1.1 Coulomb experiment equipment includes a PASCO Model EM-9070 Coulomb charge balance and a kilovolt power supply.



1. Coulomb: Charge Balance

Last updated 2012 February 25

Introduction

Augustin de Coulomb conducted the first recorded quantitative investigation [1] of the electrical force in 1784. Coulomb used a very sensitive torsion balance to measure the forces between two “point charges”, charged bodies whose dimensions are small compared to the distance between them. Consequently, the SI unit of charge is named for him.

Theory

Coulomb found that the force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The direction of the force on each particle is always along the line joining the two particles; pulling them together when the two charges are opposite, and pushing them apart when the charges are the same.

Mathematically, the vector electric force on a stationary point charge q separated by the vector displacement \vec{r} from a stationary charge Q is

$$\epsilon_0 \vec{F} = +q \frac{Q}{4 \pi r^2} \hat{r}, \quad (1.1)$$

where $\epsilon_0 = 8.85 \text{ TF/m} = 8.85 \text{ C}^2/\mu\text{m}^2\text{N}$ is the SI electric constant (and $r = |\vec{r}|$ and $\hat{r} = \vec{r}/r$ are the magnitude and direction of the separation).

In this experiment, measure the force between electrical charges by balancing electrical repulsion with gravitational attraction using the charge balance.

Equipment

The Fig. 1.1 PASCO Model ES-9070 Coulomb Balance is a delicate torsion balance that can be used to investigate the force between charged objects. A conductive sphere is mounted on a rod, counterbalanced, and suspended from a thin torsion wire. An identical sphere is mounted on a slide assembly so it can be positioned at various distances from the suspended sphere.

To perform the experiment, both spheres are charged, and the sphere on the slide assembly is placed at fixed distances from the equilibrium position of the suspended sphere. The electrostatic force between the spheres causes the torsion wire to twist. The experimenter then twists the torsion wire to bring the balance back to its equilibrium position. The angle through which the torsion wire must be twisted to reestablish equilibrium is directly proportional to the electrostatic force between the spheres.

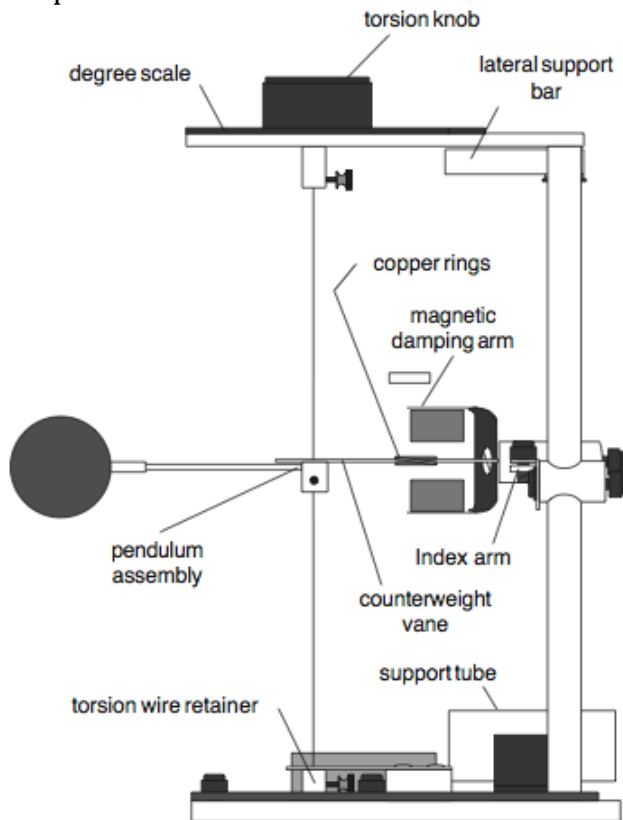


FIG. 1.2. Charge balance schematic diagram.

All the variables of the Eq. 1.1 Coulomb relationship can be varied and measured using the Coulomb Balance, shown schematically in Fig. 1.2. You can verify the inverse square relationship and the charge dependence using the balance and any electrostatic charging source. However, for best results, we recommend you charge the spheres with a stable kilovolt power supply to ensure a reproducible charge throughout the experiment. To determine the Coulomb constant with reasonable accuracy, we recommend you use an electrometer and a Faraday ice pail to accurately measure the charge on the spheres.

Additional recommended equipment include a stable kilovolt power supply for charging the spheres. Any electrostatic charger can be used to charge the spheres, but a power supply lets you replenish the charge to a fixed value throughout an experiment. Ideally the supply would have a momentary "power on" button so that you can conveniently turn it off whenever you are not charging the spheres. An

electrometer and Faraday ice pail for accurately measuring the charge on the spheres; spring balance capable of measuring a force of approximately 4 N (the weight of a 400 g mass). This is not necessary for the experiment itself, but is helpful in setting the tension of the torsion wire.

Tips

If you live in an area where humidity is always high, and if you have no facilities for controlling humidity, the experiment will be difficult, if not impossible, to perform. Static charges are very hard to maintain in a humid atmosphere because of surface conductivity. Experiments with the Coulomb Balance are straightforward and quite accurate, yet, as with any quantitative electrostatic experiment, frustration lurks just around the corner. A charged shirtsleeve, an open window, an excessively humid day – any of these and more can affect your experiment. However, if you carefully follow the tips listed below, you've got a good start toward a successful experiment.

Perform the experiment during the time of year when **humidity is lowest**. Perform the experiment in a **draft-free room**. The table on which you set up the experiment should be made of an **insulating material** – wood, Masonite, plastic, etc. If a metal table is used, image charges will arise in the table that will significantly affect the results. (This is also true for insulating materials, but the effect is significantly reduced.) Position the torsion balance at least two feet away from walls or other objects which could be charged or have a charge induced on them.

When performing experiments, stand directly behind the balance and at a maximum comfortable distance from it. This will minimize the effects of static charges that may collect on clothing. **Avoid wearing synthetic fabrics**, because they tend to acquire large static charges. Short sleeve cotton clothes are best, and a grounding wire connected to the experimenter is helpful.

Use a stable, regulated kilovolt power supply to charge the spheres. This will help ensure a constant charge throughout an experiment. When charging the spheres, turn the power supply on, charge the spheres, and then immediately turn the supply off. The high voltage at the terminals of the supply can cause leakage currents that will affect the torsion balance. A supply with a momentary “power on” button is ideal. When charging the spheres, **hold the charging probe near the end of the handle**, so your hand is as far from the sphere as possible. If your hand is too close to the sphere, it will have a capacitive effect, increasing the charge on the sphere for a given voltage. This effect should be minimized so the charge on the spheres can be accurately reproduced when recharging during the experiment.

If you are using a PASCO Electrometer to measure the charge on the spheres, connect the voltage output to a digital multimeter so that values can be measured more accurately. It is also useful to calibrate the electrometer by applying a calibrating voltage to the input and measuring the electrometer output on the digital

multimeter. Your measured values can then be adjusted as necessary.

Surface contamination on the rods that support the charged spheres can cause charge leakage. To prevent this, avoid handling these parts as much as possible and occasionally wipe them with alcohol to remove contamination. There will always be some charge leakage. **Perform measurements as quickly as possible** after charging, to minimize the leakage effects. Recharge the spheres before each measurement.

Procedure

You're now ready to experiment. The degree scale should read zero, the torsion balance should be zeroed (the index lines should be aligned), the spheres should be just touching, and the centimeter scale on the slide assembly should read 3.8 cm. (This means that the reading of the centimeter scale accurately reflects the distance between the centers of the two spheres.)

Force Versus Distance

Be sure the spheres are fully discharged (touch them with a grounded probe) and move the sliding sphere as far as possible from the suspended sphere. Set the torsion dial to 0 C. Zero the torsion balance by appropriately rotating the bottom torsion wire retainer until the pendulum assembly is at its zero displacement position as indicated by the index marks.

With the spheres still at maximum separation, charge both the spheres to a potential of 6-7 kV, using the charging probe. (One terminal of the power supply should be grounded.) Immediately after charging the spheres, turn the power supply off to avoid high voltage leakage effects. Position the sliding sphere at a separation of 20 cm. Adjust the torsion knob as necessary to balance the forces and bring the pendulum back to the zero position. Record the distance R and the angle θ . Separate the spheres to their maximum separation, recharge them to the same voltage, then reposition the sliding sphere at a separation of 20 cm. Measure the torsion angle and record your results again. Repeat this measurement several times, until your result is repeatable to within $\pm 1^\circ$. Record all your results and repeat for separations of 14, 10, 9, 8, 7, 6 and 5 cm.

Assume that force is proportional to the torsion angle. (You can separately test this assumption by calibrating the torsion balance.) To check the Eq. 1.1 force law, plot $\log \theta$ versus $\log R$ or plot θ versus $1/R^2$. Either of these methods will demonstrate that, for relatively large values of R , the force is inversely proportional to R^2 . For small values of R , however, this relationship does not hold.

The reason for the deviation from the inverse square relationship at short distances is that the **charged spheres are not simply point charges**. A charged conductive sphere, if it is isolated from other electrostatic influences, acts as a point charge. The charges distribute themselves evenly on the surface of the sphere, so that the center of the charge distribution is just at the center of the sphere. However, when two

charged spheres are separated by a distance that is not large compared to the size of the spheres, the charges will redistribute themselves on the spheres so as to minimize the electrostatic energy. The force between the spheres will therefore be less than it would be if the charged spheres were actual point charges. To correct for this deviation, reduce each angle by a factor

$$\frac{\theta'}{\theta} = \frac{1}{1 - 4(q/R)^3}, \quad (1.2)$$

where a is the radius of the spheres and R is the separation between spheres.

Force Versus Charge

With the sphere separation R held at a constant value (choose a value between 7 and 10 cm), charge the spheres to different values Q and measure the resulting force by the twist θ . Keep the charge on one sphere constant, and vary the charge on the other. Then plot θ versus Q to determine the relationship.

The charge can be varied using either of two methods. If your power supply is adjustable, simply charge the spheres to different potentials, such as 7, 6, 5, 4, and 3 kV. (When charging the spheres, they should always be at their maximum separation.) The charge on the sphere is proportional to the charging potential.

Alternately, if your power supply voltage is not adjustable, change the charge by touching one or both of the spheres with an identical sphere that is discharged. The charge will be shared equally between the charged and discharged sphere. Therefore, touch the charged sphere once to reduce the charge by half, twice to reduce the charge by 1/4, and so on.

Acknowledgments

These guidelines are based partly on the PASCO EM-9070 instruction manual.

References

[1] Augustin de Coulomb, "Premier Mémoire sur l'Electricité et le Magnétisme [First Memorandum on electricity and magnetism]", *Histoire de l'Académie Royale des Sciences* [History of the Royal Academy of Sciences], 569-577 (1785).

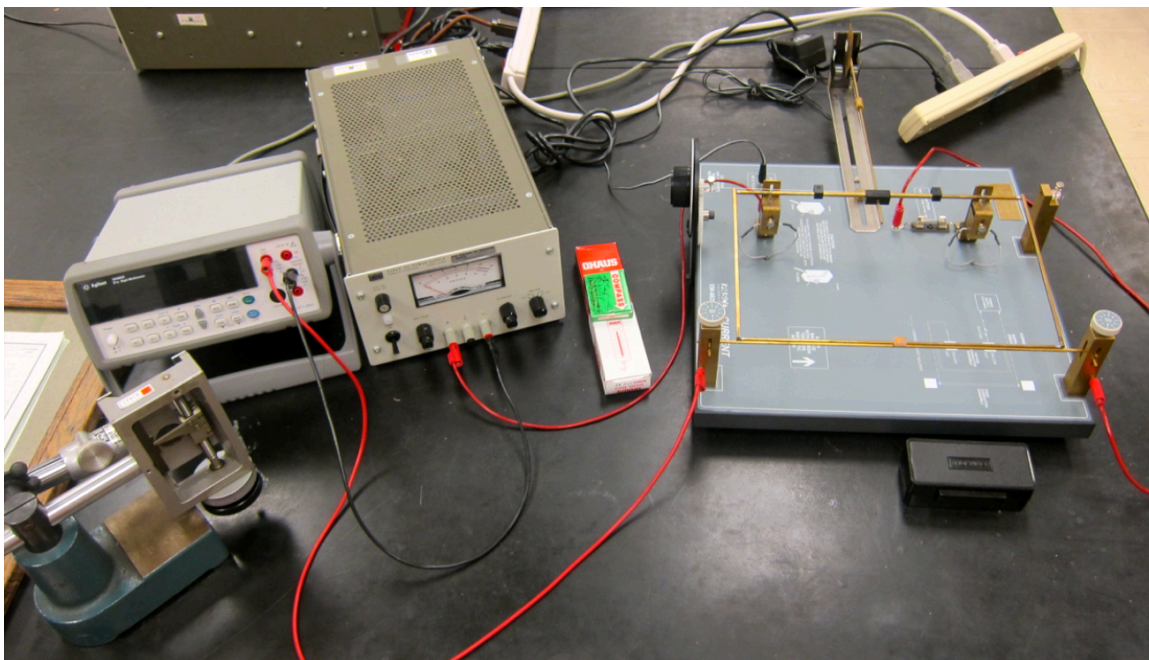


FIG. 2.1. Ampère experiment equipment includes a PASCO Model EM-8623 Ampère current balance, a power supply, a multimeter, and a traveling microscope.



2. Ampère: Current Balance

Last updated 2012 March 25

Introduction

André-Marie Ampère first systematically investigated [1] the magnetic forces associated with current-carrying wires in 1820. Consequently, the SI unit of current is named for him.

Theory

Ampère found that the force of attraction or repulsion between two lines of moving charges (or current-carrying wires) is directly proportional to the product of the currents and inversely proportional to the distance between them. While like charges repel and unlike charges attract, parallel currents attract and antiparallel currents repel. Also, while the electric force decreases inversely as the square of the distance, reflecting the spherical symmetry of a point charge, the magnetic force decreases inversely as the distance, reflecting the cylindrical symmetry of a wire.

Mathematically, the vector magnetic force per unit length on a stationary current i separated by a perpendicular separation s from a stationary current I is

$$\frac{\vec{F}}{\ell} = -\mu_0 i \frac{I}{2 \pi s} \hat{s}, \quad (2.1)$$

where $\mu_0 = 1.26 \mu\text{H/m} = 1.26 \mu\text{N/A}^2$ the SI magnetic constant. In this experiment, measure the magnetic force by balancing it against the gravitational force using the current balance.

While a current balance can be used to define the SI unit ampere, for historical reasons, the Eq. 2.1 force law is not called “Ampère's Law”; the former relates current to magnetic force, but the latter relates current to magnetic field.

Equipment

Using the Fig. 2.1 PASCO model EM-8623 current balance, you can measure the force of repulsion between identical oppositely directed currents in parallel conductors. It has three extremely helpful features. The **torsion wire suspension** system allows the operator to balance the force for any currents rather than using the usual standard masses that require that the current be adjusted to match the force applied. Any force can be obtained on the continuously adjustable dial that controls the amount of torque exerted by the wire.

Nontoxic molten gallium metal is used to make the **electrical contact reliably continuous** and nearly frictionless. This eliminates the traditional problem of intermittent contact of knife-edge connections. The gallium pots are heated electrically because gallium melts at about 30°C , although gallium will often remain in the super-cooled state for long periods of time and thus will remain liquid even below the melting temperature.

After initially setting the parallel conductor separation, the **separation can be easily and precisely changed** to any other value using the 1 mm pitch-adjusting screws to raise and lower the bottom conductor.

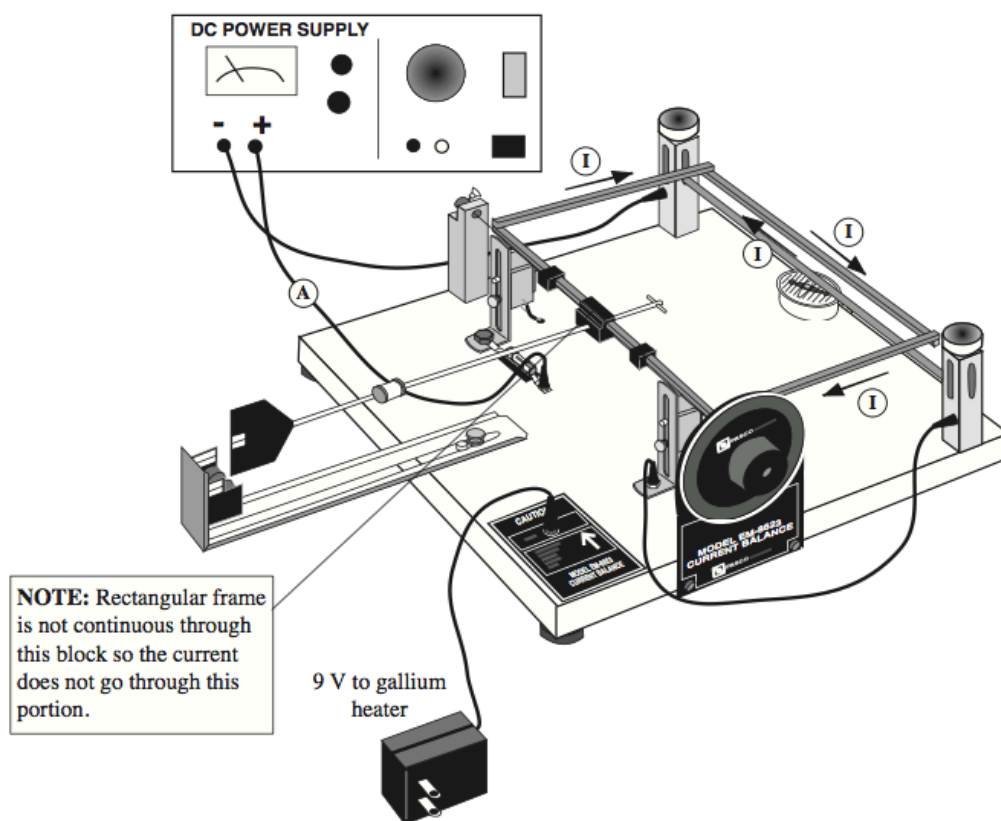


FIG. 2.2. Current balance wiring diagram.

Assembly

The following steps must be performed at the beginning of each experiment: **wiring**, **balancing** the rectangular frame so it is not touching the lower conductor and is supported only by the wire suspension, and **zeroing** the balance so the parallel conductors are a known distance apart which then can be used as a reference for all other desired separations.

Wiring

Connect the balance to a variable DC power supply as shown in Fig. 2.2 using banana plug lead wires. Use long lead wires and keep them as far away from the rectangular frame as possible (minimum distance 25 cm). This is so the magnetic field produced by the current in the lead wires will have a negligible effect on the balance.

Place the compass on the Current Balance base under the two parallel conductors. To eliminate the effect of the Earth's magnetic field, orient the parallel conductors in the magnetic N-S direction as indicated by the alignment of the compass needle. Remove any ferromagnetic materials from the vicinity of the Current Balance.

To eliminate the effects of all extraneous magnetic fields, bypass the fixed conductor and complete the current loop with a lead wire. Then orient the Current Balance until there is no deflection of the beam when a large current is turned on and off.

A 9V transformer is supplied with the Current Balance to power the gallium heater. To keep the gallium liquid, plug it into the jack. Raise and open the gallium pots and submerge the rectangular frame's supports into the pots.

Balancing

Turn the degree dial to zero degrees, making sure that the dial is in the center of its range by looking in back of the degree dial to see if the peg sticking through the large gear is halfway through the range of the gear slot. Rotate the rear wire-clamping thumbscrew so it is vertical.

Slide the counterbalance mass until the balance beam is horizontal. Make fine adjustments in balance by turning the rear wire thumbscrew slightly, which twists the back portion of the torsion wire. Because there is so little friction in the pivot, air currents may cause the balance to move. **Avoid drafts.**

Position the slide-able damping magnets so that when the balance beam is horizontal, the index reads zero (all three index lines line up).

Zeroing

In the following steps, calibrate the separation between the parallel conductors by first making the two conductors touch each other, at which point the separation is known. (The diameter of the conductor is 3.2 mm.) Then the additional separation is determined by keeping track of how many revolutions are made on the 1 mm pitch screw as the bottom conductor is lowered away from the top conductor. **To**

lower the conductor, turn the screws clockwise.

To make the bottom conductor parallel (level) with the top conductor, place a mass (200 mg) on the mass pan or twist the degree dial maximum clockwise to force the conductors together. Rotate the two separation adjustment screws alternately until there is no gap between the conductors on either end of the conductors.

Although the conductors are carefully selected to be straight, there may still be a slight bend in them. To minimize this error, insert a bent paper clip or needle in the small hole at the end of the bottom conductor and carefully rotate the conductor until the gap is eliminated or minimized.

Remove the mass from the pan or return the degree dial to the center zero position. Now the bottom conductor is parallel to the top conductor and the zero position can be determined by moving the bottom conductor up so it just barely touches the top conductor. Raise the bottom conductor by rotating the separation adjustment screws counterclockwise alternately one turn at a time until the bottom conductor just barely touches the top conductor. This should keep the conductors parallel as the bottom conductor is raised. When the conductors are just touching, the balance beam should still read zero. When this is complete, the separation between the two conductors is equal to one-rod diameter (or 3.2 mm).

If the bottom conductor cannot be raised enough by turning the separation screws, rebalance the top conductor so that it is slightly lower. This may require moving the counterbalance mass and the damping magnets.

Now whenever the balance is in the zero position, the center-to-center separation of the conductors is known to be 3.2 mm. Then any other desired separation can be known by keeping track of the number of rotations of the separation adjustment screws, which move the bottom conductor one millimeter for each complete rotation.

There is a circular scale on the top of each screw marked off in divisions of $1/20$ of a complete rotation. To keep track of the rotation of the screw, line up a corner of the square post (below the screw) with the scale. For example the number 4 is lined up with the outer corner of the square post, so one total rotation is complete when the 4 is once again lined up with the same corner and then the conductor has been raised or lowered 1 mm. Or, if it is desired to move the conductor only 0.5 mm, the screw can be rotated until the 9 is in the position formally occupied by the 4. Rotate both screws the same amount to maintain a parallel separation of the conductors.

Procedure

Fix the conductor separation and vary the current to check that the force is proportional to the current squared, $F \propto I^2$, according to Eq. 2.1. Fix the current and vary the separation to check that the force is inversely proportional to the

separation, $F \propto 1/s$, again according to Eq. 2.1. In each case, do a proportional fit to the data and determine the magnetic constant μ_0 .

Measure the distance between the parallel conductors using a **traveling microscope** with a precisely variable height. Aim the microscope at the top wire, move it to aim at the bottom wire, and record the distance the microscope moved. To ensure parallel wires, move the bottom wire upward until it just touches the top wire and then move it uniformly back down.

Place small masses on the mass pan to balance magnetic repulsion. After each measurement adjust the torsion wire to recalibrate the top wire to the desired distance above the bottom wire. This is necessary if removing the masses alters the equilibrium point after each measurement.

Acknowledgments

These guidelines are based partly on the PASCO EM-8623 instruction manual and on a report by Andrew Blaikie.

References

[1] André-Marie Ampère, "Mémoire de l'action mutuelle de deux courans électriques" [Memorandum of the mutual action of two currents], *Annales de Chimie et de Physique* [Annals of Chemistry and Physics], **15**, 59-76 (1820).

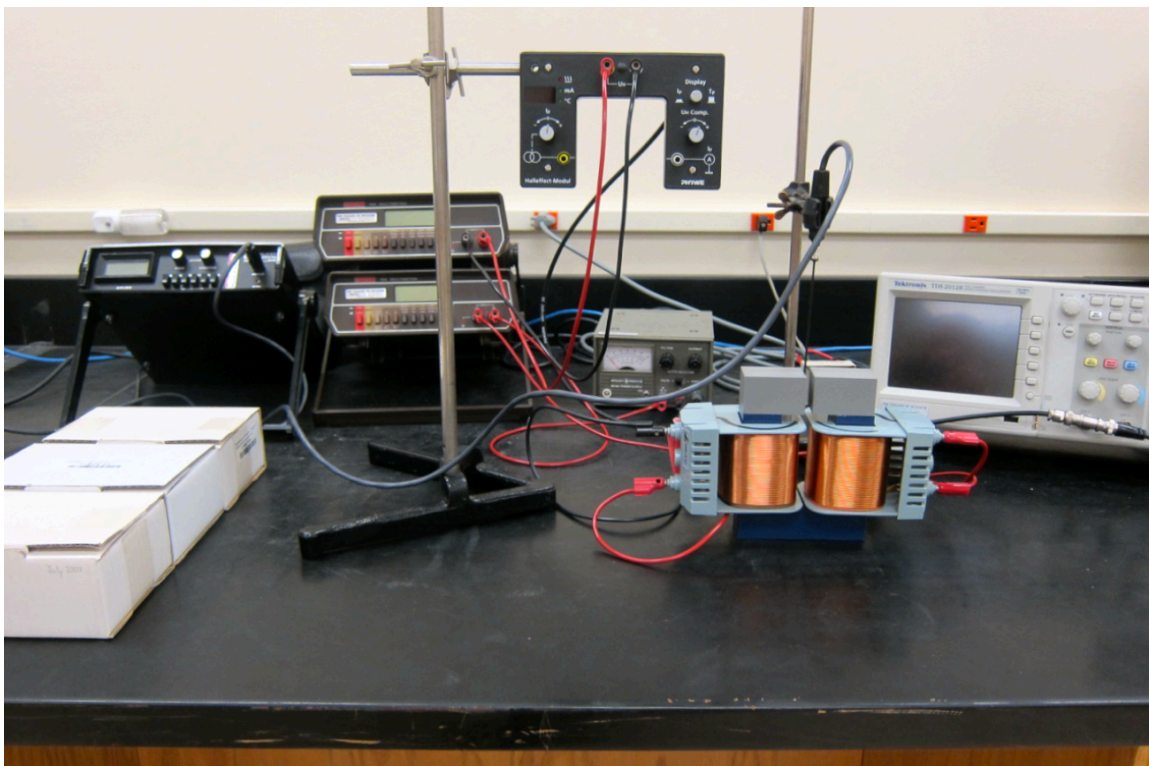


FIG. 3.1. Hall experiment equipment includes a PHYWE Systeme GmbH Hall effect apparatus an electromagnet, a teslameter, two digital multimeters, and doped Ge semiconductor samples.



3. Hall: Conductors in Magnetic Fields

Last updated 2012 February 13

Introduction

In 1879, Johns Hopkins University graduate student Edwin Hall used a magnetic field to deflect charges moving in a conductor and measured a transverse voltage [1]. Today, this *Hall effect* has wide applications, from probes that measure magnetic fields to thrusters that maneuver spacecraft.

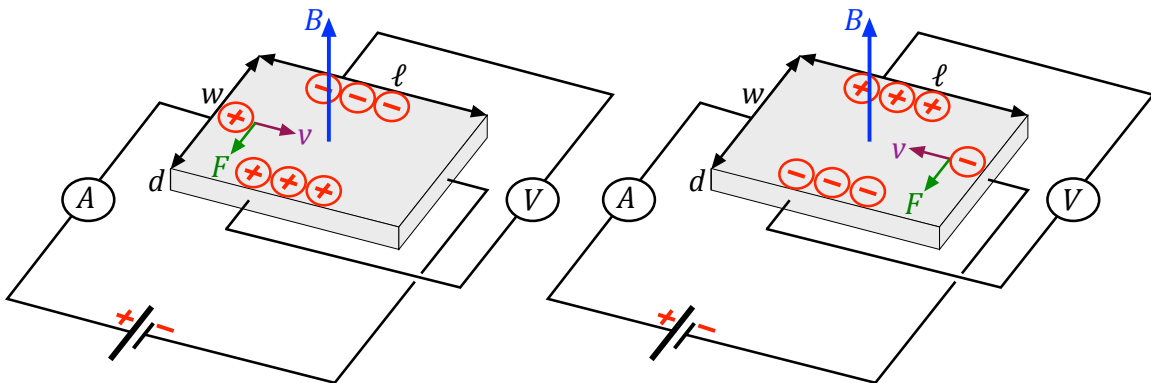


FIG. 3.2. Hall effect geometry for positive (left) and negative (right) charge carriers.

Theory

Consider a number density n of charges $e > 0$ moving with drift speed v along the length of a conduction slab of length ℓ , width w , and depth d , as in Fig. 3.2. A perpendicular magnetic field B deflects the charges creating a transverse Hall voltage

$$\Delta\phi = U_H = V \quad (3.1)$$

along the width of the slab. If the longitudinal current

$$I = \frac{Q}{t} = \frac{\rho V}{\ell/v} = \frac{(ne)(\ell wd)}{\ell/v} = nevwd, \quad (3.2)$$

then the equilibrium electric and magnetic forces balance,

$$e \frac{V}{w} = eE = evB = \left(\frac{I}{nwd} \right) B = \frac{IB}{nwd}, \quad (3.3)$$

creating a Hall voltage of magnitude

$$V = \frac{IB}{ned}. \quad (3.4)$$

The *sign* of the voltage depends on the sign of the charge carriers.

Equipment

Using the Fig. 3.1 PHYWE (pronounced **fie**-vuh) Systeme GmbH Hall effect apparatus, you can measure both the sign and the density of the charge carriers in the germanium semiconductor samples. The Ge is doped by adding impurities, which can alter the sign of the charge carries. In addition, you can study of the effects of temperature on the Hall effect.

The test piece on the board has to be put into the Hall-effect-module via the guide-groove. The module is directly connected with the 12 V output of the power unit over the ac-input on the back- side of the module.

The plate has to be brought up to the magnet very carefully, so as not to damage the crystal in particular, avoid bending the plate. The Hall voltage and the voltage across the sample are measured with a multimeter. Therefore, use the sockets on the front-side of the module. The current and temperature can be easily read on the integrated display of the module.

The magnetic field has to be measured with the teslameter via a Hall probe, which can be directly inserted into the groove in the module. This way you can be sure that the magnetic flux is measured directly on the Ge-sample.

Procedure

Set the magnetic field to a value of 250 mT by changing the voltage and current on the electromagnetic power supply. Connect the multimeter to the sockets of the Hall voltage U_H on the front-side of the module. Set the display on the module into the “current-mode”. Determine the Hall voltage as a function of the sample current from -30 mA up to 30 mA in steps of nearly 5 mA. **The module must be removed from the magnetic field and re-zeroed after every measurement.**

Set the control current to 30 mA. Connect the multimeter to the sockets of the sample voltage on the front-side of the module. Determine the sample voltage as a function of the positive magnetic field B up to 300 mT.

Be sure, that the display works in the temperature mode during the measurement. At the beginning set the current to a value of 30 mA. The magnetic field is off. The current remains nearly constant during the measurement, but the voltage changes according to a change in temperature. Set the display in the temperature mode, now. Start the measurement by activating the heating coil with the “on/off”-knob on the backside of the module.

Determine the change in voltage dependent on the change in temperature for a temperature range of room temperature to a maximum of 170°C.

Set the current to a value of 30 mA. Connect the multimeter to the sockets of the Hall voltage U_H on the front-side of the module. Determine the Hall voltage as a function of the magnetic field B . Start with -300 mT by changing the polarity of the coil-current and increase the magnetic field in steps of nearly 20 mT. At zero point, you have to change the polarity.

Set the current to 30 mA and the magnetic field to 300 mT. Determine the Hall voltage as a function of temperature. Set the display in temperature mode. Start the measurement by activating the heating coil with the “on/off”- knob on the backside of the module.

Repeat all measurements for the n-doped, p-doped, and un-doped Ge samples.

Acknowledgments

These guidelines are based partly on the PHYWE Systeme GmBH 5.3.01-01 Hall effect instruction manual and the report of Duncan Price.

References

[1] Edwin Herbert Hall, “On the New Action of the Magnet on Electric Currents”, *American Journal of Mathematics*, **2**, 287-292 (1879).

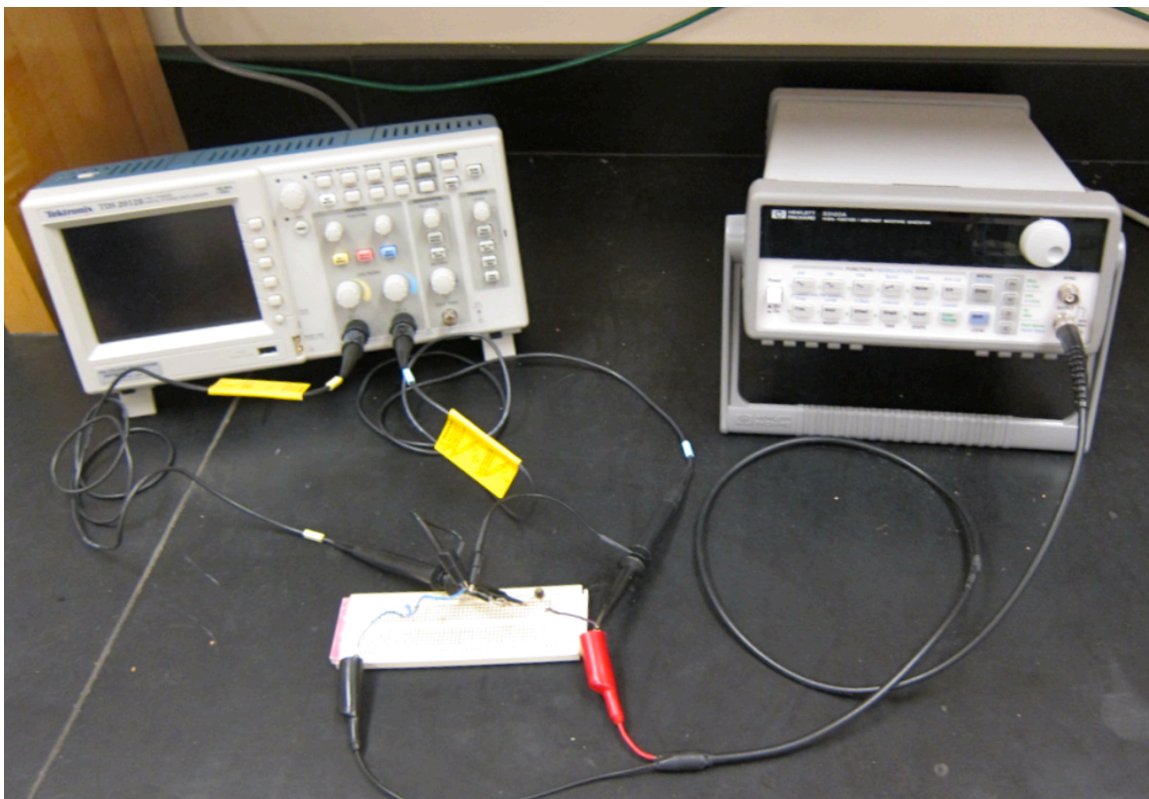


FIG. 4.1. Feigenbaum experiment equipment includes a nonlinear circuit, a waveform generator, and an oscilloscope.



4. Feigenbaum: Electronic Circuit Chaos

Last updated 2012 March 24

Introduction

In 1975, Mitchell Feigenbaum noticed that the structure and behavior of a wide class of nonlinear systems are governed by universal constants [1]. Such constants have since been measured in many physical systems, including electronic circuits [2-5], and are called Feigenbaum constants.

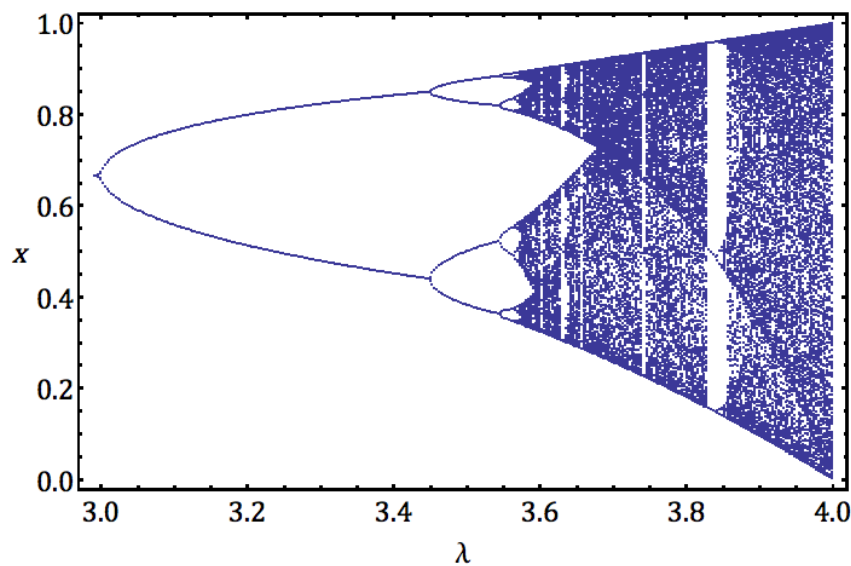


FIG. 4.2. Logistics map bifurcation diagram created with *Mathematica*.

Theory

The logistics map is a simple but nonlinear population model assuming linear birth and quadratic death. If x_n is the normalized population at generation n , then

$$x_{n+1} = \lambda x_n (1 - x_n). \quad (4.1)$$

The normalized population exhibits an extraordinary range of subtle behavior as the parameter λ varies, as indicated by Fig. 4.2, which plots the steady state population as a function of λ .

Feigenbaum showed that the map undergoes a period-doubling route to chaos, where each successive bifurcation occurs geometrically faster as λ increases. If the bifurcations occur at λ_b , then

$$\delta = \lim_{b \rightarrow \infty} \frac{\lambda_{b+1} - \lambda_b}{\lambda_{b+2} - \lambda_{b+1}} = 4.669201609102990671853203820466 \dots \quad (4.2)$$

The period doubling route to chaos can be observed and Feigenbaum's δ can be measured using a simple resistor, inductor, and diode – an RLD circuit. A diode is a non-linear circuit element that can cause a simple series connection of these three elements to exhibit a period doubling route to chaos. A signal generator producing a sine wave drives the circuit and the measured voltage across the diode (or diode and inductor) changes from periodic to chaotic as the amplitude of the drive voltage increases.

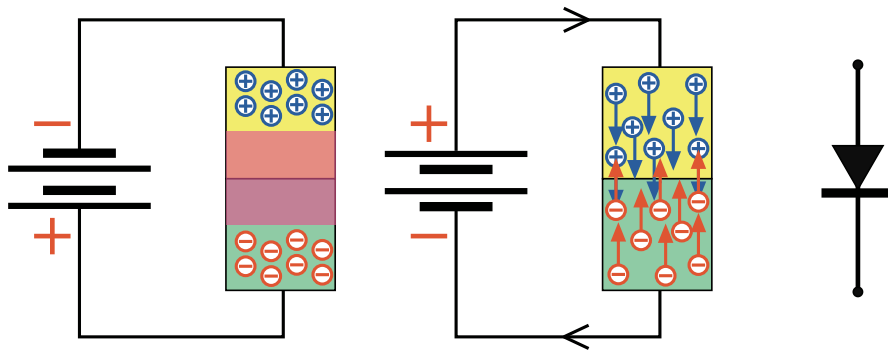


FIG. 4.3. An ideal diode allows current to flow in only one direction.

A simple **diode** consists of two **semiconductors** contaminated or **doped** with other atoms so that the **N-type** semiconductor has excess negative charges or **electrons** and the **P-type** semiconductor has a deficit of negative charges or an excess of positive **holes**. A battery or other voltage source connected one way attracts the positive holes to its negative terminal and attracts the negative electrons to its positive terminal, and no current flows across the **junction** between the semiconductors, as on the left of Fig. 4.3. However, a battery connected the other way repels positive holes from its positive terminal and repels negative electrons from its negative terminal, and the electrons fill the holes at the junction allowing current to flow, as on the right of Fig. 4.3, with new electrons and holes appearing to replace the annihilated ones. (For light emitting diodes or **LEDs** the annihilation of the electrons and holes produces visible light.)

Nonlinearities in practical diodes involve conductance, voltage-dependent capacitance, and reverse recovery times. Model a diode as a parallel connection of a

nonlinear, voltage-dependent resistor and capacitor. If the impedance of the capacitor is *smaller* than that of the resistor, neglect the resistor. The simplest such model [6] that results in a period doubling route chaos assumes a reverse diode capacitance C_r , a forward diode capacitance C_f , and a nonlinear voltage relation

$$V_D[Q] = V_Q + \frac{C_r - C_f}{2C_f C_r} |Q| + \frac{C_r + C_f}{2C_f C_r} Q \quad (4.3)$$

that obeys Kirchoff's loop rule

$$0 = V_0 \sin[2\pi f t] - L \frac{dI}{dt} - IR - V_D[Q], \quad (4.4)$$

where the current $I = dQ/dt$.

Equipment

Assemble a diode-inductor circuit like either of those in Fig. 4.4. Drive the circuit sinusoidally with the signal generator and plot the voltages on the oscilloscope. However, don't leave the apparatus on for too long, else it will grow hot and adversely affect the data.

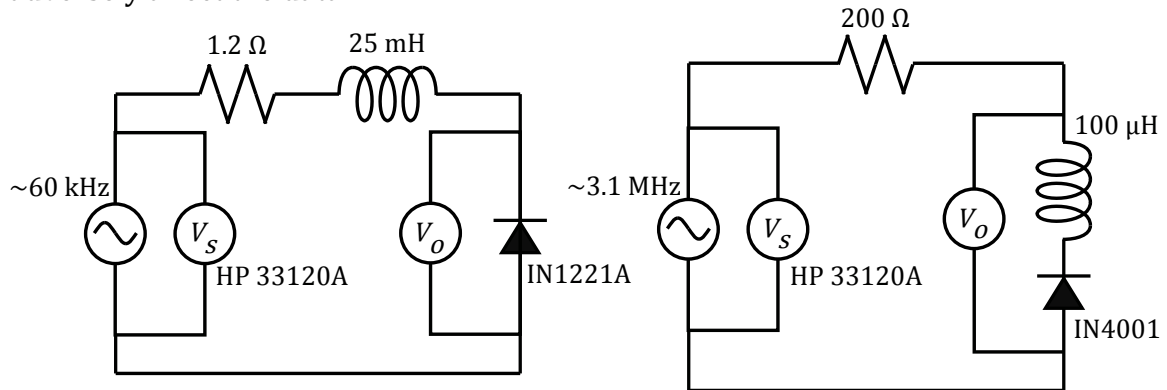


FIG. 4.4. Two driven nonlinear RLD circuits.

Procedure

Plot voltage V_0 versus time t and also voltage V_0 versus voltage V_s , as in Fig. 4.5, while slowly varying the drive amplitude. How many bifurcations can you observe? Can you find a period-3 window? What happens if you also vary the frequency f ?

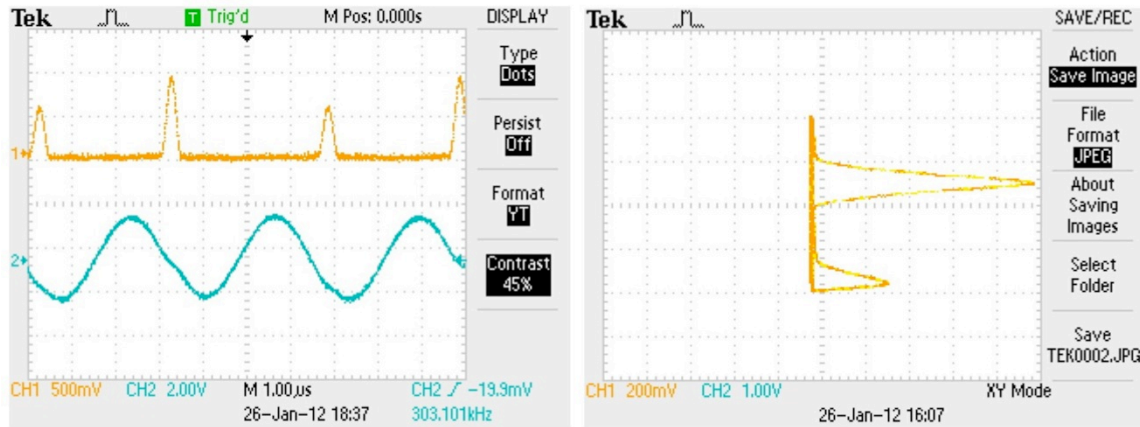


FIG. 4.5. Waveform and phase space oscilloscope traces showing period-2 behavior.

Analysis

Use Eq. 4.2 to estimate Feigenbaum's δ from the first few circuit bifurcations.

Use *Mathematica*'s `NDSolve[]` function to numerically integrate Eqns. 4.3 and 4.4 for a range of amplitudes V_0 . Begin with parameters $V_0 = 0.1801\text{V}$, $f = 700\text{ kHz}$, and $L = 100\text{ }\mu\text{H}$, $R = 100\text{ }\Omega$, and $V_Q = 0.1\text{ V}$, $C_f = 0.1\text{ }\mu\text{F}$, $C_r = 0.4\text{ nF}$. Compare with the experiment.

Acknowledgments

These guidelines are based partly on reports by Karl Smith and Tom Gilliss.

References

- [1] Mitchell Jay Feigenbaum, "Quantitative universality for a class of nonlinear transformations", *Journ. Stat. Phys.*, **19**, 25-52 (1978).
- [2] P. S. Linsay "Period doubling and chaotic behavior in a driven anharmonic oscillator," *Phys. Rev. Lett.* **49**, 1349 (1981).
- [3] T. Mishina, T. Kohmoto, and T. Hashi, "Simple electronic circuit for the demonstration of chaotic phenomena", *Am. J. Phys.* **53**, 332 (1985).
- [4] Thomas P. Weldon, "An inductorless double scroll chaotic circuit", *Am. J. Phys.* **58**, 936-941 (1990).
- [5] R. W. Rollins and E. R. Hunt, "Exactly Solvable Model of a Physical System Exhibiting Universal Chaotic Behavior," *Phys. Rev. Lett.* **49**, 1295 (1982).
- [6] T. Matsumoto, L. O. Chua, S. Tanaka, "Simplest chaotic nonautonomous circuit", *Phys. Rev. A* **30**, 1155-1158 (1984).

Mechanics

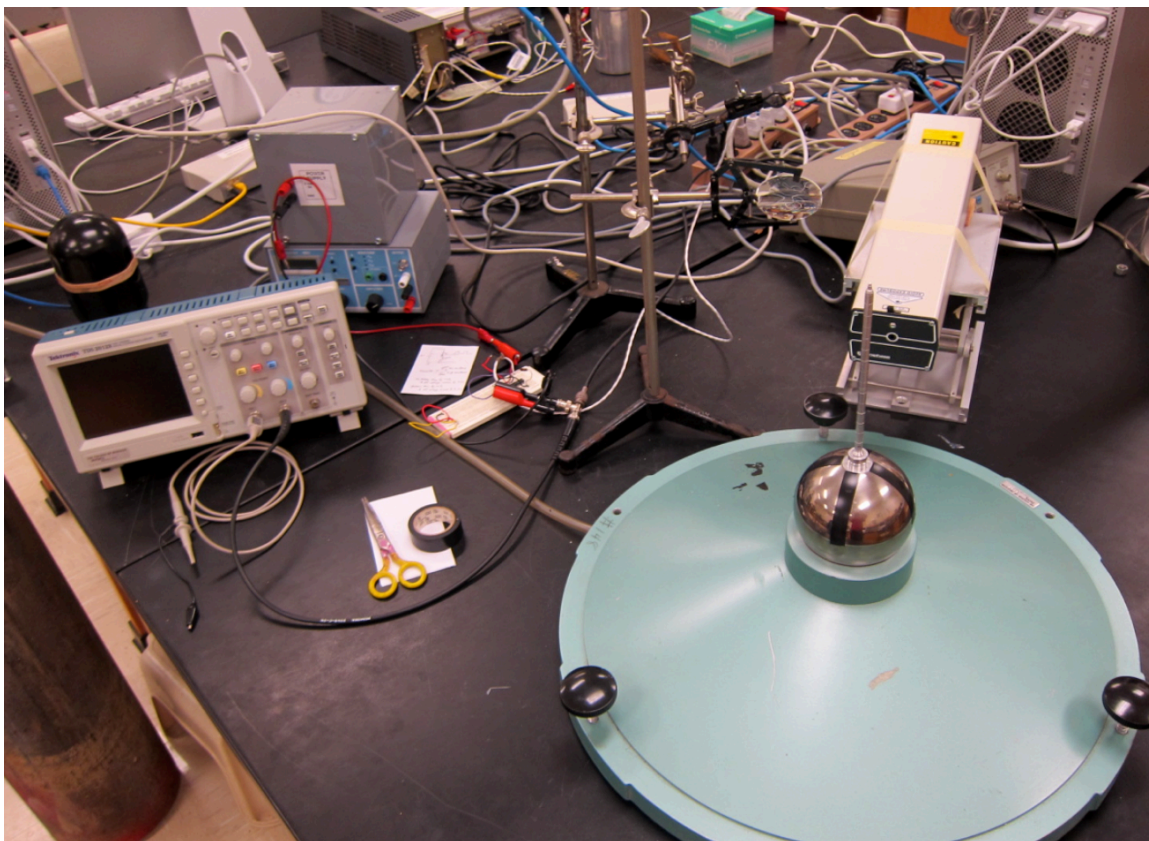


FIG. 5.1. Stokes experiment equipment includes an Ealing Air Gyroscope, a laser, a photodiode, a power supply, a Schmitt trigger circuit and counter, a nitrogen tank, and a LabVIEW computer interface.



5. Stokes: Rotational Viscous Drag

Last updated 2012 March 25

Introduction

As an object moves through a fluid, the viscosity of the fluid acts on the moving object with a force that resists the motion of the object. Two common approaches for modeling this resistive force are due to Stokes and Newton. In 1851, Sir George Gabriel Stokes published equations of viscous flow [1]. In particular, Stokes determined the resistive force, or viscous drag force, of a sphere falling under the force of gravity in a fluid, either liquid or gas, to be directly proportional to the sphere's velocity. Stokes model most accurately describes objects moving linearly in a fluid that moves with *laminar* or steady fluid flow.

However, Isaac Newton had previously argued that the drag force is proportional to the *square* of the speed of the object and acts in the directions opposite to the direction of the velocity [2]. Today Newton's model is associated with higher velocities and *turbulent*, or non-steady, fluid flow. This experiment investigates the relationship between frictional *torque* and *angular velocity*.

Theory

For Stokes' laminar flow, the frictional torque τ_z on an object of rotational inertia I_{zz} rotating about the fixed z-axis with angular velocity component ω_z can be modeled

$$I_{zz} \frac{d\omega_z}{dt} = I_{zz} \alpha_z = \tau_z = -k_S \omega_z . \quad (5.1)$$

Solve this differential equation by separating variables to find

$$\omega_z = \omega_0 e^{-t/t_S}, \quad (5.2)$$

where the decay time $t_S = I_{zz}/k_S$. For Newton's turbulent flow

$$I_{zz} \frac{d\omega_z}{dt} = I_{zz} \alpha_z = \tau_z = -k_N \omega_z |\omega_z| \quad (5.3)$$

implies

$$\omega_z = \frac{\omega_0}{1 + t/t_N}, \quad (5.4)$$

where the decay time $t_N = I_{zz}/\omega_0 k_N$. For a combination of laminar and turbulent flow

$$\omega_z = \frac{\omega_0}{e^{t/t_S} + (1 - e^{t/t_S})t_S/t_N}. \quad (5.5)$$

Equipment

This experiment uses the Fig. 5.1 Ealing air gyroscope. The steel rotor ball of the gyroscope is the rotating mass that is studied. The angular velocity, or frequency of rotation, of the rotor ball is determined using a laser. Black strips of electrical tape are placed on the upper portion of the shiny rotor, and a laser is aligned so that it reflects off this portion of the rotor. Then the laser beam is focused through a lens onto the tiny pinpoint head of a fast photodiode.

The photodiode circuitry of Fig. 5.2 acts like a voltage divider. When the laser light reflects onto the photodiode (or phototransistor), the current supplied by the power supply easily passes through the phototransistor, causing a high voltage reading to pass through the Schmitt trigger. But, when no light is incident on the phototransistor, little current makes it to the resistor, sending a lower voltage reading to the Schmitt trigger. The Schmitt trigger then shapes the voltage signals into a sharp square wave, which can then be read by a Hewlett Packard frequency counter. In addition, the frequency counter is connected to a Macintosh computer with a GPIB cable, in order for a LabVIEW algorithm to import the data acquired through the frequency counter.

Finally, a tube connects a tank of compressed nitrogen gas to the air gyroscope apparatus. Adjust the pressure of the nitrogen gas coming out of the tank to a constant 12 psi throughout the experiment, so that the rotor “sits” on a cushion of gas.

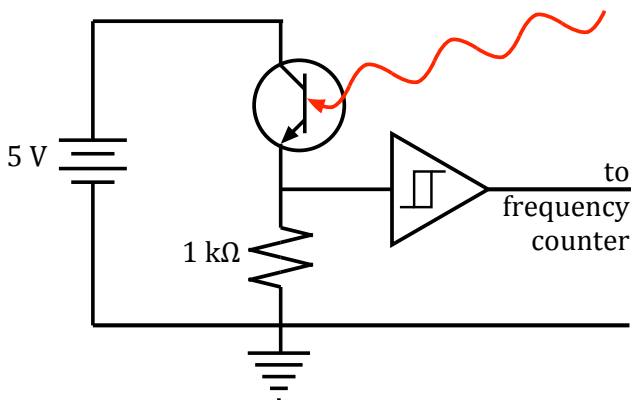


FIG. 5.2. Photodiode circuitry.

Procedure

Use the LabVIEW program “Viscous Torque” to set the controls for the frequency counter and to import data from the counter. Spin the rotor by hand to attain the highest possible initial angular speed. Then, as the rotor spins, the LabVIEW program calculates the average frequency $f = 2\pi/\omega$ in hertz over 10.0 second intervals and records a table of values for f and the corresponding time to a file you designate.

To more thoroughly investigate the relationship between frictional torque and angular velocity, alter the rotor by adding surface area to the rod, which causes the rotation to be damped more quickly. For example, use rectangular pieces of Styrofoam board, each measuring (say) 0.5 cm thick by 3.0 cm wide by 6.0 cm long, for the additional area. In order to help keep the rod upright as it rotates, add two rectangular pieces opposite one another (like wings spanning approximately 12 cm) to the rod of the rotor for each run.

Begin your analysis by creating a semi-log plot of the gyroscope’s angular average ω as a function of time t . Is the Eq. 5.2 Stokes model always valid? Perform nonlinear fits of the models to the data. Which fit has the smallest reduced chi-squared χ_r^2 ?

Acknowledgments

These guidelines are based partly on a report by Manon E. Grugel.

References

- [1] George Gabriel Stokes, “On the effect of the internal friction of fluids on the motion of pendulums”, *Transactions of the Cambridge Philosophical Society*, **9**, 8-106, Eq. 126 (1851).
- [2] Isaac Newton, *Philosophiae Naturalis Principia Mathematica* [*Mathematical Principles of Natural Philosophy*], Book 2 (Joseph Streater for the Royal Society, 1687).

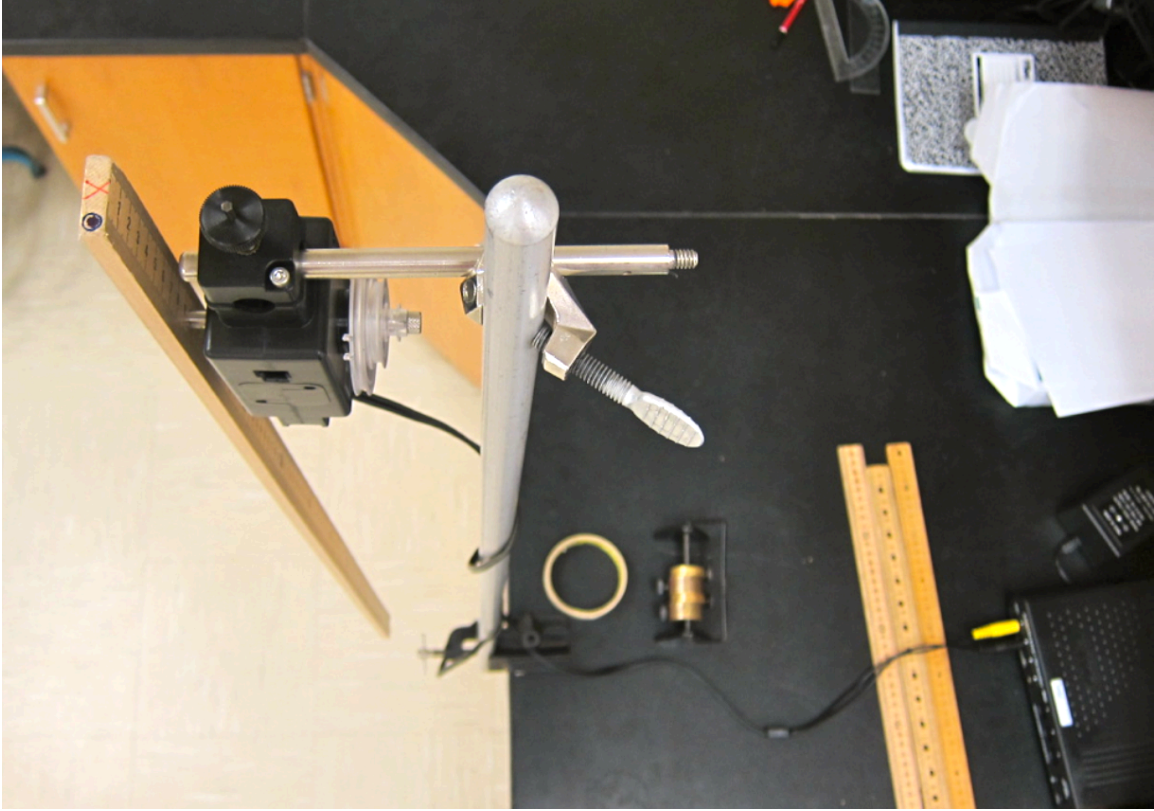
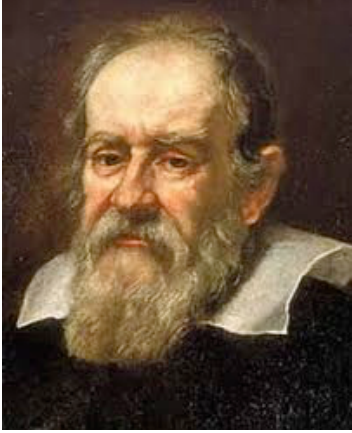


FIG. 6.1. Galileo experiment equipment includes a physical pendulum, sliding masses, a PASCO rotary motion sensor, and a ScienceWorkshop DataStudio computer interface.



6. Galileo: Physical Pendulum

Last updated 2012 February 14

Introduction

Galileo Galilei first realized that the period of a pendulum was independent of its amplitude, provided the amplitude was small [1]. This became the basis of the pendulum clock [2]. The mass of a *simple* pendulum is concentrated at one point, the pendulum bob, while the mass of a *physical* pendulum is distributed throughout its length.

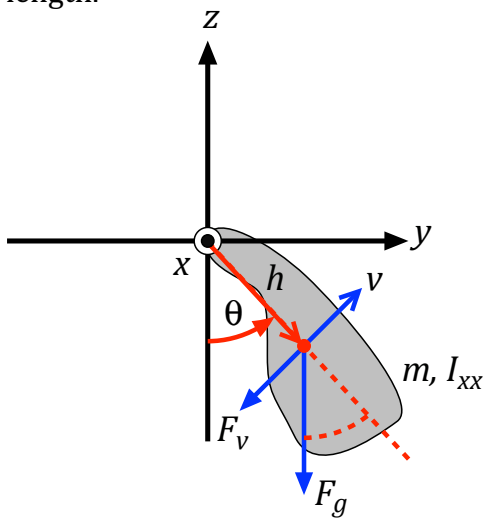


FIG. 6.2. Generic physical pendulum.

Theory

Suppose that a physical pendulum of mass m and rotational inertia

$$I_{xx} = \int (y^2 + z^2) dm \quad (6.1)$$

swings in the yz -plane and makes an angle θ with respect to the negative z -axis, as in Fig. 6.2. If a gravitational force $F_g = mgh$ and a viscous damping force $F_v = -b v$

act on the pendulum's center of mass a distance h from the pivot, then summing the torques in the x -direction implies

$$I_{xx} \frac{d^2\theta}{dt^2} = I_{xx}\alpha = \tau_x = -hb \left(h \frac{d\theta}{dt} \right) - mgh \sin \theta \quad (6.2)$$

or

$$I_{xx} \frac{d^2\theta}{dt^2} + c \frac{d\theta}{dt} + k \sin \theta = 0, \quad (6.3)$$

where $c = bh^2$ and $k = mgh$. For small angles $\theta \ll 1$ and small damping $c^2 < 4kI_{xx}$, the solution is the damped sinusoid

$$\theta = \theta_0 e^{-\gamma t} \cos[\omega' t - \varphi], \quad (6.4)$$

where the reduced frequency $\omega' = \sqrt{\omega_0^2 - \gamma^2}$, the natural frequency $\omega_0 = \sqrt{k/I_{xx}}$, the damping $\gamma = c/2I_{xx}$, and the amplitude θ_0 and phase φ depend on the initial conditions.

For large angles, the period of the undamped pendulum is

$$T = T_0 \left(1 + \frac{1}{16} \theta_0^2 + \frac{11}{3072} \theta_0^4 + \dots \right). \quad (6.5)$$

Equipment

This experiment uses the Fig. 6.1 physical pendulum, consisting of a meter stick and an adjustable mass, with a PASCO rotary motion sensor, and a ScienceWorkshop DataStudio computer interface.

Procedure

Determine the total mass m of the physical pendulum. Secure the sliding mass to a position 80 cm from the point of support. Balance the pendulum horizontally on a knife edge and record the distance h of the center of mass from the point of support.

Mount the pendulum on the potentiometer shaft, start it swinging with moderate amplitude, then start the computer data-recording program. From output, graph the displacement from the equilibrium position for the first two oscillations. Determine the period of oscillation from this graph. Repeat with a larger initial displacement. What effect does the initial displacement have on the period?

Calculate the moment of inertia I_{xx} for the pendulum using the parallel axis theorem, idealizing the sliding mass as a point. From the moment of inertia and the period of the pendulum find a value for the gravitational field g and compare your result to the accepted value.

Find the value of the damping constant b by plotting the natural logarithms of the maximum and minimum deviations of the pendulum from the equilibrium position versus time. Perform a least-squares fit to quantify the amplitude decay.

Acknowledgments

These guidelines were inspired in part by the work of Andrew Blaikie.

References

- [1] Galileo Galilei, *Dialogo sopra i due massimi sistemi del mondo [Dialogue Concerning the Two Chief World Systems]* (Landini, 1632).
- [2] Gregory L. Baker, James A. Blackburn, *The Pendulum: A Case Study in Physics* (Oxford University Press, 2005).

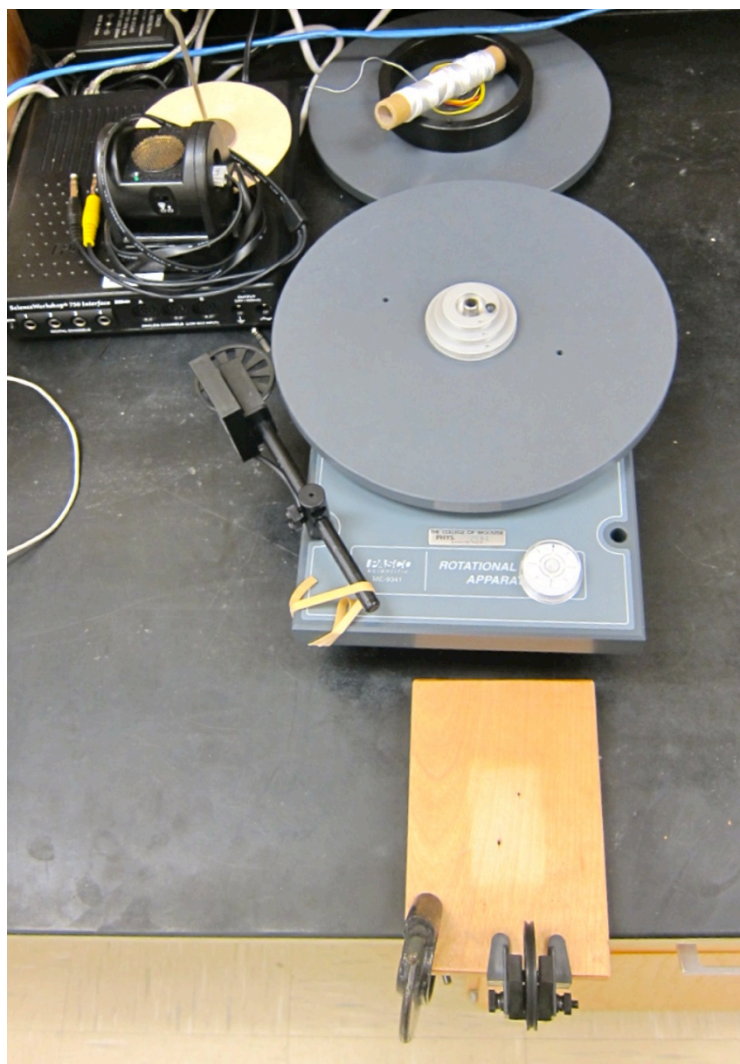


FIG. 7.1. Noether experiment equipment includes a PASCO rotational motion apparatus, a smart pulley, a motion sensor, and a ScienceWorkshop DataStudio computer interface.



7. Noether: Angular Momentum Conservation

Last updated 2012 March 25

Introduction

In 1915 Amalie Emmy Noether proved [1] that under Lagrangian mechanics, each symmetry of a physical system corresponds to a conservation law, as in Table 7.1. Something is *symmetric* if it is *invariant* under a *transformation*, and the physically relevant transformations are the space and time translations and space and spacetime rotations (or boosts) of the *Poincaré group*. As a partial example, if gravity we're weaker on Tuesdays than on other days in the week, one could violate energy conservation by pumping water uphill on Tuesdays and allowing the water to fall back downhill the rest of the week, extracting energy with a water wheel.

Table 7.1. Symmetries and conservation laws linked by Noether's theorem.

| Symmetry | Conserved Quantity |
|--------------------------|--------------------------|
| Time translation | Energy |
| Space translation | Linear momentum |
| Rotations | Angular momentum |
| Boosts | Initial center of energy |

Theory

The rotational form of Newton's second law equates torque and the time rate of change of angular momentum. For motion about a fixed axis

$$\frac{d\vec{L}}{dt} = \vec{\tau} = I\vec{\alpha} = I \frac{d\vec{\omega}}{dt}, \quad (7.1)$$

where the torque

$$\vec{\tau} = \vec{r} \times \vec{F} \quad (7.2)$$

and the angular momentum

$$\vec{L} = \sum \vec{r} \times \vec{p} = \int \vec{r} \times dm\vec{v} = I\vec{\omega} \quad (7.3)$$

and, for a uniform disk of radius R , the rotational inertia

$$I = \int r^2 dm = \frac{1}{2} mR^2. \quad (7.4)$$

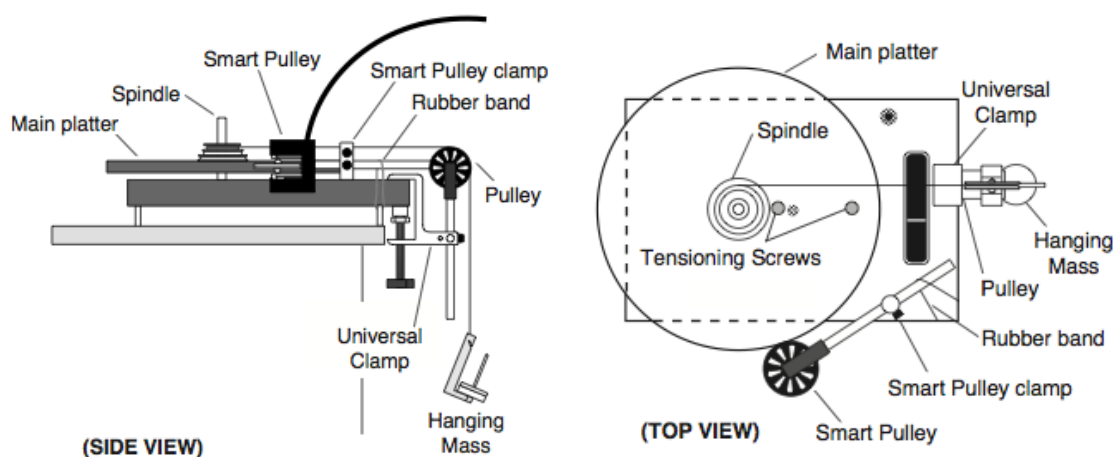


FIG. 7.2. Measuring the rotational inertia of the platters.

Equipment

This experiment uses the Fig. 7.1 PASCO Model ES-9070 Rotational Apparatus, ScienceWorkshop DataStudio computer interface. A platter spins on low-friction ball bearing supports. Torques can be applied to the platter with various moment arms and forces, and the angular velocity of the platter can be monitored over time to determine angular acceleration. Adding (by dropping) a second platter, a steel ring, or a steel block can vary the moment of inertia of the rotating system.

The Smart Pulley connects to the computer interface, providing automatic data collection, analysis, and graphing. As the platter spins, the pulley is held lightly against the rim of the platter, and therefore spins with the platter. The Smart Pulley photogate monitors the rotation of the pulley, sending its signals to the computer for timing and analysis.

Procedure

Perform an initial experiment, as in Fig. 7.2, to determine the rotational inertia of the platter(s). Then drop a second platter onto the spinning first platter and check for conservation of angular momentum just before and just after the collision.

Acknowledgments

These guidelines are based partly on the PASCO ME-9341 instruction manual.

References

[1] Emmy Noether, "Invariante Variationsprobleme [Invariant Variation Problems]". *Nachr. d. König. Gesellsch. d. Wiss. zu Göttingen, Math-phys. Klasse*, 235–257 (1918).

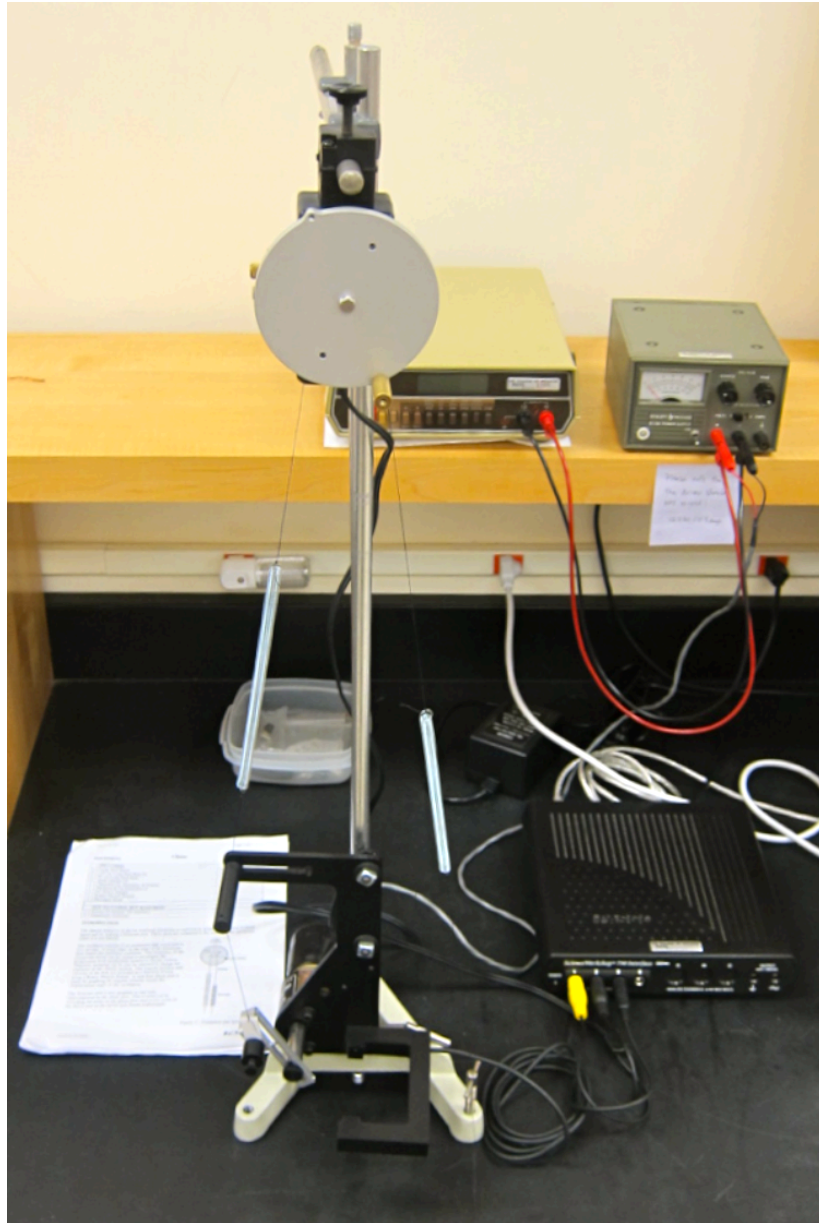
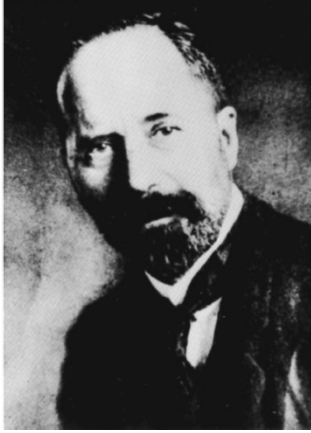


FIG. 8.1. Duffing experiment equipment includes a PASCO oscillator, a mechanical driver with power supply, a multimeter, rotary motion and photogate sensors, and a ScienceWorkshop DataStudio computer interface.



8. Duffing: Mechanical Oscillator Chaos

Last updated 2012 January 29

Introduction

In 1918, Georg Duffing published [1] an analysis of the forced, damped, nonlinear oscillator. The Duffing oscillator can be imagined as a point particle moving in rocking bistable potential. This paradigm for nonlinear dynamics can manifest both regular and *chaotic* motion, including a *strange attractor* [2] of fractional dimension. In this experiment, investigate a mechanical realization of the Duffing oscillator.

Theory

If a unit mass moves in bistable potential

$$V = - \int F dx = -\alpha \frac{x^2}{2} + \beta \frac{x^4}{4}, \quad (8.1)$$

with linear and nonlinear elastic constants $\alpha > 0$ and $\beta > 0$, corresponding to a force

$$F = -\frac{dV}{dx} = \alpha x - \beta x^3, \quad (8.2)$$

Newton's laws imply the differential equation of motion

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} = \alpha x - \beta x^3 + a \sin \omega t, \quad (8.3)$$

with viscosity γ and driving amplitude and frequency a and $\omega = 2\pi/T$.

More generally, in the absence of viscosity and forcing, the total energy

$$E = \frac{1}{2}mv^2 + V \quad (8.4)$$

is constant. Hence,

$$V = E - \frac{1}{2}mv^2 \quad (8.5)$$

implies that a plot of the negative square of the speed $-v^2$ versus the distance x gives the (scaled and shifted) *shape* of the potential.

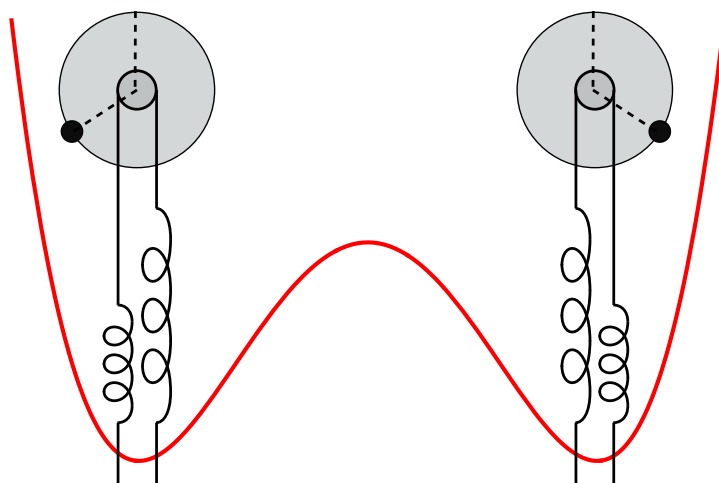


FIG. 8.2. The two symmetric pendulum equilibria superimposed on the bistable Duffing potential energy.

Equipment

The PASCO “chaos accessory” is a pendulum oscillator consisting of an aluminum disk connected to two springs. A point mass on the edge of the aluminum disk makes the oscillator nonlinear. The disk is magnetically damped. The frequency of the sinusoidal driver can be varied to investigate the progression from predictable motion to chaotic motion. Magnetic damping can also be adjusted to change the character of the chaotic motion. The angular position and velocity of the disk are recorded as a function of time using a Rotary Motion Sensor. Graphing the angular velocity versus the displacement angle of the oscillation produces a real-time phase space plot.

A strobe or Poincaré plot can also be graphed in real time and compared to the phase space plot. Achieve this by recording the point on the phase space plot once every cycle of the driver arm as the driver arm blocks a photogate. Accomplish that by using the DataStudio calculator to separately multiply the photogate state, which is either 0 or 1, by the angular position and the angular velocity and plotting the results against each other.

Several quantities can be varied to cause regular motion to become chaotic. These variables are the driving frequency, driving amplitude, damping amplitude, and the initial conditions. There are three different ways of plotting oscillations: angular position $\{t, \theta\}$, phase space $\{\theta, \omega\}$, and Poincaré plot $\{\theta, \omega\} \bmod T$ (strobe the phase space only once per period of the driving force). The phase space and the Poincaré plot are particularly useful for recognizing chaotic oscillations, because when the motion is chaotic, these graphs do not repeat.

Procedure

Part I: Mapping the Potential Well

This pendulum has two equilibrium points, one on each side where the torque caused by the weight of the point mass balances the torque from the springs, as in Fig. 8.2. To map the potential energy versus the angle of the pendulum from the vertical, with the driving force off, remove the magnetic damping by screwing the magnet away from the disk. Displace the pendulum to one side far enough that it will oscillate all the way over to the other side when released. Allow it to oscillate once. Measure the angular velocity and, after Eq. 8.5, use the DataStudio software to plot the negative square of the speed versus the angle. Because there are two equilibrium points, there should be a double well. Are the wells equally deep? Why or why not?

Part II: Resonant Frequency

Screw the magnet toward the disk until it is about 3 mm from the disk. Without turning on the power supply that powers the driver, allow the point mass to fall into the equilibrium position on either side of the pendulum. Click on START and displace the pendulum from equilibrium and let it oscillate for a few oscillations. Click on STOP. Examine the angle versus time graph. Are the oscillations sinusoidal? Are they damped?

Examine the phase plot (angular speed versus angle). What shape is it? How is affected by the amount of damping? What would it look like if there weren't any damping? Measure the period of the oscillation using the Smart Tool at the top of the angle versus time graph.

Part III: Non-chaotic Oscillations

For the rest of the experiment, hold the point mass end at the top and then let go when the driver arm is at its lowest point.

Set the driver arm amplitude at about 3.3 cm. Make sure the driver arm only breaks the photogate beam once per revolution. Adjust the magnet distance to about 4 mm from the disk. Turn on the power supply and adjust the voltage to about 4.5 V so the oscillation is simply one back-and-forth motion.

Click on START and record data for a few minutes. Examine the graph of angle versus time. Is it sinusoidal? What is the period? Is the period the same as the driving period? Why is this graph different from the graph in Part II?

Examine the graph of angular velocity versus angle (the phase diagram). Why does it look the way it does? How is it different from the phase diagram in Part II? Examine the Poincaré plot. Why does it look the way it does? How does this plot indicate that this oscillation is regular?

Gradually increase the driving frequency by increasing the voltage on power supply. Give the pendulum time to respond to the change in driving frequency. Increase the frequency until the motion of the pendulum is slightly more complicated: It should not simply have one back-and-forth movement but rather it should oscillate back-and-forth with an extra back-and-forth movement on one side. Re-start the oscillation, holding the point mass end at the top and letting go when the driver arm is at its lowest point.

Click on START and record data for a few minutes. Examine the graph of angle versus time. Is it sinusoidal? What is the period? Is the period the same as the driving period? How is it different than the previous oscillation? Examine the graph of angular velocity versus angle (the phase diagram). Why does it look the way it does? Compare it to the previous phase diagram. Examine the Poincaré plot. Why does it look the way it does? How does this plot indicate that this oscillation is regular?

Part IV: Chaotic Oscillations

Continue to gradually increase the driving frequency to the resonant frequency by increasing the voltage on power supply. To make the motion of the pendulum very complicated, you may have to adjust the distance of the magnet from the disk. The pendulum should pause suddenly at various points in its motion and spend random times on each side of the oscillation. Re-start the oscillation, holding the point mass end at the top and letting go when the driver arm is at its lowest point.

Click on START and record data for an hour. Examine the graph of angle versus time. Is it sinusoidal? What is the period? Is the period the same as the driving period? Examine the graph of angular velocity versus angle (the phase diagram). Why does it look the way it does? Examine the Poincaré plot. Why does it look the way it does? How does this plot indicate that this oscillation is chaotic?

Further Studies

The driving frequency was varied to change the oscillation from regular to chaotic. Try adjusting the magnetic damping while holding the driving frequency at the frequency that gave chaos before. Then try holding the damping and driving frequency constant while varying the driving amplitude. Check the effect of initial position on the oscillations.

Acknowledgments

These guidelines are based partly on the PASCO EX-9907 instruction manual.

References

- [1] Georg Duffing, *Erzwungene Schwingungen bei veränderlicher Eigenfrequenz und ihre technische Bedeutung [Forced vibrations at variable frequency and their technical importance]* (Friedr. Vieweg & Sohn, Braunschweig, 1918).

- [2] David Ruelle, Floris Takens, “On the nature of turbulence”, *Communications of Mathematical Physics*, **20** (3), 167–192 (1971).

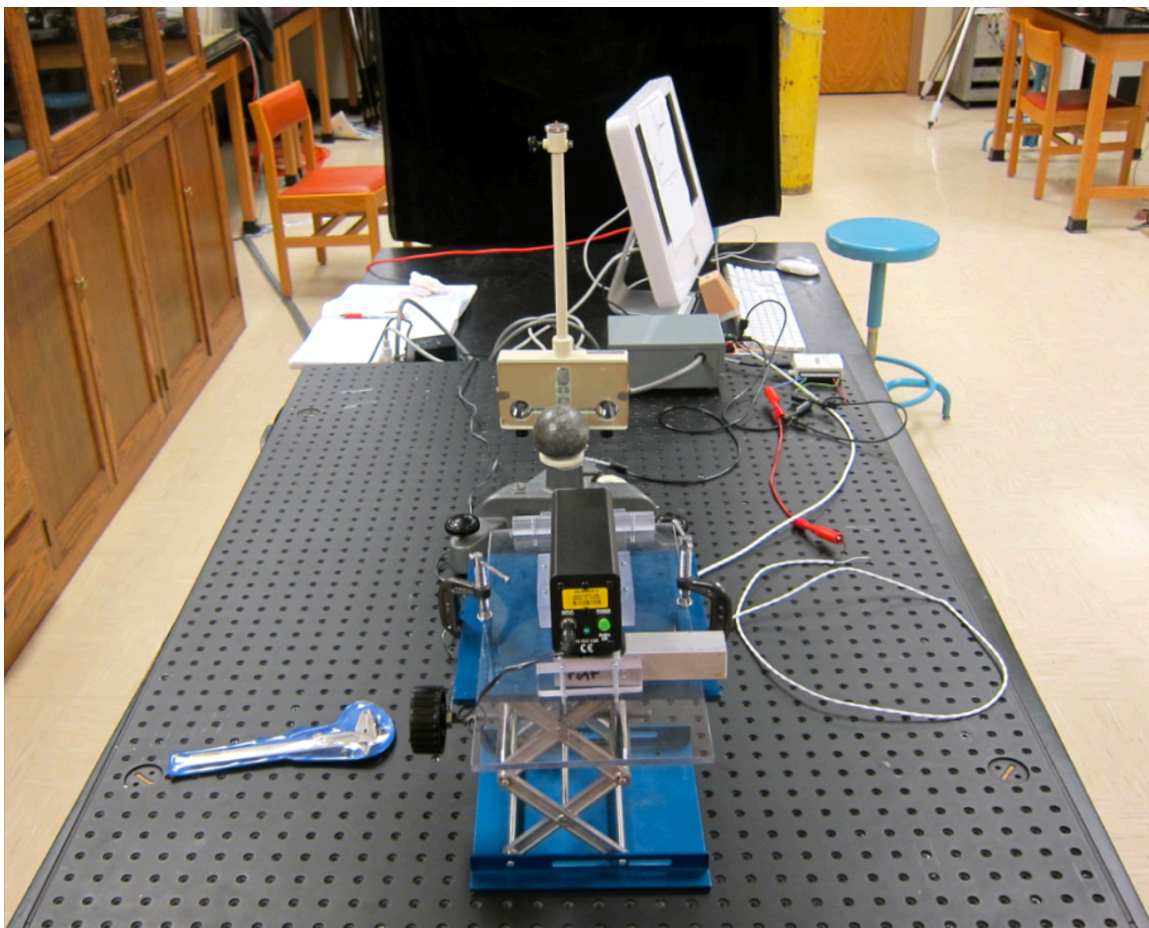


FIG. 9.1. Cavendish experiment equipment includes a Leybold torsion balance, a Hamamatsu position sensitive detector, a power supply, a laser, an optical breadboard, and a LabVIEW computer interface.



9. Cavendish: Torsion Balance

Last updated 2012 April 8

Introduction

In 1797-1798 Henry Cavendish rebuilt and perfected a torsion pendulum designed by John Michell to measure the feeble force of gravity between laboratory masses [1]. This enabled him to “weigh the earth” and determine its density. His results were later used to determine “big G ”, the proportionality constant in Newton’s law of gravity [2].

Theory

If two mass M and m are a distance r apart, then Newton’s laws imply

$$m\vec{a} = \vec{F} = -G'm \frac{M}{4\pi r^2} \hat{r} = -G \frac{Mm}{r^2} \hat{r}. \quad (9.1)$$

If the mass M is Earth and the mass m is at Earth’s surface a distance R from its center, then

$$m\vec{g} = -G \frac{Mm}{R^2} \hat{r}, \quad (9.2)$$

where Earth’s gravitational field

$$\vec{g} = -\frac{GM}{R^2} \hat{r} \quad (9.3)$$

has magnitude

$$g = \frac{GM}{R^2} \sim 9.8 \frac{\text{N}}{\text{kg}} \sim 9.8 \frac{\text{m}}{\text{s}^2} \sim \frac{22\text{mi/hr}}{\text{s}}. \quad (9.4)$$

Invert this to find Earth's mass

$$M = \frac{gR^2}{G} \sim 5.97 \times 10^{24} \text{kg} \sim 6\,000 \text{ Yg}. \quad (9.5)$$

Equipment

Balance

The chief difficulty in measuring the gravitational constant G is that the magnitude of the gravitational force is extremely small when the gravitational masses are not the size of the Earth. The torsion balance is a sensitive detector of mechanical forces with a simple design that allows it to measure forces in the horizontal position in order to negate effects from Earth's gravitational pull. It consists of a pendulum arm with two small lead spheres at either end; this arm is suspended from a torsion wire (a thin ribbon of metal). The pendulum arm has a mirror attached at its axis of rotation so that reflected light can be used to measure the angle through which the pendulum has twisted. All these components are enclosed in a case to eliminate any effects of air currents. Outside of the case, a rotary arm holds two larger lead spheres that are used to attract the smaller spheres inside the case. The rotary arm is designed so that the center of the larger spheres and the center of the smaller spheres are at the same height and so that the axis of rotation of the pendulum arm and the rotating arm are the same. The apparatus works by rotating the two large spheres from one side of the case to the other and measuring the deflection of the pendulum arm, as shown in Fig. 9.2. The angle of deflection is determined by shining a laser beam from the mirror on the pendulum and recording the location of the reflected light using a ruler at a known distance from the mirror

The Leybold apparatus used in this experiment consists of a 10 cm pendulum arm with two ~ 15 g lead spheres of radius 7.5 mm at each end suspended from a 25 cm torsion wire, which are attracted to two ~ 1500 g lead spheres. **Use latex gloves if you need to handle the large spheres.** Because of the delicateness of the torsion wire, any movements of the apparatus must be done with extreme caution. The torsion balance has two knobs at the bottom of the case that serve as a locking mechanism for the pendulum arm when it is not in use. **Do not move the apparatus at all without locking down the pendulum arm!** A knob at the top of the tube where the torsion wire hangs can be used to make rotational adjustments to the pendulum. The screws in the base supporting the torsion balance can be adjusted to be sure the balance is level. Be sure that the pendulum is hanging freely and not touching anything within the case.

Automated detection

A one-dimensional position sensitive detector (PSD) is used to detect the location of the reflected laser beam [3]. This Hamamatsu S3270 PSD is a monolithic PIN photodiode with a fast response time. The attached circuit (Hamamatsu C3683-01) processes the signal from the photodiode and outputs a voltage proportional to the displacement of the beam from the center of the PSD. Specifically, the distance between the center of the PSD and the light spot is

$$x = kV_0, \quad (9.6)$$

where V_0 is the voltage and $k = 37/20$ V/mm is the proportionality constant. Table 9.1 lists the necessary connections between the circuit and the 9-pin connector.

Table 9.1. Position detector connections.

| | Cord 1 (from external power) | | | Cord 2 (to DAQ) | |
|--------------------|------------------------------|--------|--------|-----------------------------------|-------------------------------|
| | Black | Red | White | Black | White |
| Name | $-V$ | $+V$ | GND | V_0 | V_A |
| Pin # | 3 | 4 | 5 | 2 | 9 |
| Description | $-15V$ | $+15V$ | Ground | Position converted voltage output | Summed signal output (unused) |

Procedure

Overview

The torsion balance used in this experiment is very delicate and sensitive to small forces due to gravity. It is also sensitive to small forces from vibrations from the building, from passing students, and from touching the table. You will need to take care (and take your time) with the experiment so that these noise effects do not overwhelm the small signal due to the gravitational attraction between the two spheres. The best previous student result using this particular automated detection scheme is $G = (6.3 \pm 0.3) \times 10^{-11} \text{Nm}^2/\text{kg}^2$, which is 5% from the accepted value. Cavendish's original measurement was within 1% of our currently accepted value. Can you do better?

$$\kappa \frac{\alpha}{2} = \tau = 2Fd = 2 \frac{G M m}{r^2} d. \quad (9.10)$$

Combine Eqs. 9.6, 9.8, and 9.9 to find

$$G = \frac{\pi^2 r^2 d L}{M T^2 D}. \quad (9.11)$$

Small Corrections

However, the Eq. 9.10 expression is liable to a systematic error because the small balls are also attracted, albeit with considerably smaller force, by the *remote* large balls. As a result, the net force F' is *less* by a factor of

$$\frac{F'}{F} = 1 - \left(1 + \left(\frac{2d}{r} \right)^2 \right)^{-3/2} < 1. \quad (9.12)$$

In addition, because the torsion balance is highly damped, the period T' of the decaying oscillations is not quite the undamped period T of Eq. 9.9. Instead,

$$\frac{T'}{T} = \sqrt{1 + \left(\frac{\rho}{2\pi} \right)^2} > 1, \quad (9.13)$$

where ρ is the logarithmic decrement, or the ratio of successive position maxima [3]. However, a nonlinear fit to the decaying oscillation can obviate the need for this correction.

Acknowledgments

These guidelines are based partly on the Leybold 33210 instruction manual and a report by Lily Christman.

References

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- [2] Isaac Newton, *Philosophiae Naturalis Principia Mathematica* [*Mathematical Principles of Natural Philosophy*], (Joseph Streater for the Royal Society, 1687).
- [3] C. W. Fischer, J. L. Hunt, P. Sawatzky, “Automatic recording for the Cavendish balance”, *Am. Journ. Phys.*, **55**, 855-856 (1987).

Quantum Mechanics & Optics

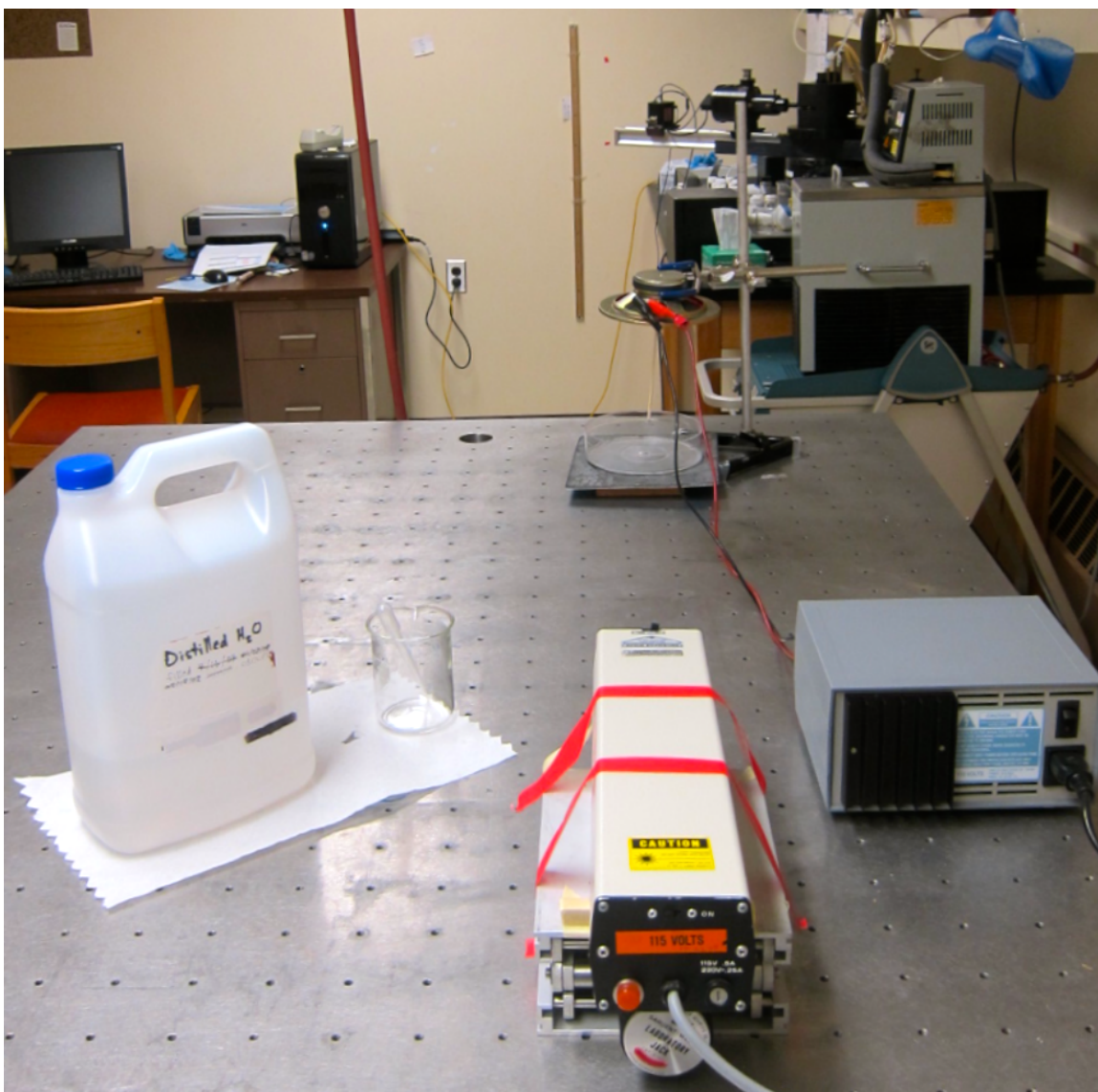
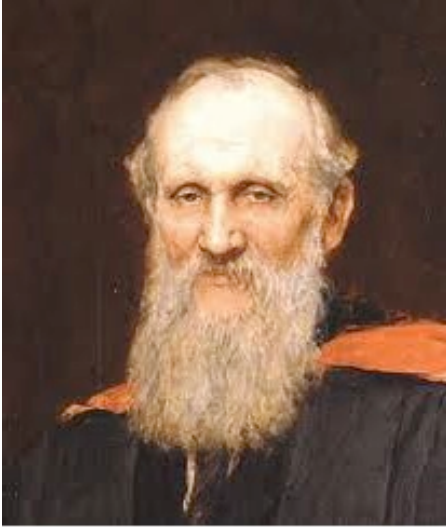


FIG. 10.1. Kelvin experiment equipment includes an audio speaker and rod, a petri dish with distilled water, a function generator, a vibration isolation table, and a meter stick.



10. Kelvin: Surface Tension Waves

Last updated 2012 March 3

Introduction

Surface tension is the force per unit transverse distance on the surface of a fluid such as water. In 1871, William Thomson (who is also known as Lord Kelvin), derived a theory [1] describing water waves, including surface tension. In this experiment, test Thomson's theory by scattering laser light from surface tension waves, which act like a reflection grating [2].

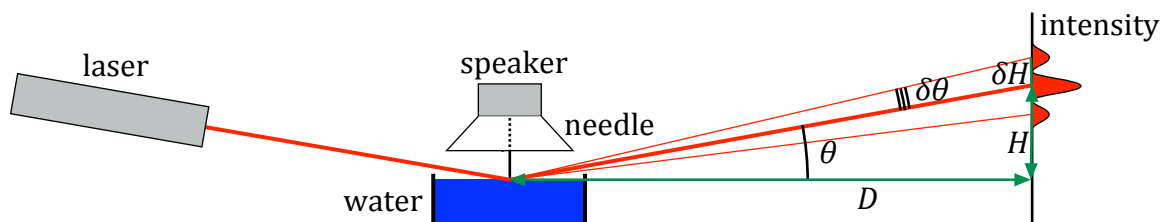


FIG. 10.2. Light scattering of surface tension ripples that form an optical grating.

Theory

The temporal and spatial frequencies $\omega = 2\pi/T$ and $k = 2\pi/\lambda$ of a dispersionless wave are proportional,

$$\omega = kc, \quad (10.1)$$

where the proportionality constant c , the wave speed, is independent of wavelength λ or period T . In contrast, Thomson showed [1] that the dispersion relation for water waves is

$$\omega = \sqrt{\left(kg + k^3 \frac{\sigma}{\rho}\right) \tanh kh}, \quad (10.2)$$

where g is the gravitational field, h is the water depth, ρ is the water density, and σ is the surface tension. If the depth is large compared to the wavelength $kh \gg 1$, so that $\tanh kh \sim 1$, and gravity is weak compared to the surface tension $\rho g \ll \sigma k^2$, then

$$\omega^2 \sim k^3 \frac{\sigma}{\rho}. \quad (10.3)$$

Such surface tension ripples act like a diffraction grating and cause light of spatial frequency $k_l = 2\pi/\lambda_l$ and grazing angle θ to interfere constructively just above and just below the specular reflection, as in Figs. 10.2 and 10.3. The path length difference to the first spot above must be a single light wavelength,

$$\lambda_l = \lambda \cos[\theta - \delta\theta] - \lambda \cos \theta. \quad (10.4)$$

Similarly, the path length differences to the first spot below must be a single light wavelength,

$$\lambda_l = \lambda \cos \theta - \lambda \cos[\theta + \delta\theta]. \quad (10.5)$$

Hence, by addition and the cosine double angle formula, the light wavelength

$$\lambda_l = \lambda \sin \theta \sin \delta\theta \quad (10.6)$$

and so the water spatial frequency [3, 4]

$$k = k_l \sin \theta \sin \delta\theta. \quad (10.7)$$

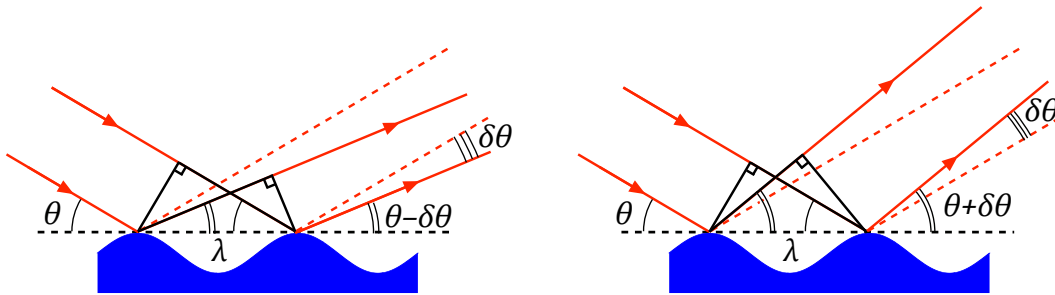


FIG. 10.3. First order spots geometry.

Equipment

This experiment uses the Fig. 10.1 audio speaker with a stick and wire glued to the center of the cone, driven by a PASCO function generator. The wire just touches the surface of distilled water in a petri dish and excites circular ripples.

Procedure

Vary the frequency ω of the audio speaker and infer the spatial frequency k of the surface tension waves from the spot separation $\delta\theta$ on a distant wall. Plot $\rho\omega^2$ versus k^3 to check the Eq. 3 dispersion relation and infer the surface tension of water. Also try a log-log plot of ω versus k .

References

- [1] William Thomson (Lord Kelvin), *Phil. Mag.*, (4) **43**, 375 (1871).
- [2] G. Weisbush, F. Garby, *Am. J. Phys.* **47**, 355 (1979).
- [3] W. M. Klipstein, J. S. Radnich, S. K. Lamoreaux, *Am. J. Phys.* **64**, 758 (1996).
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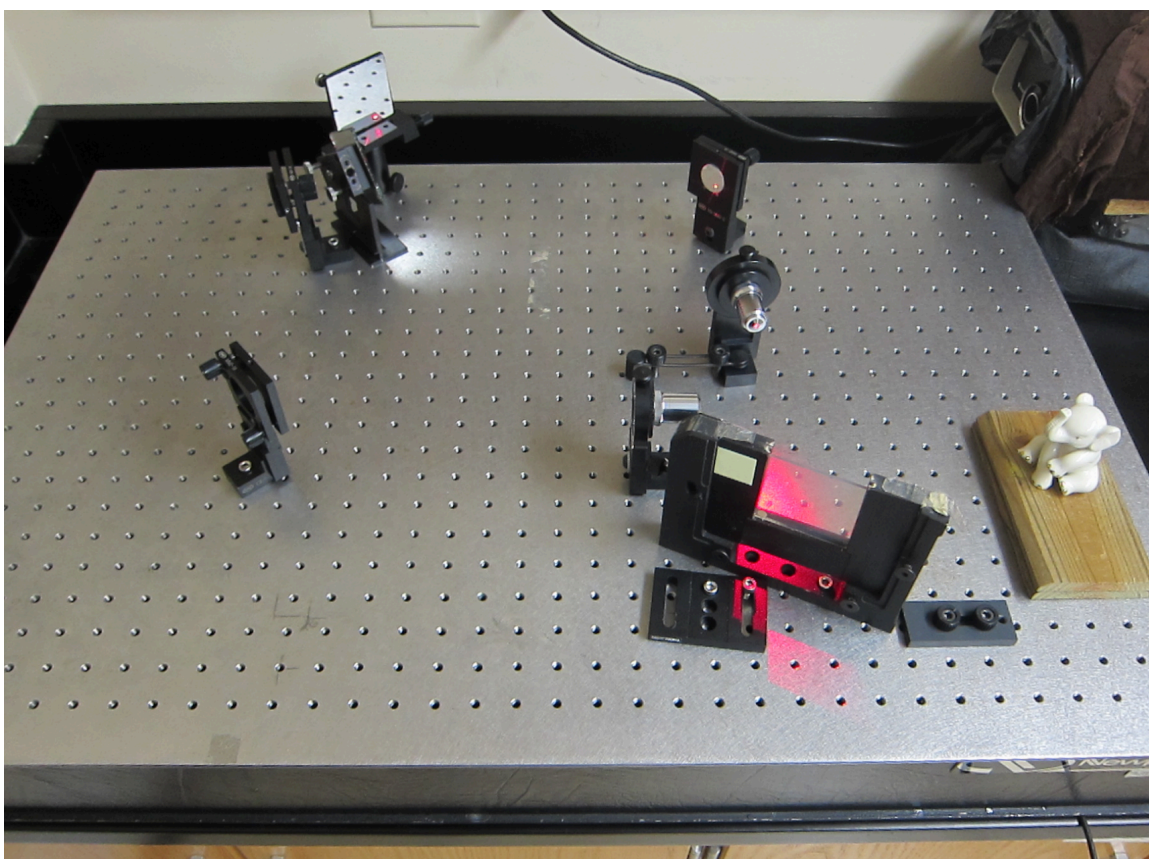


FIG. 11.1. Gabor experiment equipment includes 60 mW laser, an optical breadboard, a beam splitter, mirrors, diffusers, and a holoplate holder.



11. Gabor: Holography

Last updated 2012 May 11

Introduction

Amplitude A , frequency $\omega = kc$, and phase φ completely determine a sinusoidal light wave, $A \cos[kx - \omega t + \varphi]$. Black & white photographs record amplitude (as a function of position). Color photographs record amplitude and frequency. *Holograms* record amplitude and phase. As Dennis Gabor (born Gábor Dénes) first showed in 1947 [1], holograms consequently enable the reconstruction of three-dimensional images. Made practical by the 1960 invention of the laser, holograms are developed today in both art and engineering [2].

Theory

To record a hologram, laser light is split into an object beam that reflects off the object onto the film and a reference beam that illuminates the film directly, as in Fig. 11.2. The interference of these two beams records the phase information that encodes the object's depths. When an identical reference beam illuminates the developed film, a three-dimensional image of the object appears.

Mathematically represent the object beam as (the real part of) the complex field $E_O = |E_O| \exp[i(kx - \omega t + \varphi)]$ and the reference beam by the complex field E_R . The *irradiance* or intensity in the combined field is

$$I = |E_O + E_R|^2 = E_O E_R^* + |E_R|^2 + |E_O|^2 + E_O^* E_R. \quad (11.1)$$

The *transmittance* of the developed film is proportional to the irradiance,

$$T = \kappa I = \kappa E_O E_R^* + \kappa(|E_R|^2 + |E_O|^2) + \kappa E_O^* E_R. \quad (11.2)$$

During reconstruction, when the reference beam illuminates the developed film, the hologram field

$$E_H = TE_R = \kappa E_0 |E_R|^2 + \kappa(|E_R|^2 + \kappa |E_0|^2)E_R + \kappa E_0^* E_R^2. \quad (11.3)$$

The first term in the hologram field is proportional to the original object beam and represents a virtual image of the object. The second term is a modified reference beam. The third term is the conjugate object beam and represents a real image of object. If the object and reference beams form a significant angle, then the virtual, reference, and real wave fronts emerge at distinct angles.

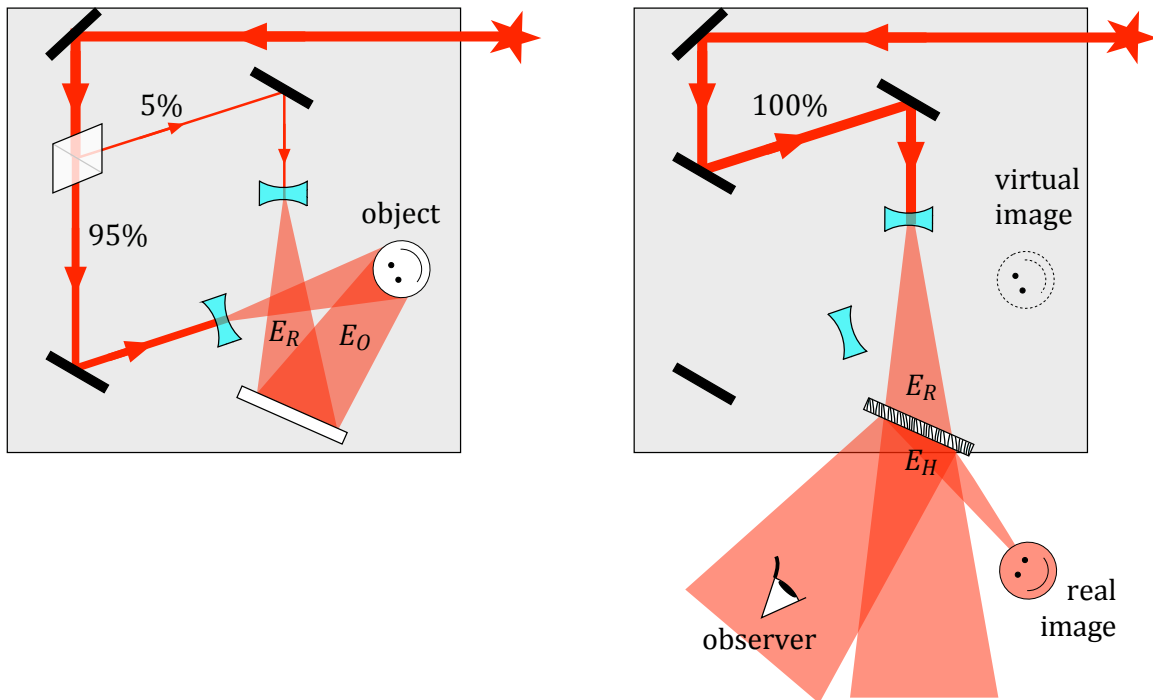


FIG. 11.2. Off-axis hologram recording (left) separates the reference beam, the converging real beam, and the diverging virtual beam during reconstruction (right).

Equipment

This experiment uses a moderately intense 60 mW laser, beam splitter, mirrors and lenses screwed into an optical breadboard, holographic film and developing chemicals in a photographic dark room.

Procedure: Single Exposure Hologram

Safety Precautions

Never look directly into the laser light. It can harm your retinas. Also never insert mirrors or any reflectors into the beam after the laser is turned on. Stray reflections can enter someone's eyes.

Make sure all the lights (including tiny automatic ones) are turned off or masked before the photographic plate is taken out of its box. It will take a few minutes before your eyes get used to the dark and you can actually start seeing things under the green light. Be sure to close the lid of the box tightly and **put it away before proceeding.**

Be aware of potential hazards of chemicals used for developing. Use rubber gloves or tongs and do not directly touch the chemicals. During developing, make sure you do not splash water on to other chemicals. (It may be safer to cover them up before you start rinsing.) **If you get chemicals in your eyes, use the eyewash in the Senior I.S. lab.**

Be careful to avoid vibrations while exposing. **Try not to bump anything!**

Preparation and Exposure

Fix or bolt the laser to its optical bench. Set mirror to reflect the laser beam onto the diverging lens. The diverged beam should fall on the object and then onto where the plate will be mounted. (The beam should strike the plate at an angle.) A used plate can be placed to ensure the exact position of the new plate. Once all the components are aligned, block the laser with a beam stop. Cover the glow from the laser body with a plastic bag but don't make the laser overheat.

The unexposed plate should never be taken out of the package or exposed to normal light. Do a "dry run" with only the green safelight on, so you know exactly what to do. Place the photographic plate on the holder with the emulsion side facing the object. (The emulsion side is sticky when a wet finger is placed on one corner; or if you put your lips to it!) **Wait for vibrations to die down for a few minutes.** Remove the beam stop (shutter) and count to 15-20 seconds and replace the stop. Now the plate has been exposed for 15-20 seconds. (Exposure time must be adjusted depending on laser power, quality of object, and the kind of hologram you are making.) Remove the exposed plate and follow developing procedures.

Developing Procedure

Keep all the chemicals ready in the appropriate trays before exposing the plate. **Developing should be done as soon after the exposure as possible.** Use the faint green light and wear latex gloves.

Mix 50 ml of "A" and 50 ml of "B" to make enough developer. Place the plate in the developer and shake gently for 1 to 2 minutes. Remove the plate and place it in the tray containing "stop bath". Soak for about 2 minutes. Remove once again and soak the plate in the "fixer" for about 2 minutes. Take the plate out and show it under running water for about 3 minutes. Lights can be turned on at this point. Optionally, rinse the plate in Photoflo solution for about a minute before drying. This will prevent water marks.

Blow-dry or stand it to dry with one edge resting on a paper plate. Do not wipe with paper towels or tissues. Place a little sticker at the bottom right hand corner of the holoplate with your initials on it. Cover all the trays containing chemicals with foil. Clean up after yourself.

Viewing the Hologram

Put the dried plate back in the plate holder, as it was during exposure. Replace the 5% beam splitter insert with the 100% insert, and fine-adjust it to direct all the energy to the reference beam, illuminating the hologram plate. **What happens if you reverse the hologram plate in its holder?**

Procedure: Double Exposure Holographic Interferometry

Overview

Holographic interferometry is an amazing visual technique. It allows us to directly observe the slight deformations that an object undergoes whenever small stresses of any kind are applied to it. The technique is extremely sensitive – so sensitive, in fact, that it often reveals peculiarities in the object's inner structure, such as minute flaws.

This procedure illustrates how holographic interferometry makes evident in an image even the slightest movement of object's surface, even that caused by slipping a rubber band around an aluminum can. For this double-exposure hologram, the can will be stressed during one exposure but unstressed during the other, yielding an image that shows “before” versus “after”.

Alignment checklist.

Use the same apparatus as for the single exposure hologram. Use the 5%-reflecting insert for the beam splitter. Spray a light coat of flat-white paint onto the aluminum can. It will dry in a few minutes and will eliminate the harsh reflections of the shiny aluminum.

Put a few touches of glue on the bottom of the can to attach it to the isolation platform. The side of the can facing the plate holder should sit over the X's in the photo. Make a double-exposure hologram, giving each of the two exposures about 7-10 seconds with 8E75HD film, 20 seconds with 120-type film. Before turning out the lights to make the double exposure, slip a rubber band around the aluminum can. The rubber band should be weak and the fit not too tight.

In the dark, in between the first and second exposures, you must not disturb or push on the film plate, or on any part of the apparatus, just very delicately slip the rubber band off the can. Wait for any vibrations to subside before exposing. **The film processing is the same as for the single exposure hologram.**

Viewing the Holographic Interferogram

Put the dried plate back in the plate holder, as it was during exposure. Replace the 5% beam splitter insert with the 100% insert, and fine-adjust it to direct all the energy to the reference beam, illuminating the hologram plate

On the holographic image of the can you will see a pattern of irregularly curved stripes and rings, called interference fringes, as in Fig. 11.3. These indicate flexing of the can — actually a change (movement) in the flexing between the first and second exposures. Notice that the area near the rubber band is where the can flexed the most, and the bottom of the can is where it hardly flexed at all.

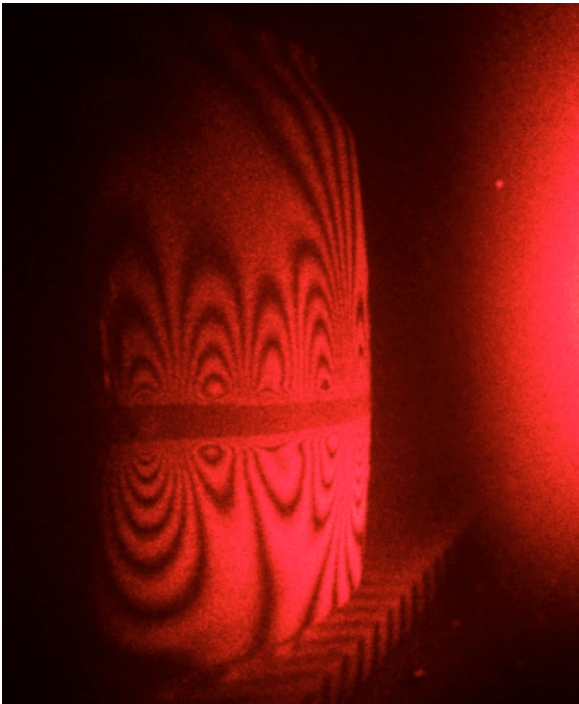


FIG. 11.3. Interference fringes indicate the flex of the can due to the rubber band.

Computing the Deformation

The number of fringes that you count just up to that point determines the amount of movement at any specific point on the plate. This is somewhat like tree rings, but counting upward from the base of the can to whatever point you are trying to measure. In this apparatus, the fringes you count represent successive increments of 370 nm of movement. For example: Starting at the base, if you have to pass over five irregularly shaped fringes in order to get to some point of interest, then you know that point moved $5 \times 370 \text{ nm} = 1.85 \mu\text{m}$ when the stress was relaxed between the two exposures.

The movement you calculate by this count is motion that has occurred perpendicular to the surface of the can. Moving around the can, some areas are bulged out and some are dimpled inward. (See if you can tell which is which.)

Acknowledgments

These guidelines are based partly on the report of Tom Gilliss.

References

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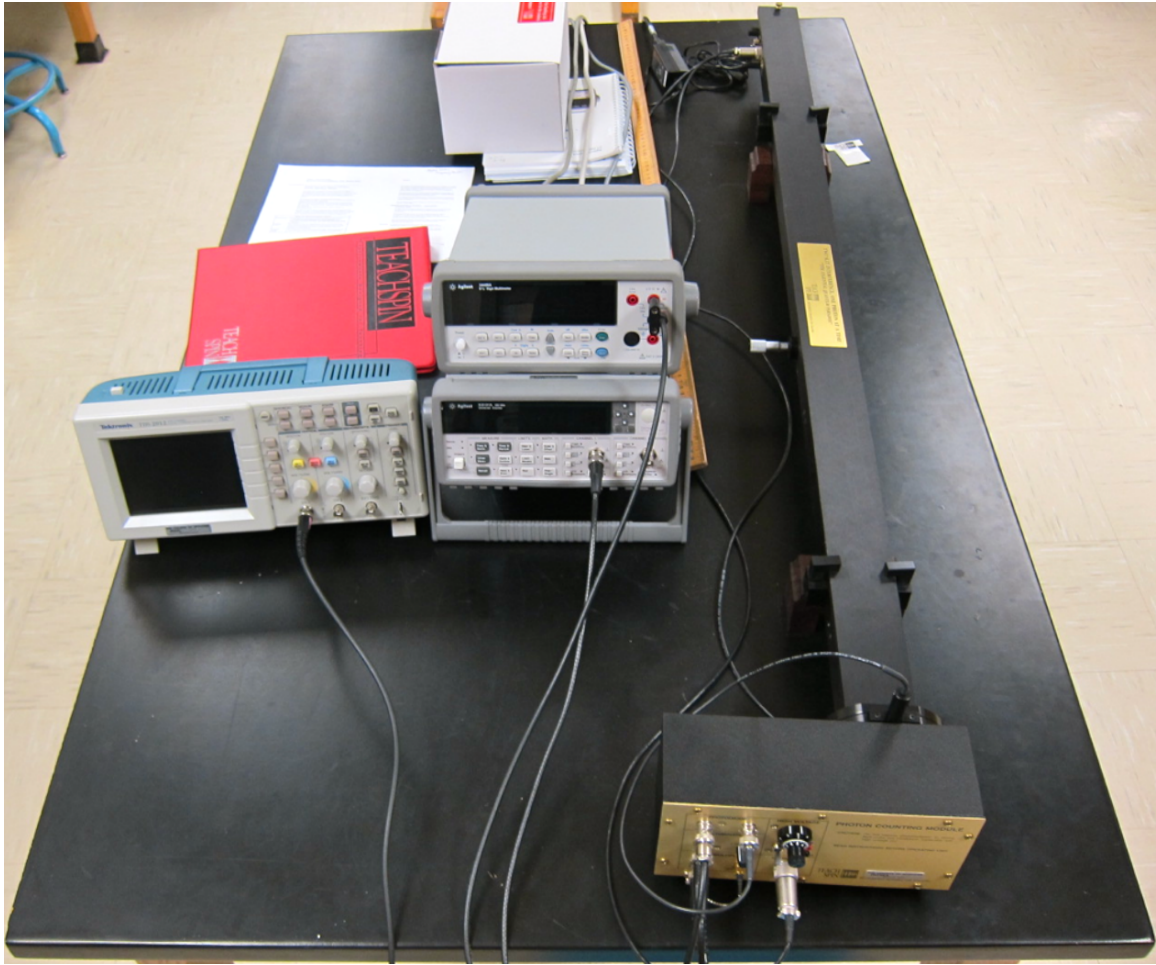


FIG. 12.1. Feynman experiment equipment includes a TeachSpin single-photon interference apparatus, a multimeter, a counter, and an oscilloscope.



12. Feynman: Single Photon, Double Slit

Last updated 2012 January 6

Introduction

By 1804, Thomas Young had compared the results from water waves and a beam of light passing through a double slit and determined that light behaved as a wave rather than a particle [1]. In 1905, Albert Einstein explained the photoelectric effect [2] by concluding that light must be divided into individual packets, or quanta. Then is light a particle or a wave? As early as 1909, Geoffrey Ingram Taylor [3] showed that even the weakest light source led to interference fringes, leading to Paul Dirac's famous statement that "each photon then interferes only with itself" [4]. In 1961, Claus Jönsson obtained similar results with *electrons* [5-6]. In 1974, Pier Giorgio Merli and colleagues [7] obtained the famous interference pattern when *one* electron at a time passed through a double slit, as later did Akira Tonomura and colleagues [8], an experiment recently voted the most beautiful of all time by physicists.

Of the double slit experiment, one particle at a time, Richard Feynman famously wrote [9], "We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery. We cannot make the mystery go away by explaining how it works ... In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics."

Theory

In classical physics, we know that a wave going through a single slit will spread out, a process known as diffraction. The situation is shown in Fig 12.2. For light of wavelength λ from a distant source passing through a **single** slit of width a , the intensity at a location x on a screen a distance L away from the slit is

$$I = I_{\max} \left(\frac{\sin \alpha}{\alpha} \right)^2, \quad (12.1)$$

where I_{\max} is the intensity at the central maximum, and α depends on both the wavelength and the slit width according to

$$\alpha = \frac{\pi a}{\lambda} \sin \theta. \quad (12.2)$$

The angle θ is the amount of deviation from the original straight beam path and is given by $\tan \theta = x / L$.

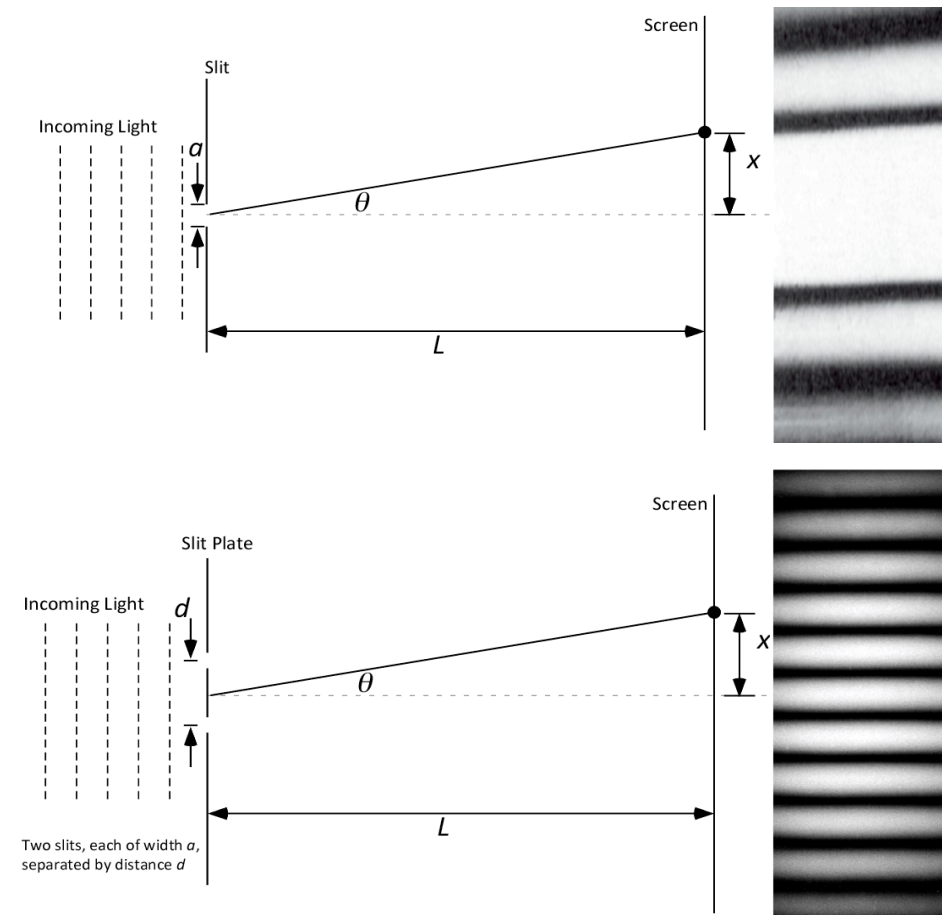


FIG. 12.2. Single and double slit interference patterns and geometry.

When light passes instead through **two** slits, the light still spreads according to diffraction, but a new pattern is overlaid on the old due to the interference of light from the two different sources. For two slits, each of width a , separated by a center-to-center distance d the intensity at the screen will now be given by

$$I = I_{\max} \left(\frac{\sin \alpha}{\alpha} \right)^2 \cos^2 \beta, \quad (12.3)$$

where I_{\max} and α are defined as before, and

$$\beta = \frac{\pi d}{\lambda} \sin \theta. \quad (12.4)$$

This type of diffraction, where both the light source and the screen are a relatively large distance from the slit, is known as Fraunhofer diffraction.

Equipment

To detect one photon at a time requires an extraordinarily sensitive light detector, known as a photomultiplier tube (PMT). Normal room light (even in a dark corner) typically consists of more than 10^{14} photons per second. **If the PMT is exposed to this type of light, it will be seriously damaged and will almost certainly have to be replaced. Do not open the shutter to the PMT unless you are absolutely certain that it is safe and the manual explicitly says to open the shutter.**

Before beginning, be sure the shutter is closed. The shutter is located right between the detector box (at the right end of the apparatus) and the long channel. The shutter is attached to the rod sticking up here with an electrical cable coming out the end. If the lid of the apparatus is closed and everything is off, you may pull the rod up (about 2 cm) to open the shutter, and immediately push the rod all the way back down to close the shutter. (It is a good idea to do this, just so that you know how the shutter works.) Also check that on the detector box the toggle switch in the HIGH-VOLTAGE section is turned **off**, and that the 10-turn dial near it is set to 0.00, fully counter-clockwise.

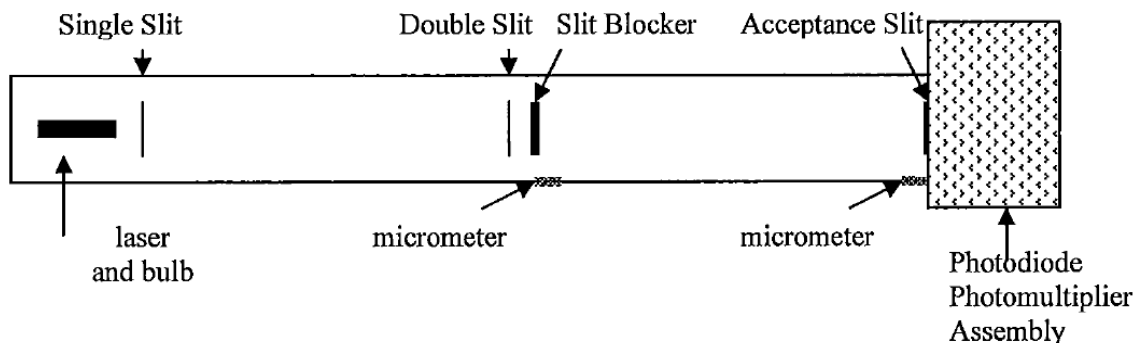


FIG 12.3. The double slit apparatus.

Procedure

There are three sections for this experiment: You will be seeing two-slit interference visually, by opening up an apparatus and seeing the exact arrangements of light sources and apertures that operate to produce an interference pattern. You'll be able to examine every of the apparatus, and make all the measurements you'll need for theoretical modeling. You will be able to perform the two-slit experiment quantitatively, recreating not only Young's measurement of the wavelength of light, but also getting detailed information about intensities in a two-slit interference pattern which can be compared to predictions of wave theories of light. You will be able to perform the two-slit experiment **one photon at a time**, continuing the same kind of experiments, but now at a light level so low that you can assure yourself that there is at most one relevant photon in the apparatus at any time.

Section 1: Familiarizing yourself with the apparatus and double slit interference

Confirm that the shutter to the PMT is **closed**. With the shutter closed, it's safe to open the cover of the long two-slit assembly. (Lift and turn the four latches holding the cover down, then lift the far-left end of the cover and slide it sideways and leftwards by about a centimeter to disengage the right end of the cover from its light-tight slot, and lift the whole cover off.)

This apparatus consists of a long channel about 1 m long with a light tight removable cover. A schematic is shown in Fig. 12.3. At one end, the light source can be either a laser or small bulb. At the other end are two light detection systems—the photodiode (used for most light sources) and the photomultiplier (an extremely sensitive detector used only for the weakest light sources).

Just after the light source is a single slit. Light from this slit is used as the source for the double slit. (Using a single slit as the source ensures that the light has spatial coherence, a requirement for observing interference.) The central maximum of the single slit pattern should already be adjusted to fall on the double slit, about 50 cm farther down the channel. A movable slit blocker is immediately after the double slit; using the micrometer, this slit blocker can be adjusted to block the light from either one of the double slits, turning the interference on and off. At the end of the channel just in front of the light detection assembly is another moveable single slit called the acceptance slit. This slit can be moved across the detection area to map out how the intensity of light varies from one side to the other.

Visual investigation:

Turn on the diode laser and use one of the business cards to trace the path of the laser down the channel. **You need to make detailed observations in your lab notebooks about what you observe in this section.** You should be able to see the beam undergoing single-slit diffraction after it passes through the 0.085 mm slit. The brightest part of this diffraction pattern should then fall on the double slit (two slits each of width 0.085 mm, with a center-to-center separation of 0.353 mm.) Although the slits are all easily moved, it is not so easy to align everything, so do not move them if you do not want to realign them. Use the viewing card downstream of

the double slit—you should observe two clearly distinct ribbons of light just after the two slits, which then merge together some distance farther downstream to form the characteristic double slit interference pattern.

While observing the light from the double slit, use the micrometer to adjust the slit-blocker so that it blocks the light from either the left or the right slit. How does the pattern change?

For later sections of the experiment, you will need to be able to set 5 specific settings for the slit-blocker using only the micrometer (leaving the box cover closed). Determine the micrometer settings now for: one position for which both slits are blocked; another for which light emerges only from the farther of the two slits; a third (anywhere in a wide range) that allows both ribbons of light to emerge; a fourth for which light emerges only from the nearer of the two slits; and finally, a fifth setting (and highest reading) which again blocks the light from both slits. You must be able to read the micrometer and set these positions later on, so practice now if you are not confident.

Finally, investigate the exit or detector slit. This slit is of the same size as the source slit but is movable via micrometer like the slit-blocker. The purpose of this detector slit is to allow light from a narrow slice of the interference pattern to pass along to the end of the long apparatus and into the detector box. By translating the detector slit laterally along the interference pattern in space, you can select which part of the pattern will have its light sent on to the detector. Thus by scanning the micrometer screw of the detector slit, you can scan over the interference pattern, eventually mapping out its intensity distribution quantitatively. For now, ensure that the detector slit is located somewhere near the middle of the two-slit interference pattern.

Section 2: Quantitative operation with the laser source

Either close the lid to the box or turn off room lights to avoid noise in your results. Set the slit-blocker to setting 3 (both beams of light emerging). The shutter should be closed, protecting the PMT. In this configuration, the less-sensitive photodiode detector is in place to measure the light intensity after the detector slit. The photodiode operates similarly to a solar cell—specifically, it outputs a current proportional to the intensity of light hitting it. Using the detector slit, we can map out the intensity of the interference pattern.

A cable should already connect the photodiode to the input BNC connector on the detector box. The output BNC next to this input carries a voltage signal from the photodiode; connect this output to a digital multimeter. You should observe a steady positive voltage.

Note the dark signal or zero offset of the photodiode system by turning off the laser and noting the diode voltage. If you have any room lights on, test the effect of

turning them off as well. You will need to subtract this zero-offset from the other measurements of the photodiode voltage.

Testing

Turn the laser on, and observe the photodiode signal as you vary the location of the detector slit. You should observe a systematic variation of the signal as you scan over the interference pattern. Find the highest of the maxima, known as the central fringe or zeroth-order fringe. The voltage at this central maximum should be 1 to 5 V; if it is not then the alignment of the apparatus will need to be improved.

Now, with the detector slit at the central max, adjust the slit-blocker from its current position of passing the light from *both* slits to one of the locations that passes light from only *one* slit. Record the photodiode readings in both cases. Adjust the slit-blocker to pass only light from the *other* slit and record the photodiode voltage. After correcting for the zero-offset, you should find the light has decreased by *more* than 50%, even though we are only blocking 50% of the light. What fraction do you find, and why?

Now with the slit-blocker passing light from both slits, adjust the detector slit to one of the minima just to the side of the central maximum, taking care to find the very bottom of the minimum. Again record the photodiode voltage for the slit-blocker passing both beams, blocking beam *A*, and blocking beam *B*. What happens to the intensity in this case?

Measurement

Now we are ready to make quantitative measurements of the intensity at different points on the interference pattern for two cases—both slits open and one slit (either one) open. (You can also perform this experiment for the other slit open, but it is not necessary.) Set the slit-blocker to a given position, and systematically vary the detector slit location while recording the photodiode voltage. You need to determine what spacing of detector-slit locations to use, and so **you must plot the data as you acquire it** by hand in your lab notebook. (Yes, really. Watching the graph take shape is the best way to learn this.)

Before proceeding, adjust the slit-blocker so it passes light from both slits and move the detector slit back to the central maximum position. Turn off the laser.

Section 3: Quantitative operation—single photon mode

Again, a **WARNING**: the photomultiplier is so sensitive to light that it should not be exposed to moderate levels of light even when the PMT is off, and it cannot be exposed to anything but extremely dim sources when the PMT is on. (Ordinary dim room lights are too strong for a PMT off, and moonlight is too strong for a PMT on.) The PMT should be used **ONLY** when the cover of the apparatus is closed and **ONLY** when the light bulb (**not** the laser) is the light source.

On the detector box, check that the High Voltage toggle switch for the PMT is off and turn the 10-turn dial (which sets the voltage) all the way to zero. Check again that the PMT shutter is down and closed.

Changing the light source

Now, open the apparatus cover. Turn the laser off if it is still on, and slide the laser source all the way to the far side of the channel, so that it is out of the path of the light from the bulb. Turn the bulb power dial all the way to 0 and turn the bulb on with the 3-way toggle switch; now dial the bulb power up gradually until you see the bulb light up. *To protect the bulb and extend its life, minimize the time spent with the dial above 6 and toggle it on and off only when the dial is set to low values.*

The apparatus should be already aligned so that the bulb light will pass through the double slits. To check, darken the room completely and make the bulb output more visible by removing the green filter from the bulb. Set the brightness to half scale, and use a white card to follow the white light emission to the source slit. Trace the narrow ribbon coming out of the source slit further downstream (the ribbon will broaden) until you reach the double slit. The center of the broad white light ribbon should be roughly centered and illuminating the double slits. (If the apparatus does not appear aligned, please do **not** attempt to adjust it without first asking for help.)

Put the green filter back on the light bulb. The filter passes only light with a wavelength between 541 and 551 nm, which means that almost all the light from the bulb is blocked. The green light should seem extremely dim; now dim the bulb **more** and set the bulb intensity down to about 3. (The room lights can be turned on at this point, if desired.)

Carefully close the cover of the box, being sure the right end is pushed all the way in place and the left end is securely down. Be sure there is no gap at the left end. Engage all four latches. The box must be light tight for operation with the PMT.

Preparing the PMT

Connect the oscilloscope to the photomultiplier output of the detector box using a BNC cable. Set the scope to about 50 mV/div and 25 ns/div. The trigger should be set for a rising edge with a level about 20 mV.

With the PMT shutter still *closed*, turn the high voltage dial to 0.0, and turn on the high voltage toggle switch. Each full turn of the dial corresponds to 100 V. Set the high voltage dial to 5.5 (550 V). On the oscilloscope you should observe occasional positive pulses, perhaps a few per second. These pulses show you the dark rate of the PMT—the signals appear just as photons would but there are no photons in the apparatus. Now, finally open the shutter, and you should see a much greater number of pulses (hundreds or thousands per second). If you adjust the bulb intensity carefully (not too bright!), you should see the rate of pulses change accordingly.

To make the pulses easier to count electronically, we use a pulse discriminator. Set the discriminator to 4, and connect the output ttl signal to the other channel of the oscilloscope (and set the channel display to about 2 V/div). You should see a square pulse for each analog pulse, as long as the analog pulse reaches a minimum (about 50 mV) value. If you see too many square pulses, or none, ask for assistance.

Disconnect the output ttl signal from the oscilloscope and connect it to the Agilent 53131A Universal Counter instead. **For all counting measurements, you should make several measurements and average the results.**

To check the behavior of the PMT, record a set of dark count measurements. First, record the count rate when light bulb is dialed down to zero. This is the dark rate. Then turn the bulb back up to about 3 or 4 so that you have around 1000 counts/second. Block the beam completely using the slit-blocker and record this background rate (which will probably be slightly higher than the dark rate).

Open up both slits and move the detector slit to check if you can observe interference fringes. (You have to stop and wait for the counter to report a value or two at each detector slit location; you can not move the detector slit continuously.) If you can observe maxima and minima you are ready to take data. Do not adjust the bulb intensity after this point, or your data will be meaningless.

Take the same type of data as in Section 3, recording the photon count rate as a function of detector position for both the situation when both slits are open and when only one slit is open. (For the one slit open, use the same slit A or B as you did earlier.) Make a few additional background rate measurements (bulb on, both slits blocked) to get a measure of how many photons reach the detector without passing through either of the two slits. Again, make a rough graph in your notebook as you acquire the data in order to be sure that you are making measurements at the appropriate spacing.

Photon Rate

The PMT is not 100% efficient, meaning it does not detect every photon. For the green light, in fact, it counts only about 4% of the incident photons.

Use your central-maximum reading for the event rate and this 4% efficiency estimate to calculate the rate of arrival of all photons at the PMT. From this arrival rate, compute the average time interval between the arrivals of successive photons. Next, compute the time of flight of a photon in the apparatus, that is, the time it takes to traverse the distance from the bulb source to the detector. Comparing the time of flight of a given photon to the typical time interval between the arrivals of successive photons, compute the fraction of the time there is even one photon in flight through the apparatus. This fraction is the probability of finding a photon in the box at any one time; the probability that there are ever **two** relevant photons in the box is truly tiny, of the order of the **square** of this probability. This is the sense in which this apparatus allows you to work 'one photon at a time'; and yet even at

this low rate of photons, you can see phenomena that you attribute to constructive and destructive interference. But, interference of what, with what?

Make a graph of voltage (or photon count rate) versus detector slit position for all four of your data sets (laser light: one slit, two slits; green light: one slit, two slits). Plot the two laser light data sets on one graph, and the two green light data sets on a different graph.

Fit the green light double slit data to Eq. 12.3. You will need to adapt this equation slightly so that it is in terms of your detector slit location. Then define this fit equation in Igor and provide approximate values for the parameters. This apparatus has slits with approximate values $a = 0.085$ mm, $d = 0.353$ mm, and slit lengths of about 10 mm. The detector is 50 cm from the slit plate. The laser light has a wavelength of 670 ± 5 nm, while the filtered bulb light is not as monochromatic and covers the wavelength range from 541 to 550 nm.

Now, using the same parameters for a , d , and λ , fit the green light single slit data. Fit the laser light data in the same way, using the same values for a and d , since those characteristics of the system should not change.

Are you able to fit all four sets of data? How does the wavelength change affect the spacing of the interference maxima? Does your model predict the correct intensity for the maxima?

Questions to ponder and discuss

Why are there detector locations at which the photon arrival rate **increases** when one of the slits is **closed**? How can light, which so clearly propagates as a wave that we can (with this very apparatus) measure its wavelength, also be detected as individual photon events? How can individual photons in flight through this apparatus nevertheless “know” whether one, or both, slits are open, in the sense of giving photon arrival rates which **decrease** when a second slit is opened?

What is the best evidence your experiment provides that light has a wave-like nature? What is the best evidence your experiment provides that light has a particle-like nature? What is the concept of duality, and why was it invented? Does duality satisfy you, or is the behavior of light paradoxical? Or, is it the character of your description of light that is paradoxical?

Finally, have you confirmed Dirac’s dictum that photons only ever interfere with themselves or does the phenomenon of photon bunching undercut that confirmation?

Acknowledgments

These guidelines are based partly on the TeachSpin “Two-Slit Interference, One Photon at a Time” expanded operating manual.

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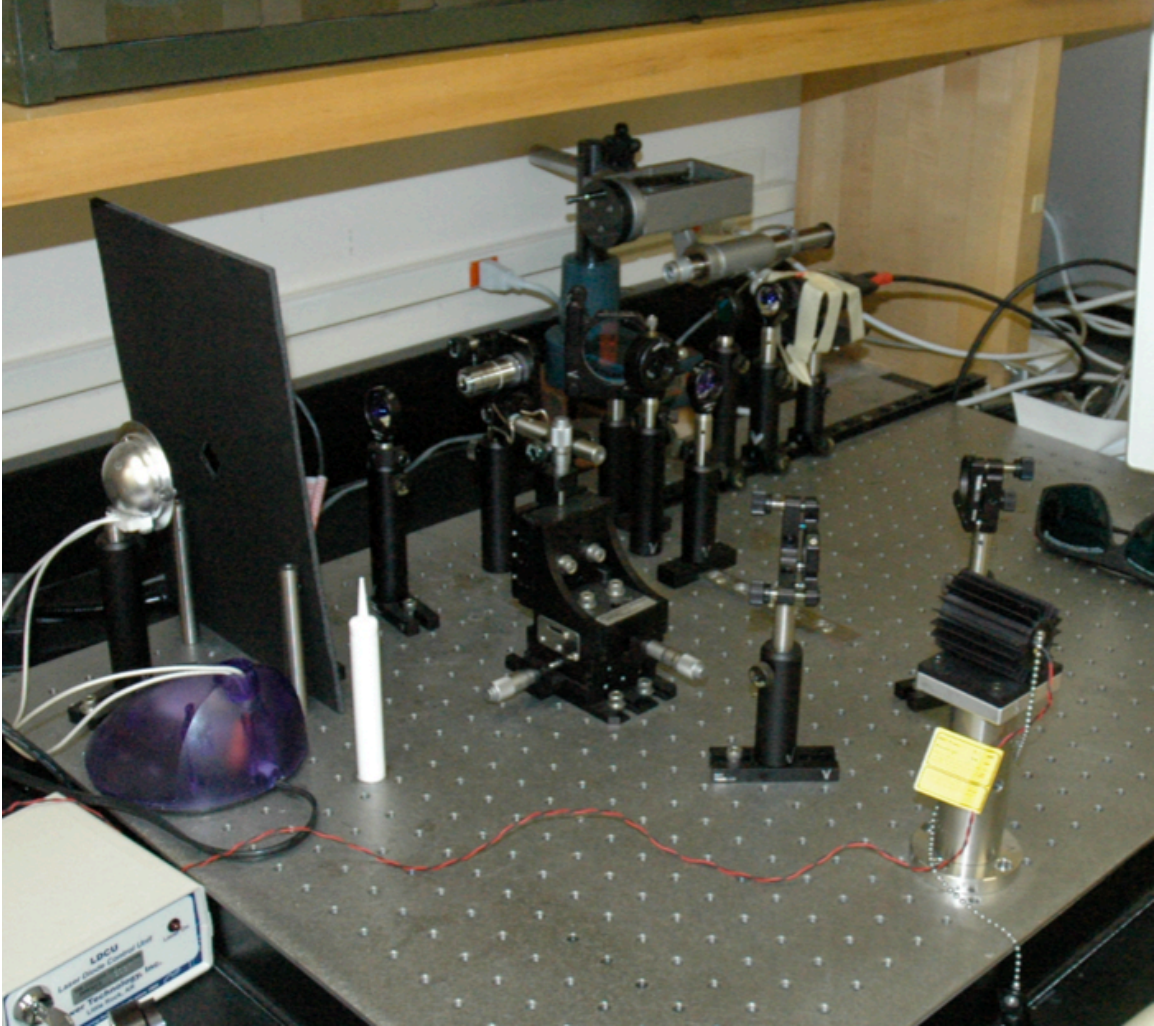


FIG. 13.1. Ashkin experiment equipment includes a 30 mW diode laser, a focusing lens, a web cam, an optics breadboard, and a QuickTime computer interface.



13. Ashkin: Optical Tweezers

Last updated 2012 January 6

Introduction

In 1970, just ten years after the invention of the laser, Arthur Ashkin detected gradient forces on micrometer-sized particles in laser beams [1]. By 1986, he and his team had trapped microscopic glass beads using a tightly focused laser beam [2]. Shortly thereafter, they used the optical trap, which Ashkin referred to as *optical tweezers*, to trap a living *E. coli* bacterium. Since then, optical tweezers have been used for a multitude of research in single molecule biophysics, including blood cell analysis, studies on the mechanics of molecular motors, as well as unzipping DNA [3-4]. The particles within the trap can be manipulated with very fine control; we can apply forces of piconewtons and measure displacements in the nanometer to micron range. The most powerful optical tweezers can trap and control single atoms.

Theory

For large particles, ray optics is sufficient to understand the trapping mechanism. The photons from the laser source carry momentum, which can be transferred when the photons in a light ray are refracted by the dielectric interface of the particle [5]. In order to conserve momentum, the particle is deflected in the opposite direction as the refracted light ray, as shown in Fig. 13.2, where the dark gray arrows denote the *recoil* force on the particle due to the light, which is equal in magnitude and opposite in direction to the *deflecting* force on the light ray due to the particle. Since the beam increases in intensity near its center, the net force on the particle is directed inwards, towards the peak, creating a transverse trap. For a focused beam, as in Fig. 13.3, there is also net axial restoring force, creating an axial trap.

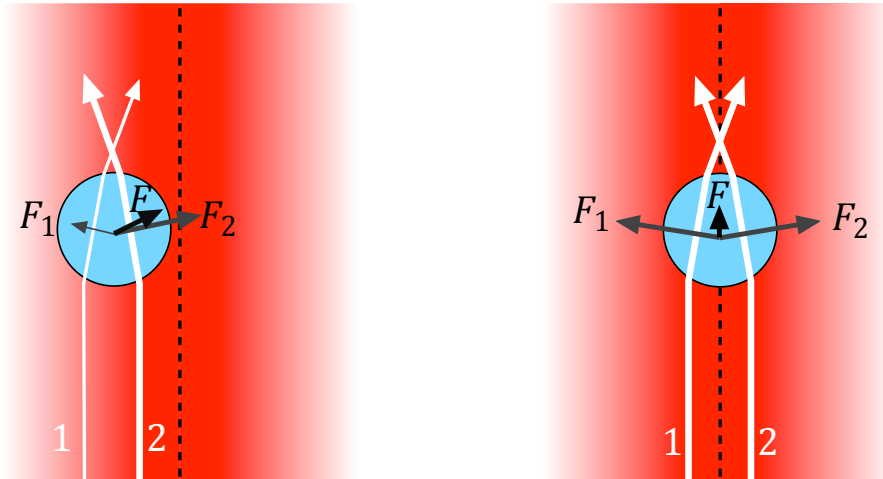


FIG. 13.2: Unfocused laser beam with a Gaussian profile is more intense at the center, so the recoil due to light refracting through a sphere forces the sphere toward the beam's center, where it is transversely stable.

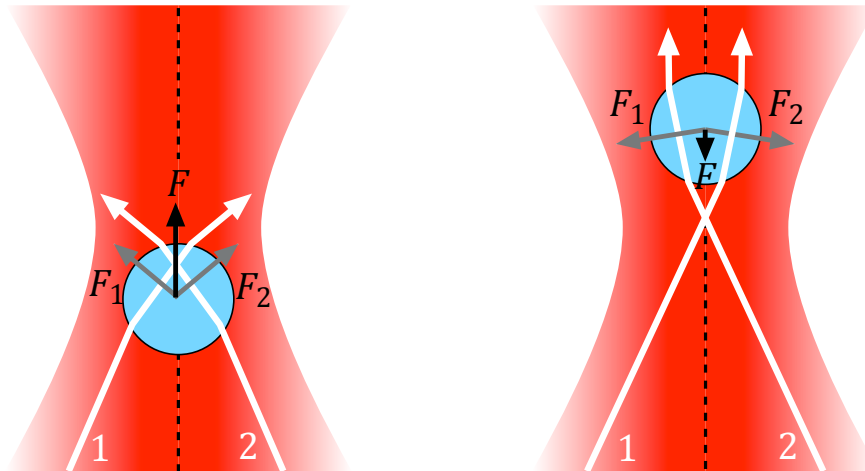


FIG. 13.3: Focused laser beam is also axially stable, so the recoil due to light refracting through a sphere forces the sphere toward the focal point.

The trapping potential for the sphere is approximately parabolic (like a mass on a spring), so statistically, the particle position should be related to its thermal energy by

$$\frac{1}{2}k_x\langle x^2\rangle = \frac{1}{2}k_BT, \quad (13.1)$$

where k_x is the spring constant for the trap along the x axis, x is the deviation of the particle from its average position x_0 , $\langle \cdots \rangle$ denotes averages over the number of measurements N , k_B is Boltzmann's constant, and T is the absolute temperature.

Equipment

A 30 mW diode laser with a wavelength of $\lambda = 658$ nm is used for this optical trap. **Although this laser is in a small package, it is more than 100 times as powerful**

as an ordinary laser pointer! Do not bend down to put your eyes at beam level; do not put objects (especially shiny ones like watches and rings) into the beam path. Safety goggles should be worn whenever you need to change the laser beam path.

The laser beam is sent through a number of lens and a linear polarizer and reflected from a dichroic mirror D so that it is incident on the microscope objective. (The dichroic mirror transmits light in the blue spectrum and reflects red light.) A 1.25 numerical aperture (NA) objective lens sharply focuses the laser beam onto the sample slide placed immediately after the objective. The slide contains a cell for the polystyrene (PS) spheres and is mounted on a three-dimensional translation stage. The sample is illuminated by light from a 20 W light bulb focused with a two-lens optical condenser, as in Fig. 13.4.

Finally, the sample is imaged using a CCD web camera. The imaging branch is directly behind the dichroic mirror. A lens focuses the image of the sample on a CCD camera connected with a firewire cable to an iMac computer. The camera image can be viewed using iMovieHD. For further details on the system, consult references [6] and [7].

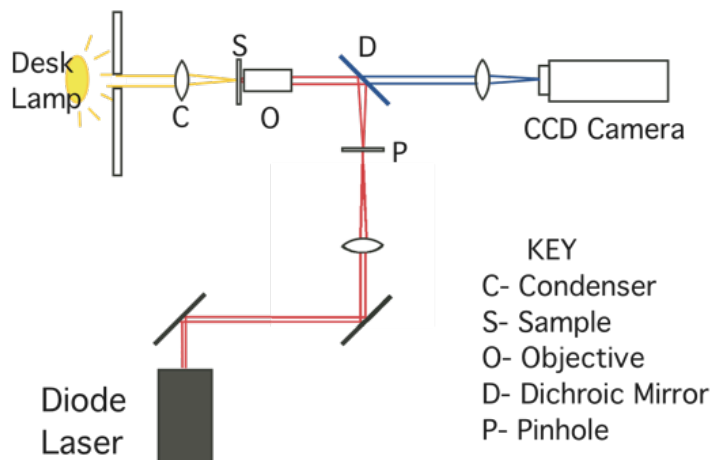


FIG. 13.4: Schematic of the optical tweezers apparatus.

Procedure

Turn on the laser to allow it to warm up; place the beam block just after the laser to keep it from going through the objective. Using a dropper, place a small quantity of the PS sphere solution along the top of the cell. Capillary forces will pull the solution into the cell. **Place a drop of oil between the slide and the objective lens to aid focusing.** Adjust the imaging system to focus on a sample of onion roots. Next observe the PS spheres. Finally, unblock the laser. Watch for a PS sphere to be trapped in the tweezers.

When the trap is working, make measurements of the particle's position over time. By graphing a histogram of particle positions, take the variance of the distribution to be the spring constant k_x or k_y . The trap strength should depend linearly on the

power of the laser. Adjust the power by rotating the polarizer on the laser path, and graph the spring constant versus power to check Eq. 13.1.

Acknowledgments

We thank Henry Timmers and Heather Moore for assistance with this entry.

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Thermal Physics

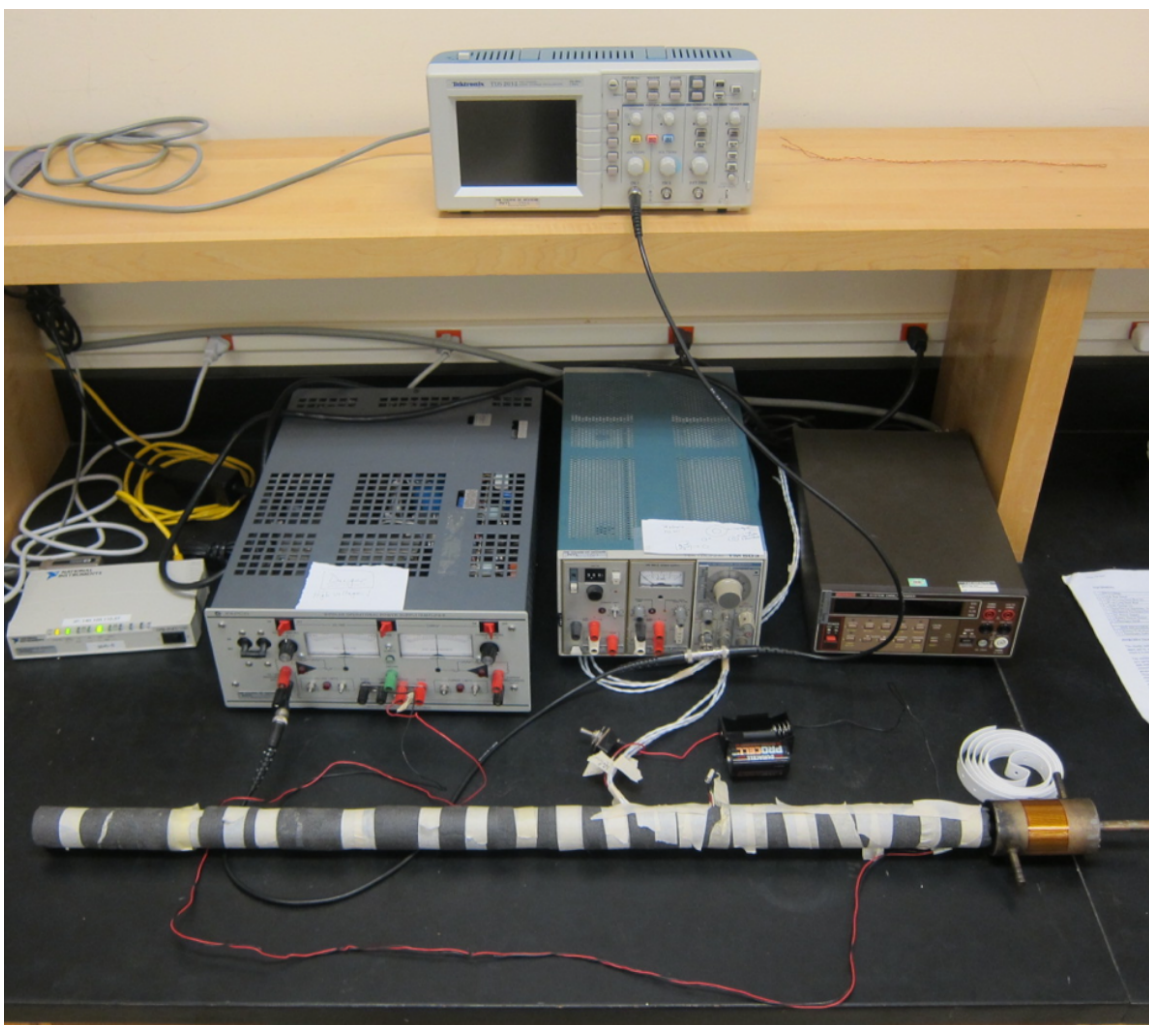


FIG. 14.1. Ångström experiment equipment includes a brass rod and thermistors, a power supply, a function generator, a multimeter, an oscilloscope, and a LabVIEW computer interface.



14. Ångström: Thermal Conductivity

Last updated 2012 March 4

Introduction

In 1861, Anders Jonas Ångström published a clever method [1] for determining the thermal conductivity of a body. If one end of a bar of the material is heated periodically and the temperature as a function of time is monitored at two points along the bar, a simple formula gives the conductivity. A later refinement [2] employs the Fourier analysis of a simple square wave heating function, where the heater is on only every other half cycle.

Theory

Sinusoidal Input

Let $T[x, t]$ be the bar's temperature *relative to the air*. Sinusoidally vary the temperature at the rod's end according to

$$T[0, t] = T_0 \cos[\omega t], \quad (14.1)$$

where T_0 is the amplitude and ω is the frequency. The heat will diffuse along the bar and convect away through the air (via Newton's cooling law) so that

$$\frac{\partial T}{\partial t} = D \nabla^2 T - \epsilon T, \quad (14.2)$$

where the thermal diffusivity $D = \kappa / s\rho$ is the ratio of the thermal conductivity κ to the product of the specific heat s and the density ρ , and the emissivity $\epsilon = RC / s\rho\mathcal{A}$ is the ratio of the product of the emission coefficient R and the circumference C to the product of the specific heat s and the density ρ and cross sectional area \mathcal{A} .

Expect the temperature fluctuations to vary sinusoidally with exponentially decaying amplitude along the bar, so guess a solution of the form

$$T[x, t] = Ae^{-ax} \cos[\omega t - bx] \quad (14.3)$$

and substitute into Eq. 14.2 to find

$$D = \frac{\omega}{2ab} \quad (14.4)$$

and

$$\epsilon = (a^2 - b^2)D, \quad (14.5)$$

which can be readily inverted to solve for the parameters a and b .

Monitor the temperature at two points distances $x_L > x_R$ from the end. Based on Eq. 14.3, the amplitude ratio

$$\frac{A_L}{A_R} = e^{-a(x_L - x_R)} > 1 \quad (14.6)$$

and the phase difference

$$\varphi_L - \varphi_R = b(x_L - x_R) > 0. \quad (14.7)$$

Invert these relations to find the parameters

$$a = \frac{\log[A_L/A_R]}{x_R - x_L} > 0 \quad (14.8)$$

and

$$b = \frac{\varphi_L - \varphi_R}{x_R - x_L} > 0. \quad (14.9)$$

Hence, the thermal conductivity is

$$\kappa = s\rho D = \frac{s\rho\omega}{2} \frac{(x_R - x_L)^2}{(\varphi_L - \varphi_R) \log[A_L/A_R]}. \quad (14.10)$$

Square Wave Input

Think of a square wave input as an infinite superposition of sinusoids [3]. Fourier analyze the time series to isolate the contributions of the individual sinusoids

$$T[t] - \langle T \rangle = \sum_{n=1}^{\infty} (\alpha_n \cos \omega_n t + \beta_n \sin \omega_n t) = \sum_{n=1}^{\infty} \gamma_n \cos[\omega_n t + \varphi_n], \quad (14.11)$$

where $\omega_n = n\omega_1 = n2\pi/\tau$ and the amplitudes

$$\alpha_n = \frac{2}{\tau} \int_{t_0}^{t_0+\tau} T[t] \cos \omega_n t \, dt \quad (14.12a)$$

and

$$\beta_n = \frac{2}{\tau} \int_{t_0}^{t_0+\tau} T[t] \sin \omega_n t \, dt. \quad (14.12b)$$

In Eq. 14.10, use the angular frequency ω_n and the phases $\varphi_n = \arctan[\beta_n/\alpha_n]$ and amplitudes $\gamma_n = \sqrt{\alpha_n^2 + \beta_n^2}$ for the n th harmonics.

Equipment

The Fig. 14.2 equipment includes a cylindrical rod of brass, an alloy of copper and zinc, approximately 1 m in length and 1 cm in diameter. A **dangerous** Kepco power supply powers a thermfoil heater attached to a reservoir in thermal contact with the rod. A Tektronix function generator provides the square wave signal to the power supply with a frequency of about 1 mHz. Foam insulation and bubble wrap surround the entire rod to limit loss of heat through radiation and conduction.

Two YSI 44004 Precision thermistors are housed in small holes drilled into the rod about 15 cm apart and held in place by a thermally conductive epoxy. The thermistors are wired into a series circuit with a reference resistance of 15 k Ω , and a 1.5 V dry cell battery. The thermistors resistance depends on their temperature, and the reference resistor provides a means of determining the current through the circuit. Each resistor is connected to a channel of the Hewlett Packard 3421A Data Acquisition Unit, which measures the voltage across each one. LabVIEW automates the data collection.

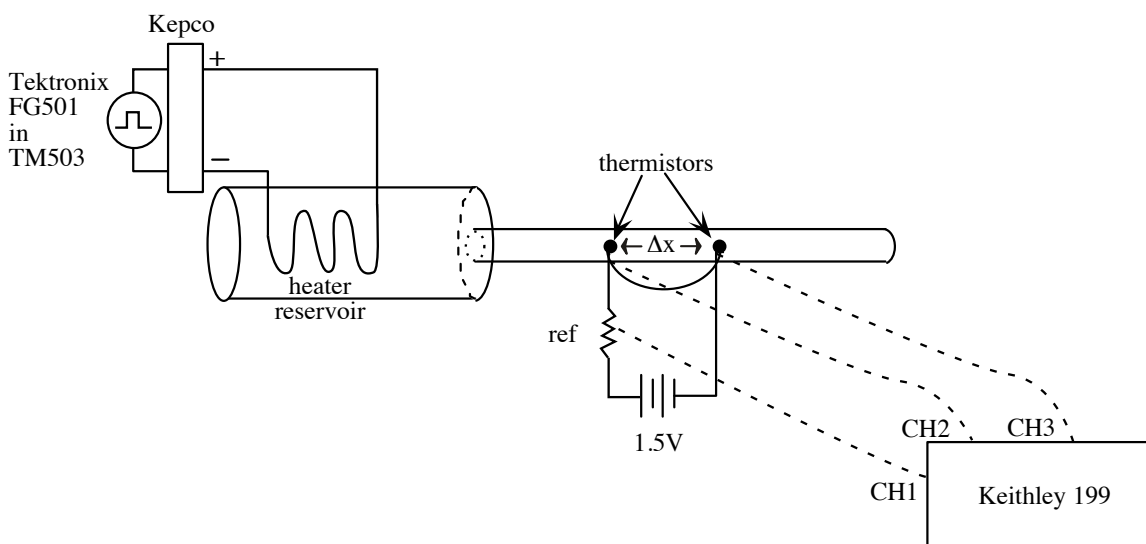


FIG. 14.2: Ångström apparatus schematic.

Procedure

The system needs hours to reach a steady state, and data collection requires many hours overnight. To facilitate the Fast Fourier Transform, adjust the driving frequency and collection time to obtain a large power-of-2 number of data points and an integer number of forcing periods. Use Eq. 14.10 to compute the rod's thermal conductivity using each of the first few harmonics, and compare with the accepted value.

To begin taking data, first power on the apparatus. Turn on the system digital multimeter scanner by pressing the red power button on the bottom left. This measures the resistance across the thermistors. Make sure that the input is set to rear, where the cables are attached. Second, turn on the function generator by pulling the black "pwr" tab on the left side. This controls the function that powers the thermofoil. The only part of its interface that is useful is the right third, however.

Next, turn on the oscilloscope, whose power button is on the top left. It should be receiving the output from the function generator as well as the power supply, so one can see the square wave being generated. Next, turn on the power supply by flipping its large double switch on the left. It will make a menacing racket, but that is normal. Be wary of it, since it produces high voltages. It should be responding to the function generator's input.

A small D-cell battery also powers the thermistors. Pop it into its holder and flip the attached switch to apply a voltage across the thermistors. The switch is there to make sure the battery has a long life, since a dying battery could result in confusing and wrong readings.

Next, check to make sure that the computer interface slightly behind the computer is turned on. On the computer's desktop should be a file labeled "ThermCondLab.K199". Double click on it to open a LabVIEW interface. To start

taking data, set the interface's giant switch to "Go" **before** pressing run (the white arrow in the top bar). If you run the LabVIEW program before flipping the switch to "Go", the system will only take one data point. It will continue to display data on the chart, which you can use for debugging and setup. **However, just because there is data on the plot does not mean that the system is storing it.** In order to begin recording data, flip the switch to "Go" before running the program. To finish taking data, simply flip the interface's switch to "Stop". After doing so, do not press the LabVIEW stop sign button, simply wait. After a few seconds, a dialogue box will appear offering to save the data file. Save this in the location of your choice. If you do not get this dialogue box before you halt the LabVIEW program, the computer will not save any data. To clear the chart of any data points plotted on it (which is not the computer's actual saved data but only a display of that data), simply right click the chart, select "Data Operations" and then "Clear Chart".

Adjust the function generator so that it outputs a square wave. Change its frequency so that the period is very large (on the order of a several minutes). Check to make sure that the thermistors are both showing clear periodic change. At this point run the experiment overnight to collect a long trial that contains little fluctuations in room temperature.

The raw data may have to be "cleaned up" because occasionally the interface measures a single point to be positive or negative infinity, an obvious error that could thwart efforts to Fourier analyze the data. Fix these anomalous points by replacing them by the average of the two adjacent points.

Acknowledgments

The guidelines are based partly on the unpublished work of B. R. Russell and D. T. Jacobs and on reports by Amy Lytle and Karl Smith.

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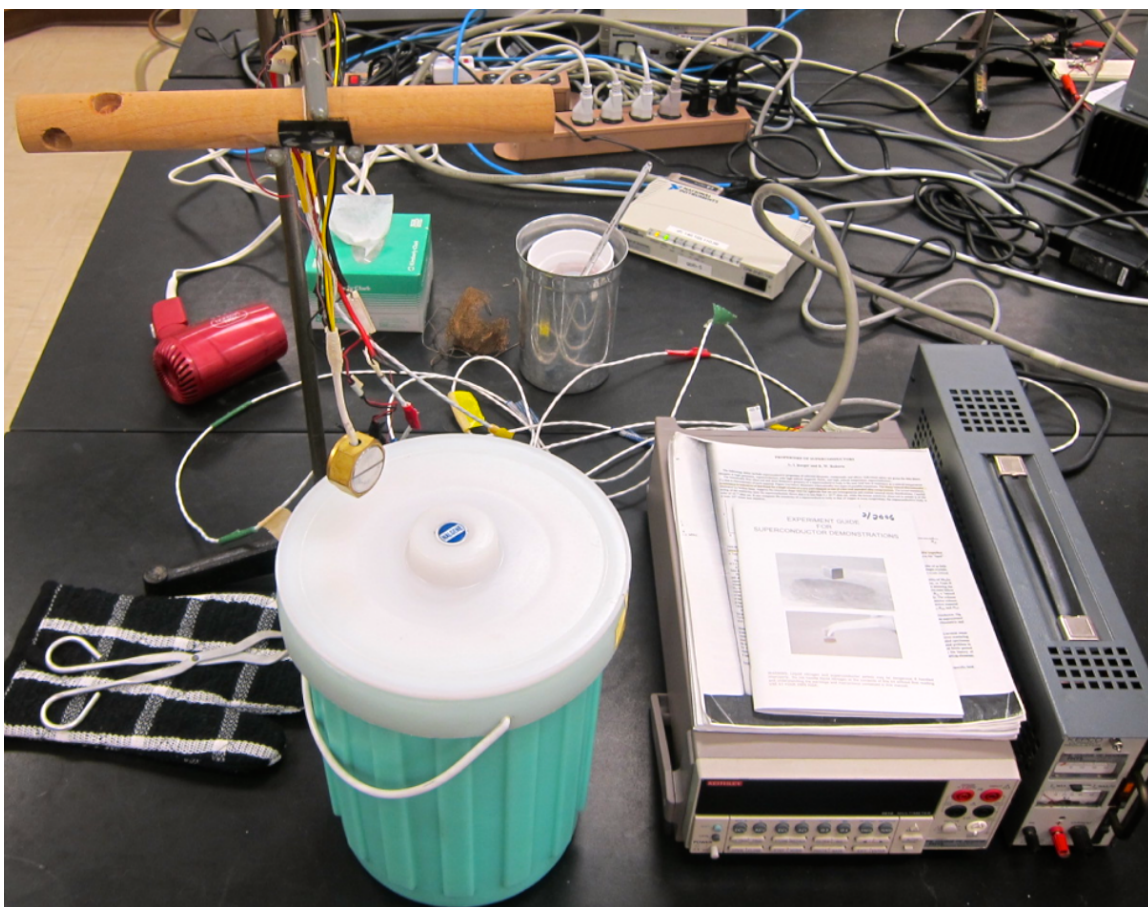


FIG. 15.1. Kamerlingh Onnes experiment equipment includes an YBCO superconducting disk in a brass casing, a constant current supply, a multimeter, and a liquid nitrogen Dewar.



15. Kamerlingh Onnes: Superconductivity

Last updated 2012 March 4

Introduction

Heike Kamerlingh Onnes devoted his career to achieving low temperatures and exploring the behavior of matter there. In 1898 his rival James Dewar beat him to liquefying hydrogen, but in 1908 he beat Dewar to liquefying helium. Three years later, Kamerlingh Onnes discovered superconductivity [1, 2] in mercury. In his laboratory book 56 at 4 PM on 1911 April 8, he wrote “Kwik nagenoeg nul”, which means “Quicksilver [resistance] practically zero [at 3 K]”. His team quickly demonstrated that below critical temperatures T_c of a few kelvins lead and tin were also superconductors.

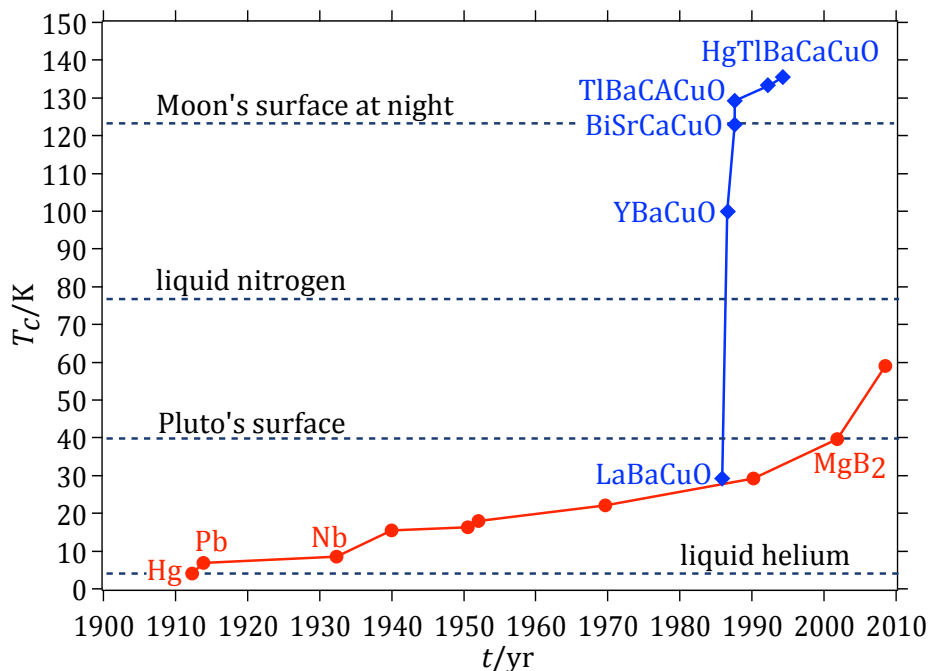


FIG. 15.2. History of highest superconducting critical temperatures.

Theory

Superconductivity's zero electrical resistance – and especially its expulsion of magnetic fields in the *Meissner effect* – are non-classical quantum effects. Not until 1956 did John Bardeen, Leon Cooper, and Robert Schrieffer (BCS) present a satisfactory microscopic theory of superconductivity [3] in which spin-half fermionic electrons interact weakly via quantized lattice vibrations called phonons to form spin-zero bosonic pairs that condense into a superfluid current. However, the BCS theory does not explain the ceramic copper-oxide high- T_c superconductors discovered in 1986-1987 [4], which make practical this experiment, as is clear from Fig. 15.2.

Electrons are *fermions* with spin angular momentum $\hbar/2$. According to the Pauli exclusion principle, fermions exclude one another, so that no two are ever in the same state. In low-temperature superconductors, electrons form nonlocal spin zero *Cooper pairs*, which are *bosons*. Bosons can condense into a single ground state with a nonzero current. The ionic lattice mediates the weak interaction that binds the Cooper pairs, as in Fig. 15.3. Because the effective binding energy is only a few meV over hundreds of nm, this phenomenon is too weak to work at high temperatures, and the mechanism(s) of high temperature superconductivity are still not understood, although the exchange of spin waves may be involved.

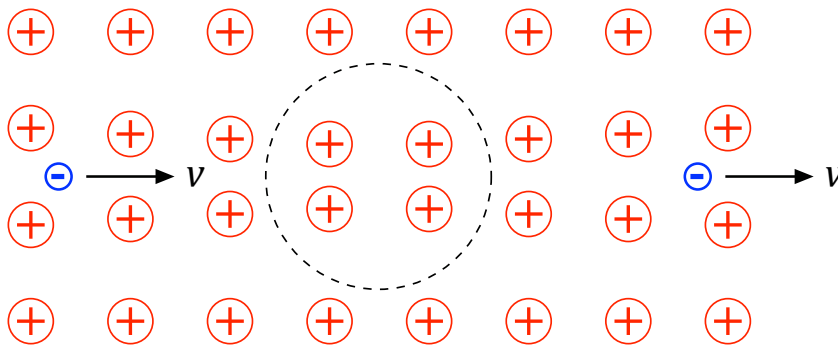


FIG. 15.3. Like one bicycle drafting another, the bunching of positive ions in the wake of a negative electron attracts another electron and weakly binds them.

Equipment

Four-Point Probe

This experiment uses a four-point electrical probe, which is a very versatile device used widely in physics for the investigation of electrical phenomena. Colorado Superconductor Inc. has especially designed two four-point superconducting devices from the $\text{YBa}_2\text{Cu}_3\text{O}_7$ and the $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$ materials for such investigations

When a simple measurement of the electrical resistance of a test sample is performed by attaching two wires to it, one inadvertently also measures the resistance of the contact point of the wires to the sample. Typically the resistance of

the point of contact (called contact resistance) is far smaller than the resistance of the sample, and can thus be ignored. However, when one is measuring a very small sample resistance, especially under variable temperature conditions, the contact resistance can dominate and completely obscure changes in the resistance of the sample itself. This is the situation that exists for superconductors.

The effects of contact resistance can be eliminated with the use of a four-point probe. A schematic of a four-point probe is shown in Fig. 15.4. In this diagram, four wires (or probes) have been attached to the test sample. A constant current is made to flow the length of the sample through probes labeled 1 and 4 in the figure. This can be done using a current source or a power supply as shown. Many power supplies have a current output readout built into them. If not, an ammeter in series with this circuit can be used to obtain the value of the current. A 5 W power supply capable of producing up to 0.5 A is required for the experiments described for our superconducting devices.

If the sample has any resistance to the flow of electrical current, then there will be a drop of potential (or voltage) as the current flows along the sample, for example between the two wires (or probes) labeled 2 and 3 in the figure. The voltage drop between probes 2 and 3 can be measured by a digital voltmeter. The resistance of the sample between probes 2 and 3 is the ratio of the voltage registering on the digital voltmeter to the value of the output current of the power supply. The high impedance of the digital voltmeter minimizes the current flow through the portion of the circuit comprising the voltmeter. Thus, since there is no potential drop across the contact resistance associated with probes 2 and 3, only the resistance associated with the superconductor between probes 2 and 3 is measured.

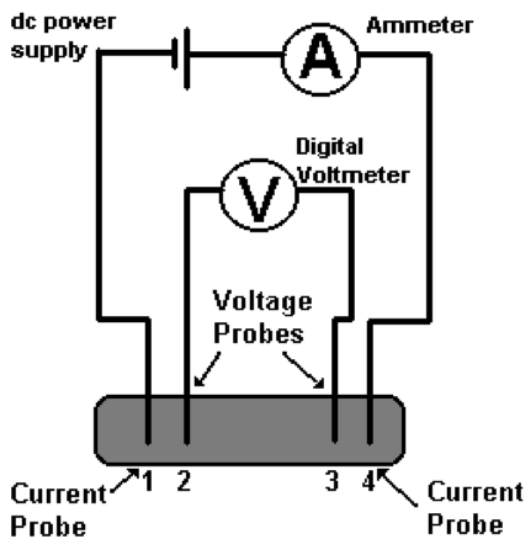


FIG. 15.4. Four-point probe schematic.

The four-point probe is encapsulated in rugged brass casing. On one side of the casing, the superconductor disk is visible. An aluminum end cap has been inserted on the backside of the brass casing to seal and to protect the probe connections with

the superconductor. Please do not attempt to remove the end cap. A matched thermocouple has also been attached to the superconductor in this casing. This thermocouple is a type 'T'.

The $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$ superconductor four point electrical probe casing is larger than the $\text{YBa}_2\text{Cu}_3\text{O}_7$ casing. The aluminum backs of the probes have either BSCCO (pronounced “bis-coe”), or YBCO (pronounced “ib-coe”) printed on them, for further identification.

The Fig. 15.5 illustration shows the salient features of the four-point probe devices. The two black wires are current leads for the input of current from the power supply and are labeled probes 1 and 4. The two yellow wires are the voltage measurement probes for measuring the voltage drop across the superconductor with the help of a digital voltmeter and are labeled probes 2 and 3. The red and blue wires are leads for the thermocouple. **Be careful not to break these fragile wires!**

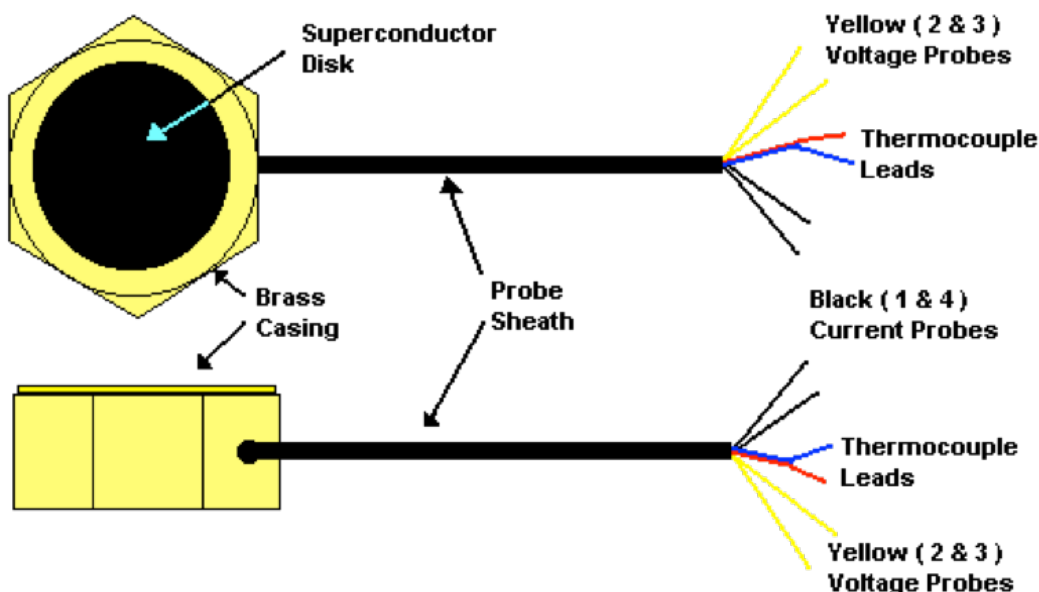


FIG. 15.5. Four-point probe casing and wiring.

Ancillary Equipment

The measurement of electrical resistance as a function of the superconductor's temperature yields fundamental insights into its properties. The critical temperature (as well as the critical current density and critical magnetic field for the breakdown of superconductivity) can be obtained through variations of this basic experiment.

In addition to the four-point probe, this experiment requires a constant current source or a power supply operating in the current limited mode. The output should not exceed 0.5 A. This is connected between the black current probes (probes 1 and 4). An ammeter placed in series with this circuit will measure the current I_{14} . **When connecting and disconnecting probes, make sure the power supply is turned**

all the way down. If the probes are disconnected or connected to an active power supply, they will be damaged.

A digital voltmeter with a 0.01 mV resolution measures the voltage drop V_{23} across the yellow voltage probes (probes 2 and 3). A Dewar of liquid nitrogen is deep enough to completely immerse the four-point probe device.

The voltmeters should be connected as shown in Fig. 15.4 to record of the voltage drop. The thermocouple reading can also be measured simultaneously. The output from the voltmeters connected to probes 2 and 3, and to the thermocouple, is sent directly to a computer running a LabVIEW program to display, store, and further analyze the data.

Precautions

When pouring liquid nitrogen be careful to prevent any splashing. Be careful not to touch the device or wires when they are cold. Follow the safety directions. No more than 0.5 A of current should pass through the device at any time. Use a hair dryer to carefully dry the four-point probe after use. Store it with a desiccant. The probe and thermocouple wires are very brittle when cold. Please handle them with care.

Procedure

Set up the equipment as described above, but do not as yet immerse the device (four point probe) in liquid nitrogen. The voltage V_{23} across probes 2 and 3 should measure between 2 and 5 mV. Ensure that the pushbutton on the front of the multimeter is depressed so that the multimeter will report data from its rear (where the four-point probe is attached) rather than its front.

Launch the LabVIEW program “SupercondK2000”, which will produce a text file with resistance versus temperature. With the multimeter on, choose “Refresh” from the “Visa resource name” drop-down menu, and then reopen the drop-down menu and select the multimeter name (for example, GPIB0::16::INSTR). The program uses the empirical calibration equation

$$T = 286.9 - 49.765V + 5.9041V^2 - 0.90151V^3 + 0.060915V^4 \quad (15.1)$$

to convert voltage in volts to temperature in kelvins.

Carefully immerse the device in liquid nitrogen. Use the white, sheathed wire bundle to suspend the device in the liquid. Ensure that the current I_{14} remains constant at 0.5 A. The nitrogen boils furiously. Wait until the boiling subsides.

Record the voltage V_{23} and across the thermocouple junction, which should be zero. The thermocouple temperature reading should be 77 K.

Remove the device from the liquid nitrogen but keep it in the Dewar as it warms with the top partially covered by the insulating glove (else the brass ring outside the probe will warm too fast and contaminate the data). As the device warms, continuously monitor the value of V_{23} . Record the thermocouple reading each time V_{23} is recorded. Initially, V_{23} remains constant even as the thermocouple reading increases. Later, the voltage V_{23} between the probes abruptly increases. The thermocouple reading corresponding to this jump in voltage is the critical temperature T_c of the superconductor. The ratio of the voltage V_{23} between probes 2 and 3 to current I_{14} flowing between probes 1 and 4 is the instantaneous resistance of the superconductor between probes 2 and 3.

What effect would one expect if the critical temperature were measured with the device placed inside a functioning electromagnet? Why is the transition in resistance gradual at the critical temperature? Why does a simple two-probe measurement of device resistance below its Critical Temperature exhibit a non-zero value?

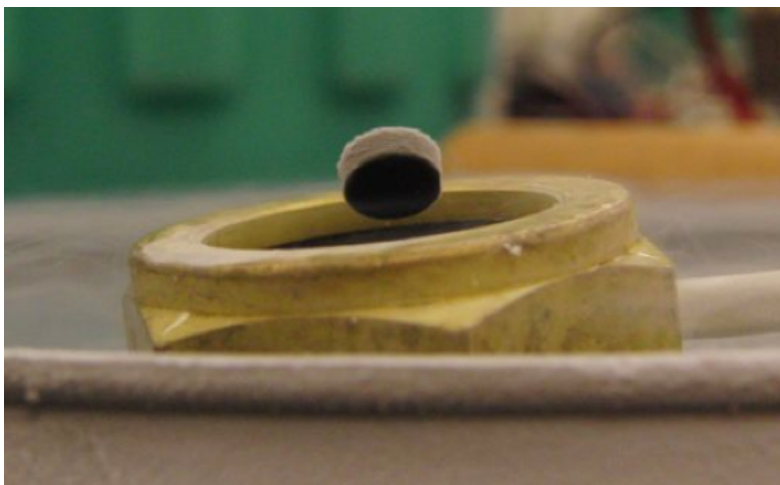


FIG. 15.5. A rare-earth magnet hovers above an YBCO superconductor.

For fun and amazement, **float a neodymium magnet on top of the superconductor**. Why doesn't it fall? Why doesn't it slide off?

Acknowledgments

These guidelines are based partly on the Colorado Superconductor instruction manual for superconducting demonstrations and on reports by Duncan Price and Dan Axe.

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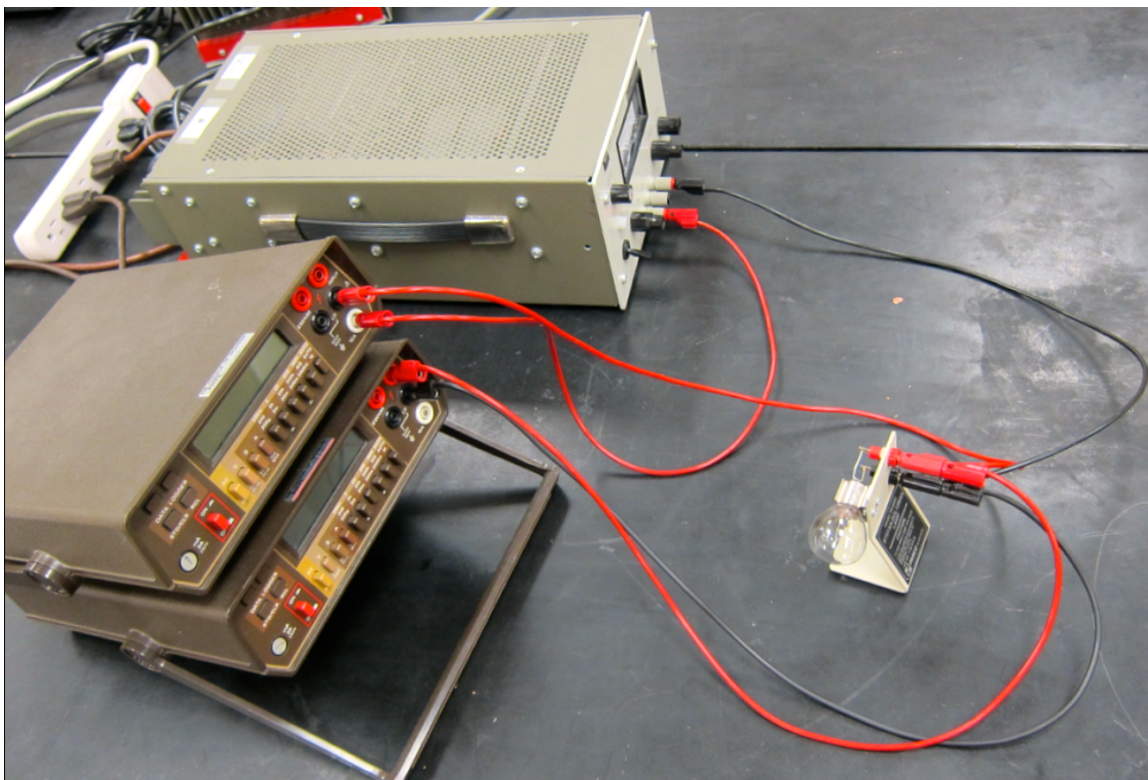


FIG. 16.1. Stefan-Boltzmann experiment equipment includes a tungsten lamp, a power supply, and two multimeters.



16. Stefan-Boltzmann: Thermal Radiation

Last updated 2012 March 24

Introduction

In 1879, Jožef Štefan empirically inferred from data taken by John Tyndall that the power radiated by a thermal or “black” body is proportional to the fourth power of its absolute temperature [1]. Five years later, in 1884, one of Štefan’s students, Ludwig Boltzmann theoretically derived this relation from the principles of thermodynamics [2].

Theory

The power P radiated by a body in thermal equilibrium at temperature T is

$$P = \epsilon \sigma A T^4, \quad (16.1)$$

where A is the surface area and

$$\sigma = \frac{2\pi^5 k^4}{15c^2 h^3} = 56.7 \frac{\text{nW}}{\text{m}^2 \text{K}^4} = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4} \quad (16.2)$$

is the Stefan-Boltzmann constant. The dimensionless emissivity $\epsilon = 1$ for an ideal “black” body and $\epsilon < 1$ for a non-ideal “gray” body.

If the body is a tungsten filament at temperature T_f , the power input includes an electric current I_f driven by a voltage V_f and the radiation absorbed from the ambient temperature T_a . This is balanced by the power radiated away and lost by conduction. Hence,

$$P_e + P_a = P_{\text{in}} = P_{\text{out}} = P_r + P_c \quad (16.3)$$

and so

$$P_e = I_f V_f = \epsilon \sigma A T_f^4 - \epsilon \sigma A T_a^4 + \kappa \ell (T_f - T_a), \quad (16.4)$$

where κ is the conductivity and ℓ is a characteristic length scale [4].

The resistivity ρ of tungsten increases with absolute temperature T . It has been well tabulated [3] and is approximately fit by the power law

$$\frac{\rho}{\text{n}\Omega \cdot \text{m}} = 0.06331 \left(\frac{T}{\text{K}} \right)^{1.197}, \quad (16.5)$$

which can be inverted to enable the resistivity to act as a thermometer for the temperature of the filament. Since the resistivity is proportional to the resistance,

$$\rho_f = \rho_a \frac{R_f}{R_a} = \rho_a \frac{V_f}{I_f R_a}, \quad (16.6)$$

where the resistance R_a at ambient (room) temperature can be found by sending small currents I_a through the filament and measuring the corresponding voltages V_a .

Equipment

The Fig. 16.1 apparatus includes a PASCO Stefan-Boltzmann lamp, two multimeters and a power supply configured according to the Fig. 16.2 schematic. The lamp is a light bulb with a tungsten filament that can withstand up to 13 V or 3 A.

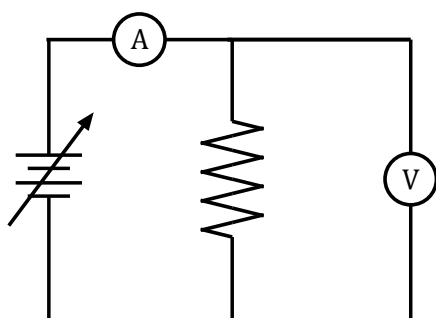


FIG. 16.2: Stefan-Boltzmann circuit schematic.

Procedure

First find the tungsten filament's ambient resistance R_a . Then use the Eq. 16.5 quadratic fit to the filament's temperature T_f as a function of its resistivity ρ_f to test Eq. 16.4. Does the tungsten filament obey the Stefan-Boltzmann law?

Acknowledgments

These guidelines are based partly on a report by Austin Carter.

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FIG. 17.1. Brown experiment equipment includes a Brookhaven Instruments Photon Correlation Spectrometer (goniometer, photodetector, correlation board), a laser, a multimeter, polystyrene spheres, and a Brookhaven computer interface.



17. Brown: Photon Correlation Spectroscopy

Last updated 2012 March 24

Introduction

In June 1827, botanist Robert Brown used a magnifying glass to observe tiny particles, which were ejected from pollen immersed in water, moving randomly [1,2]. In 1905, Albert Einstein explained the motion of such particles as resulting from random collisions with water molecules [3]. The small particles move in a random walk, now referred to as *Brownian motion*. Einstein used Brownian motion to determine the size of the water molecules and help confirm the existence of atoms. (This was just one of five key insights Einstein published in 1905, his *Annus Mirabilis*.) In this experiment, you will use the Brownian motion of tiny objects in water to determine their sizes.

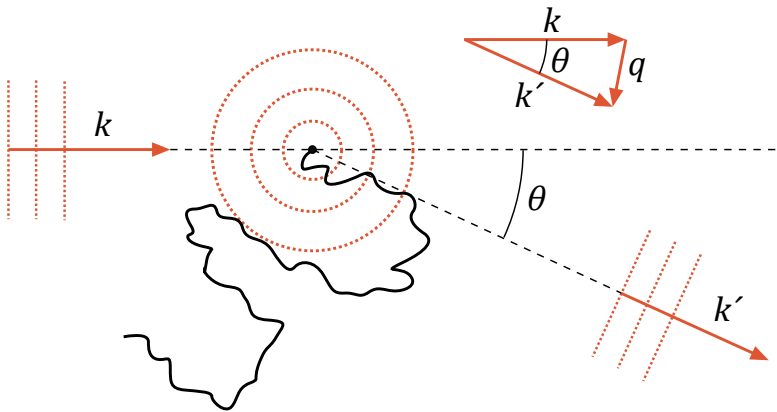


FIG. 17.2. Scattering geometry.

Theory

In *dynamic light scattering* or *photon correlation spectroscopy*, a coherent beam of light scattering from suspended particles undergoing Brownian motion can reveal their sizes. Represent the incident electric field by (the real part of)

$$E = E_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}. \quad (17.1)$$

Then the scattered wave far away at the detector is approximately [4]

$$E_s \approx E_{s0} e^{i(\omega t - \vec{q} \cdot \vec{r})}, \quad (17.2)$$

where from the Fig. 17.2 geometry, the spatial frequency change

$$\vec{q} = \vec{k}' - \vec{k} \quad (17.3)$$

has magnitude

$$q = 2k \sin \frac{\theta}{2} \quad (17.4)$$

because the scattering is elastic with $k' = k = 2\pi/\lambda$. The Brownian particle executes a random walk with Gaussian probability density

$$\rho[r, t] = (4\pi Dt)^{-3/2} e^{-r^2/4Dt}, \quad (17.5)$$

where the Einstein-Stokes diffusion constant

$$D = \frac{k_B T}{3\pi\eta d} \quad (17.6)$$

and T is the fluid's temperature, η is its viscosity, and d is the particle's diameter [3]. The normalized *electric field correlation function* is the average $\langle \cdots \rangle$ of the product of the electric fields at different times,

$$C_1[\tau] = \frac{\langle E_s^*[t] E_s[t + \tau] \rangle}{\langle I \rangle}, \quad (17.7)$$

where the intensity $I = E_s^*[t] E_s[t]$ so that $0 \leq |C_1[\tau]| \leq 1$. Substituting the Eq. 17.2 electric field gives

$$C_1[\tau] = e^{i\omega\tau} \langle e^{i\vec{q} \cdot \vec{r}[t]} e^{-i\vec{q} \cdot \vec{r}[t+\tau]} \rangle = e^{i\omega\tau} \langle e^{-i\vec{q} \cdot \vec{r}[\tau]} \rangle \quad (17.8)$$

by choosing $t = 0$ and $\vec{r}[0] = \vec{0}$ due to the stationarity of the random process. Using the Eq. 17.5 probability density to perform the averaging yields

$$C_1[\tau] = e^{i\omega\tau} \int e^{-i\vec{q}\cdot\vec{r}} \rho[r, \tau] d^3r = e^{i\omega\tau} e^{-Dq^2\tau}, \quad (17.9)$$

because the integral is merely the Fourier transform of a Gaussian (which is necessarily another Gaussian). The normalized *intensity correlation function* is the average of the product of the intensities at different times,

$$C_2[\tau] = \frac{\langle I[t]I[t + \tau] \rangle}{\langle I[t] \rangle^2}, \quad (17.10)$$

so that $1 \leq C_2[\tau]$. Averaging over many Brownian particles yields the Siegert relation [4,5]

$$C_2[\tau] = 1 + |C_1[\tau]|^2 = 1 + e^{-2Dq^2\tau}, \quad (17.11)$$

so that $1 \leq C_2[\tau] \leq 2$ for Brownian motion. For detectors of non-negligible area, a decoherence correction factor multiplies the Eq. 17.11 exponential.

Hence, a (shifted) exponential fit to the intensity correlation function $C_2[\tau]$ gives the diffusion constant D , which then gives the particle diameter d .

Equipment

A Melles Griot HeNe laser provides high intensity monochromatic light. A glass sample vial contains an aqueous sample of standard polystyrene $(C_8H_8)_n$ spheres. The Brookhaven goniometer contains a sample chamber where the vial is immersed in a liquid that has the same refraction index as glass. Water from a water bath circulates through the goniometer to maintain a constant temperature. The detector includes a photomultiplier tube (PMT) and a pulse amplifier-discriminator (PAD). The PMT converts scattered photons into electrical pulses. The PAD filters out random noise and amplifies the pulses so the correlation board in the Dell Vostro 400 computer can process them.

Procedure

Use the Brookhaven apparatus and software to compute the sizes of several samples via Eq. 17.11 and Eq. 17.6 and compare with the labeled sizes.

Turn on the water bath and set it to a temperature that allows for good data, such as 25.0°C. The water in the bath has previously been poisoned with sodium azide NaN_3 to prevent any organisms from growing when the machine is not in use. Manually adjust the temperature by rotating the knob next to the digital output to the position marked ϑ . Press on the button beneath the digital output screen and adjust the dial marked as ϑ to set the temperature on the output screen. Upon the release of the button the water bath heats or cools to that temperature. As the water temperature

adjusts, a green light shines continuously, but once the water reaches the set temperature it flashes to signify stability.

Once the water is stable, use the BIC vacuum filter to pump decahydronaphthalene $C_{10}H_{18}$ through the sample chamber for fifteen minutes to ensure thorough mixing. This mixing not only eliminates any air pockets within the chamber, but it also allows for the temperature of the decahydronaphthalene to be even throughout. The cis-trans mixture of decahydronaphthalene is a solution with a refractive index very similar to glass to prevent unnecessary light scattering.

While the decahydronaphthalene is being mixed, launch the Brookhaven software program and adjust the settings for the first trial of the experiment by choosing the “Parameters” tab from the top left. After the parameters are set, set the experimental vials in order for an easy transition between trials. For example, try vials containing polystyrene spheres of diameters 304 nm, 96 nm, 51 nm, a mixture of 96 nm and 304 nm, and a mixture of 51 nm and 304 nm.

Turn on the laser by using the key on the power supply box. The Melles Griot HeNe laser provides high intensity monochromatic light into the sample chamber. The shutter furthest from the sample chamber, the filter wheel, must be closed, indicated by the dial reading “C”, whenever no data is being taken. When taking data, adjust the filter wheel to “633” when taking data and leave the other dial at “200”.

Before each vial is used in a trial, wipe it to remove any unwanted dust or fingerprints. Place the vial inside the sample chamber and put the lid back on top. Set the software parameters to collect data for five minutes. Once the program is finished collecting data, remove the vial, wiped it again and place it back inside for a second trial. Repeat for two trials of each diameter of polystyrene sphere.

Save the data from each trial. In the program, select the “File” tab, click on “Save Correlation Function As”, click on “Export Function Data”, and continue to save from there. Ensure each trial is clearly labeled in both the saved folder and the parameters to avoid mixing data. Clear program data between runs.

For safety, wear latex gloves throughout the experiment, as naphthalene $C_{10}H_8$ is the main ingredient in traditional mothballs.

Acknowledgments

These guidelines are based partly on a report by Andrew Sopher.

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Appendix

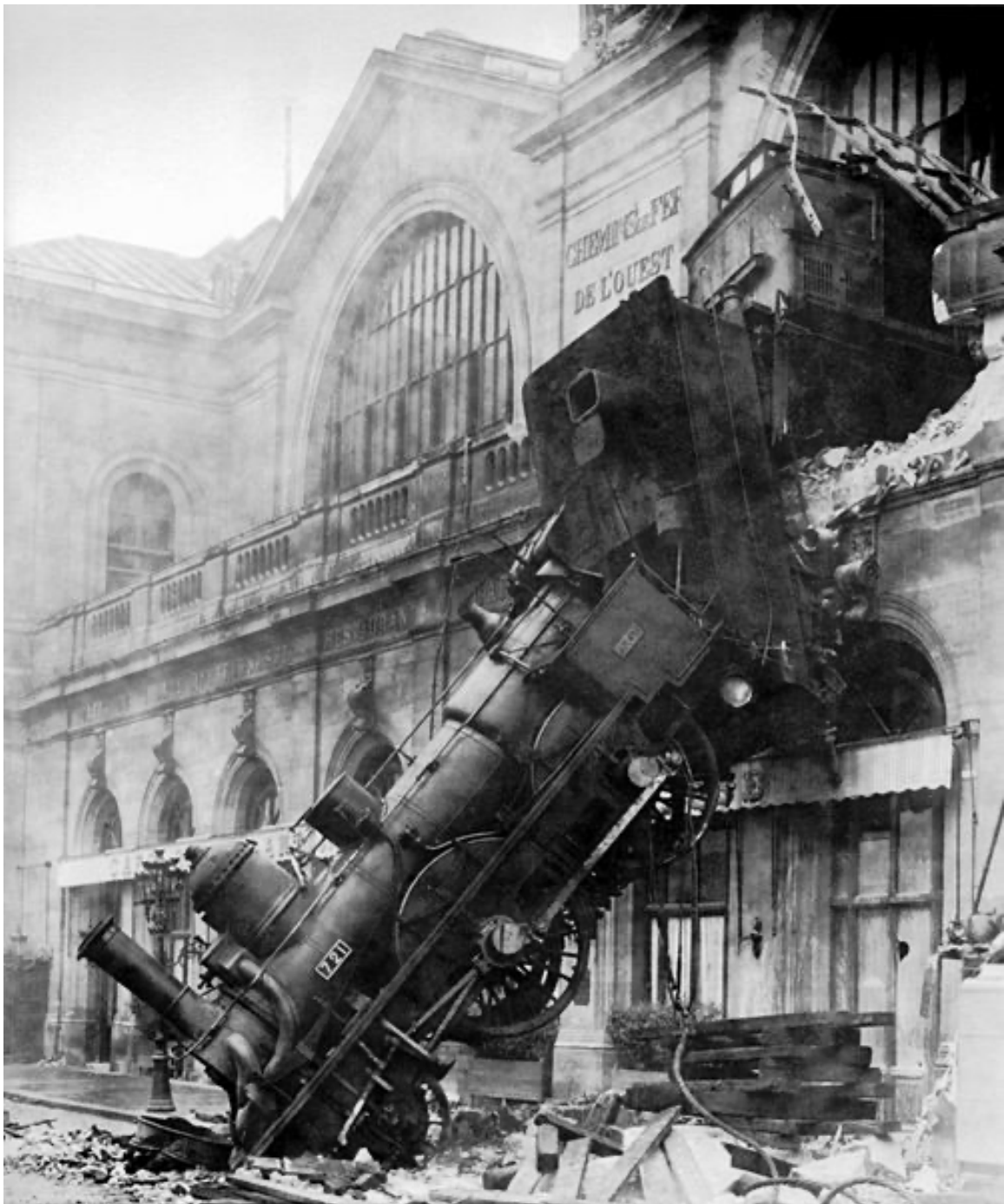


FIG. 18.1. The infamous 1895 derailment at Gare Montparnasse in Paris [1].

18. Error Analysis

Last updated 2012 January 11

Measurement

Whenever you take experimental data, you measure quantities to some precision. There is an uncertainty associated with every measurement you make! Experiment provides a way of testing theory; you must know how well you have determined a quantity to know how well you have verified theory. As scientists who may be designing new drugs, electronic devices, spacecraft, or even bridges, you must understand the tolerances involved in your designs to assure safety, reliability, and usefulness. Hence, we must propagate uncertainties.

Every data value has an uncertainty. So in a laboratory experiment in which you measure, for example, current I , length L , or distance D , each quantity has a corresponding uncertainty δI , δL , or δD . Uncertainties on digital or finely-spaced analog scales are usually taken to be $1/2$ of the smallest digit. Uncertainties in lab masses are $\pm 1\%$ unless otherwise specified.

Repeated Measurements:

We often repeat measurements to improve the uncertainty of our result. Normally we make a measurement on a representative sample of objects rather than for every possible object. For example, given 10 000 uniform glass beads, we would measure the mass of perhaps 100 of them rather than measuring the mass of the entire population of 10 000 beads. We then can calculate the *sample mean*, given by

$$\bar{x} = \frac{1}{N} \sum_{n=1}^N x_n. \quad (18.1)$$

To evaluate the spread of the values, we can calculate the *sample variance* by

$$\sigma^2 = \frac{1}{N-1} \sum_{n=1}^N (x_n - \bar{x})^2, \quad (18.2)$$

or we can take the square root of the variance to obtain the standard deviation

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{n=1}^N (x_n - \bar{x})^2}. \quad (18.3)$$

The $N - 1$ in the denominator reflects the fact that the calculation of the mean \bar{x} has effectively reduced the number of degrees of freedom by one.

Uncertainty Propagation

Often you will measure quantities x and y and use a theoretical formula to compute a quantity $z = f[x, y]$. From multidimensional calculus, if x and y undergo infinitesimal changes dx and dy , then z undergoes an infinitesimal change

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy. \quad (18.4)$$

Similarly, if x and y are uncertain by small amounts Δx and Δy , then z is uncertain by a small amount

$$\begin{aligned} \sigma_z^2 &= \frac{1}{N} \sum_{n=1}^N \Delta z_n^2 \\ &\approx \frac{1}{N} \sum_{n=1}^N \left(\frac{\partial z}{\partial x} \Delta x_n + \frac{\partial z}{\partial y} \Delta y_n \right)^2 \\ &= \frac{1}{N} \sum_{n=1}^N \left(\frac{\partial z}{\partial x} \Delta x_n \right)^2 + \frac{1}{N} \sum_{n=1}^N \left(\frac{\partial z}{\partial y} \Delta y_n \right)^2 + \frac{1}{N} \sum_{n=1}^N 2 \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \Delta x_n \Delta y_n \\ &\approx \left(\frac{\partial z}{\partial x} \right)^2 \frac{1}{N} \sum_{n=1}^N (\Delta x_n)^2 + \left(\frac{\partial z}{\partial y} \right)^2 \frac{1}{N} \sum_{n=1}^N (\Delta y_n)^2 + 0 \\ &= \left(\frac{\partial z}{\partial x} \right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial y} \right)^2 \sigma_y^2, \end{aligned} \quad (18.6)$$

where the cross terms sum to nearly zero as half are likely positive and half negative. Thus,

$$\sigma_z = \sqrt{\left(\frac{\partial z}{\partial x} \right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial y} \right)^2 \sigma_y^2} = \sqrt{\left(\frac{\partial z}{\partial x} \sigma_x \right)^2 + \left(\frac{\partial z}{\partial y} \sigma_y \right)^2}. \quad (18.7)$$

For a function $w[x, y, z, \dots]$ with three or more variables, this result generalizes to

$$\delta w = \sqrt{\left(\frac{\partial w}{\partial x} \delta x \right)^2 + \left(\frac{\partial w}{\partial y} \delta y \right)^2 + \left(\frac{\partial w}{\partial z} \delta z \right)^2 + \dots}, \quad (18.8)$$

where $\delta x, \delta y, \delta z, \dots$ are the respective uncertainties for x, y, z, \dots . We use the symbol δ here rather than σ because the equation is valid for uncertainty in general and not simply for the strict definition of the variance.

Given a function of the form

$$w = x^M y^N z^P \dots, \quad (18.9)$$

simplify Eq. 18.8 by computing the partial derivatives of w and dividing by w to obtain

$$\frac{\delta w}{w} = \sqrt{\left(M \frac{\delta x}{x}\right)^2 + \left(N \frac{\delta y}{y}\right)^2 + \left(P \frac{\delta z}{z}\right)^2 + \dots}, \quad (18.10)$$

which is often more convenient to use in practice.

Examples

Example 1:

Suppose we want to calculate the power P from the current I and the resistance R , where $P = I^2 R$. Using Eq. 18.8, the *absolute* uncertainty in the power is

$$\delta P = \sqrt{\left(\frac{\partial P}{\partial I} \delta I\right)^2 + \left(\frac{\partial P}{\partial R} \delta R\right)^2} = \sqrt{(2IR \delta I)^2 + (I^2 \delta R)^2}, \quad (18.11)$$

since $\partial P / \partial I = 2IR$ and $\partial P / \partial R = I^2$. Using Eq. 18.10, the *fractional* or *relative* uncertainty in the power is

$$\frac{\delta P}{P} = \sqrt{\left(2 \frac{\delta I}{I}\right)^2 + \left(\frac{\delta R}{R}\right)^2}, \quad (18.12)$$

without calculating any derivatives. Obtain the absolute uncertainty by multiplying both sides by the power P to find

$$\delta P = P \sqrt{\left(2 \frac{\delta I}{I}\right)^2 + \left(\frac{\delta R}{R}\right)^2}. \quad (18.13)$$

Example 2:

Suppose we want to calculate the magnitude of Earth's magnetic field from

$$B = \frac{mg}{IL \sin \theta} - \frac{\mu_0 I}{2\pi d \sin \theta}, \quad (18.14)$$

where I , L , m , d , and θ have respective uncertainties δI , δL , δm , δd , and $\delta \theta$. Unfortunately, Eq. 18.10 is not valid here because the expression for B is not of the form of Eq. 18.9. Hence we must use the Eq. 18.8 general form

$$\sigma_B = \sqrt{\left(\frac{\partial B}{\partial I} \delta I\right)^2 + \left(\frac{\partial B}{\partial L} \delta L\right)^2 + \left(\frac{\partial B}{\partial m} \delta m\right)^2 + \left(\frac{\partial B}{\partial d} \delta d\right)^2 + \left(\frac{\partial B}{\partial \theta} \delta \theta\right)^2} \quad (18.15)$$

and compute each of the partial derivatives

$$\frac{\partial B}{\partial I} = -\frac{mg}{I^2 L \sin \theta} - \frac{\mu_0}{2\pi d \sin \theta}, \quad (18.16a)$$

$$\frac{\partial B}{\partial L} = -\frac{mg}{IL^2 \sin \theta}, \quad (18.16b)$$

$$\frac{\partial B}{\partial m} = \frac{g}{IL^2 \sin \theta}, \quad (18.16c)$$

$$\frac{\partial B}{\partial d} = \frac{\mu_0 I}{2\pi d^2 \sin \theta}, \quad (18.16d)$$

$$\frac{\partial B}{\partial \theta} = -\frac{mg \cos \theta}{IL \sin^2 \theta} + \frac{\mu_0 I \cos \theta}{2\pi d \sin^2 \theta}. \quad (18.16e)$$

Symbolic algebra software like *Mathematica* can assist in these calculations.

Example 3:

Suppose we perform N measurements x_n , each with most likely uncertainty σ . What is the uncertainty in the mean \bar{x} of these measurements? Combine Eq. 18.1 and Eq. 18.8 to find

$$\begin{aligned}\sigma_{\bar{x}} &= \sqrt{\sum_{n=1}^N \left(\frac{\partial \bar{x}}{\partial x_n} \sigma_{x_n} \right)^2} = \sqrt{\left(\frac{\partial \bar{x}}{\partial x_1} \sigma_{x_1} \right)^2 + \left(\frac{\partial \bar{x}}{\partial x_2} \sigma_{x_2} \right)^2 + \dots} \\ &= \sqrt{\left(\frac{\partial}{\partial x_1} \left(\frac{1}{N} \sum_{n=1}^N x_n \right) \sigma_{x_1} \right)^2 + \left(\frac{\partial}{\partial x_2} \left(\frac{1}{N} \sum_{n=1}^N x_n \right) \sigma_{x_2} \right)^2 + \dots} \\ &= \sqrt{\left(\frac{\partial}{\partial x_1} \left(\frac{1}{N} x_1 \right) \sigma_{x_1} \right)^2 + \left(\frac{\partial}{\partial x_2} \left(\frac{1}{N} x_2 \right) \sigma_{x_2} \right)^2 + \dots} \quad (18.17) \\ &= \sqrt{\left(\frac{1}{N} \sigma_{x_1} \right)^2 + \left(\frac{1}{N} \sigma_{x_2} \right)^2 + \dots} = \sqrt{\sum_{n=1}^N \left(\frac{1}{N} \sigma \right)^2} = \sqrt{N \left(\frac{1}{N} \sigma \right)^2} \\ &= \frac{\sigma}{\sqrt{N}}\end{aligned}$$

This is a famous result [2,3]: averaging N results each with uncertainty σ reduces the uncertainty of the average by $1/\sqrt{N}$. It is called *the standard deviation of the mean* or the *standard error*.

Other Helpful Information

Units

All dimensional quantities must be reported with appropriate units. Failure to do so was the root cause of the 1999 loss of the \$330 million *Mars Climate Orbiter*. NASA worked in metric units but one of its subcontractors provided thruster performance data to the team in imperial units. Because of the incorrect data, the spacecraft entered Mars orbit too low and likely burned up in Mars' atmosphere [4].

Sig Figs

All measurements are approximations. The number of significant figures (or “sig figs”) in a measurement is the number of figures that are reliably known. **Do not carry excessive significant figures when rounding numerical results.**

The first significant digit in the value of δz determines the number of significant digits that you report for z . Never record more digits in a quantity than allowed by

the first significant digit in the value of the uncertainty. For example, if $\delta z = 0.09$, then you can report z to the hundredths place (like 1.25 ± 0.09). You will notice that, most of the time, the number of digits that propagation of uncertainty tells you to report is the same as what you would report using the rules for significant numbers. This provides a good check.

When using scientific notation, put your value and uncertainty together in parentheses and place the multiplier outside, like $(2.225 \pm 0.004) \times 10^{-9}$. Always be sure to report the value and uncertainty with the *same* multiplier in scientific notation.

Negligible Terms

Sometimes, you may be able to neglect certain terms in your expression for the uncertainty. If one of the fractional uncertainty $\delta x_1/x_1$ is significantly smaller than the other $\delta x_n/x_n$ in the expression for δz , you may be able to neglect the contribution of $\delta x_1/x_1$ to the uncertainty δz .

In Example 1, suppose that the resistance of a Vishay precision resistor is $R = 10.000 \pm 0.001 \Omega$, but the current $I = 4.3 \pm 0.2 \text{ A}$ is measured with an old ammeter. Because $\delta I/I = 0.2/4.3 = 0.047$ and $\delta R/R = 0.001/10 = 0.0001$, we know I to 4.7%, and we know R to 0.01%. Since $\delta R/R \ll \delta I/I$, from Example 2, write $\delta P/P = 2\delta I/I$. Because P depends on the square of I , the uncertainty of I contributes twice as much as if P depended on just I . Thus the fractional uncertainty of P should be $\delta P/P = 2 \times 0.047 = 0.093 = 9.3\%$. We calculate P to be 184.90 W (saving all our digits until we know how many we need) and then we can calculate the absolute uncertainty of P by $\delta P = P \times \delta P/P = 184.90 \times 0.093 = 17.2$. We should have only one digit in the uncertainty so δP is rounded to 20, and we report the final value as $P = 180 \pm 20 \text{ W}$.

In your reports, you should always show all the calculations necessary to propagate the uncertainties. You must clearly explain any terms that you choose to neglect and why. Keeping track of the fractional uncertainty values will help you track down any calculation errors you make when propagating errors, since it is unlikely that you will have a final error of 50% if all your original measurements were good to 2%.

Difference Versus Uncertainty

Now that you have propagated the uncertainties, you can compare your result to the accepted value. First compute

$$\text{Percent Difference} = \frac{|\text{Experimental Value} - \text{Accepted Value}|}{\text{Accepted Value}} \times 100\%. \quad (18.18)$$

The Percent Difference tells you how far your experimentally determined value is from the accepted value. To see if your experimental uncertainty could account for

this difference, you must compare the Percent Difference to the Percent Experimental Uncertainty. The latter is calculated from the fractional uncertainty by

$$\text{Percent Experimental Uncertainty} = \text{Fractional Uncertainty} \times 100\%. \quad (18.19)$$

If your Percent Experimental Uncertainty is *greater than* the Percent Difference, then you have determined the quantity z to within experimental uncertainty. If your Percent Experimental Uncertainty is *less than* the Percent Difference, then you have not determined the quantity z with great surety (assuming the Accepted Value is correct). You may have mistakenly neglected the uncertainty in some quantity or you may have underestimated the magnitude of uncertainty in one or more measured variables. You could also have an unidentified systematic error. When discussing possible errors, you should always consider whether your proposed error could make a significant change in your calculated value and in which direction the error tends to influence the value. That is, does it make your result closer to or farther from the expected value? You should discuss these sources of uncertainty in your experiment report regardless of whether you verified the quantity or not.

References

- [1] Studio Lévy & fils [Studio Lévy and Sons] (1895). This image is in the public domain because its copyright has expired.
- [2] P. R. Bevington, D. K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences, Third Edition* (McGraw-Hill, 2002).
- [3] J. R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements, Second Edition* (University Science Books, 1997).
- [4] Arthur G. Stephenson *et al.*, “Mars Climate Orbiter Mishap Investigation Board, Phase I Report”, NASA (1999 November 10).

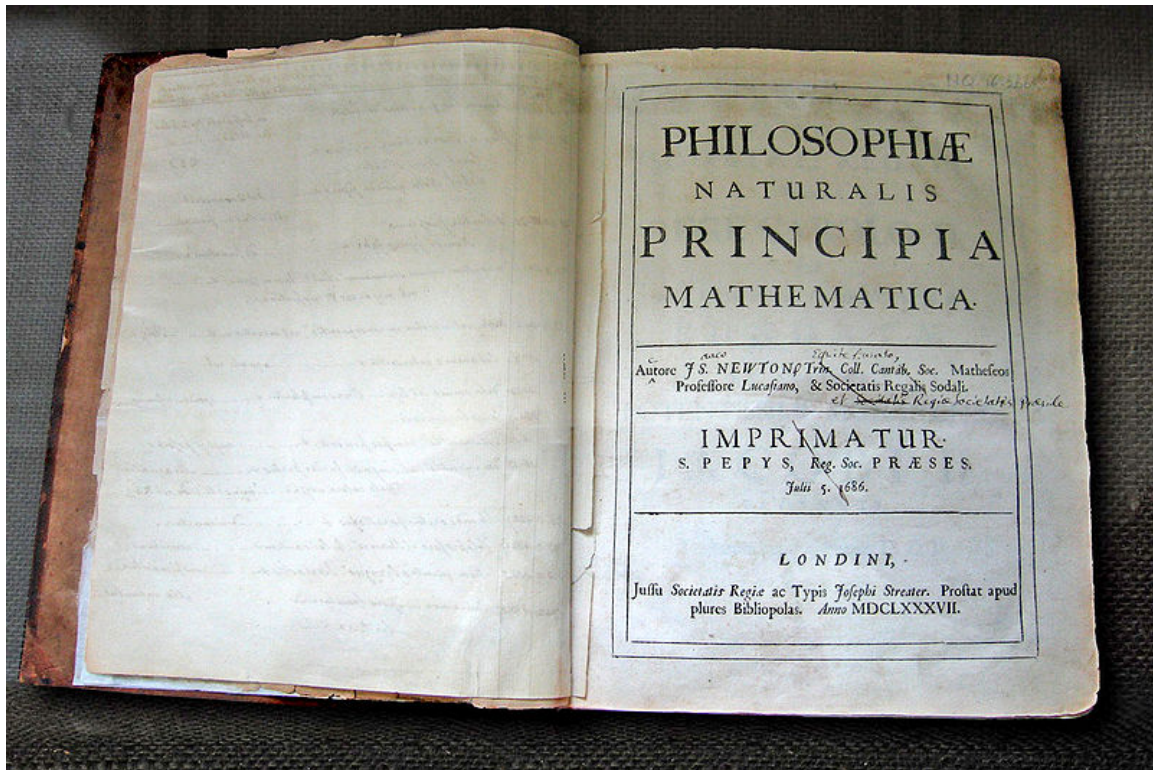


FIG. 19.1. Isaac Newton's personal first edition copy of his *Mathematical Principles of Natural Philosophy* with his own handwritten corrections for the second edition [1].

19. Scientific Writing

Last updated 2012 January 10

Formal Reports

Guidelines

Good scientific writing should be everything good writing is and more!

Reports should be written as if **intended for a reader who has studied physics but who is not familiar with the particular experiment described**. It should be as much like a report of original research as possible. Therefore, one should employ high standards of organization, neatness, and the proper use of the English language. Jokes about one's difficulties with apparatus are out of place in the formal report, although they might appear in the lab notebook. You are expected to *discuss* and illuminate the material; do *not* say "data were taken as shown in the graphs; I learned a lot in the lab." The style of writing is the more detailed mode used in a thesis, and your reports provide experience useful for writing your Senior I.S. thesis.

The American Institute of Physics (AIP) *Style Manual* is a valuable reference for basic information on writing a scientific report as well as the format, abbreviations, and organization required by all AIP journals (and, with slight changes, most other physics journals in the world). You should become familiar with the *Style Manual* and follow it where possible. The manual is online at <http://www.aip.org/pubserv/membership/AM/> and a hard copy is in the reference section of the Timken Science Library. Your reports will be more detailed than journal articles but will follow the same format: title and author, abstract, introduction, theory, experimental (equipment, procedure, data), results and discussion, and a conclusion followed by references. Of course, your first papers are not expected to be fully suitable for *The Physical Review*, but as the semester progresses, you should approach that goal.

Two useful books on writing lab reports are Matt Young's *The Technical Writer's Handbook: Writing with Style and Clarity* (T11 .Y68 2002) and Lobban and Schefter's *Successful Lab Reports* (Q183.A1L63 1992), both in the reference section of the Timken Science Library. A copy of Young's book is also by the printer in the Jr. I.S. lab so that you can easily consult it. The book covers many helpful topics and is somewhat whimsically written and organized. Some sections of this book may be assigned as reading during the semester.

As you write your report, you will need more information about the physics involved than is provided in this manual. There are many reference books and much other useful material for Junior I.S. in Timken Science Library. I expect that you will consult library sources to further your understanding of the problem at hand. *The reports should reflect that you have read and thought beyond the information provided in the lab manual.*

Submission

Most physics and mathematics journals today encourage electronic submission and resubmission in LaTeX. Practicing this process, **you will upload all your Jr. I.S. reports via Woodle as PDF documents generated from LaTeX using the American Physical Society's REVTeX macros.** There will be a LaTeX tutorial early in the semester, and an example LaTeX document is below. The **first submission** of each report will be in the one-column, double-spaced **preprint style**. The **second submission** of each report, responding to detailed feedback from the instructor, will be in the two-column, single-spaced **galley style**.

One of the most effective ways to learn is to correct one's own work, and one of the best ways to learn technical writing is to re-write. Revision does not simply mean correcting spelling and grammar errors that you missed in the original draft. Revision should involve looking critically at your writing, working hard to make your writing even clearer and easier to understand, making the analysis as complete as possible, and making the entire report more effective.

Laboratory Notebook

You must obtain a new bound laboratory notebook. The type with grid lines is preferable for tabulating data and sketching preliminary graphs. Please number the pages of your notebook and *make a table of contents on the first page*.

All data is to be taken in the notebook, not on loose paper to be recopied later. Never remove pages from your notebook. When you make an error, mark it out with a single line; it's often helpful to write a note explaining the mistake ("I forgot to convert into cm!").

The following information should be recorded **directly** into your lab notebook **while** you perform an experiment (or write a simulation):

- The title of the experiment and the dates you performed it.
- A sketch of any electrical schematics.
- A brief apparatus description. A diagram is most helpful.
- Explanatory notes and experimental conditions (the procedure).
- The data, usually in table form with columns carefully labeled and units given. Include uncertainty estimates with your raw data. You do not need to include tables of data taken by a computer, but you should include graphs of that data.
- Data analysis. Calculations are best done directly in the notebook. Arithmetic is unnecessary, but formulas used and the values substituted into them should be shown. *A sample calculation* showing values, units, and errors is essential. If you use computer algebra or a computer simulation, tape a printout or screenshot in your notebook along with annotations.
- Anything else you feel is important.

Some students are overly concerned about keeping their notebook “nice” and apparently error-free. One way to help overcome this tendency is to use the right page for your neatly written observations and results of any calculations, while you use the left page for rough calculations or sketching an idea for a circuit.

Some students write only numeric data and final results and don’t record any intermediate steps or their thought process during the experiment. **That is unacceptable.** It’s OK for your entries to be brief, but it must be clear exactly what you did so that another student could recreate it. Record your thoughts, plans and conclusions, not just numbers from the meters. The very act of writing can help to clarify your thoughts. **Do not underestimate how quickly you will forget experimental details!**

Think of the lab notebook as a professional diary. It is where you record your thoughts, observations, derivations, comments on the literature you read, print-outs of programs, careful diagrams of the equipment and its settings as well as electrical schematics, procedures, data, reproducibility, systematic errors, analysis, comparison to theories, and summaries of where the project is at a certain point in time. Each page should have a complete date (including year) and time. As entries are made, reference to prior pages should be made as appropriate ("I have changed the procedure outlined on page 45 to the following...") and then to indicate on that earlier page that one should look at the new material.

The lab notebook should have sewn in pages (not spiral bound). Pages are never removed from a notebook; if you have an error, then that is useful information and should be noted on the page. It is ok to put a big red X through something that is wrong but indicate why and do not scratch it out so it is unreadable. If a number is misrecorded, then put a single line through it and write above the correct value.

Leave the first few pages of the notebook blank for an index that is generated after you are well into the notebook. The index is by topic and well organized; it is not a table of contents.

No loose paper is inserted in the notebook. Tape paper onto the pages of the notebook using Scotch magic transparent tape that will last decades. Never assume that you will find what you need in a saved computer file; if it is important then put the results in your notebook. Computer systems fail and programs change so that your data may not be retrievable in 10 years.

You keep a notebook not only for yourself so you know exactly what you did and how it turned out but also for others who may need to replicate your results. You protect yourself from charges of manipulating or fabricating data if you have a good notebook. It is part of the ethics of being a scientist.

*As my mentor, Sandra Greer, would comment: **If it isn't in your notebook then you didn't do it.***

—Don Jacobs

% THIS IS AN UNRENDERED LATEX EXAMPLE ARTICLE WITH COMMENTS

```
\documentclass[aps,twocolumn,amscd,amsmath,amssymb,verbatim]{revtex4}
% replace "twocolumn" by "preprint" to toggle from galley to preprint styles
\usepackage{graphicx} % for figures
\usepackage{epstopdf} % so can use EPS or PDF figures

\begin{document}

% %%%%%%%%%%
\title{This is the Title of an Example Article}
\author{Jane Doe}
\affiliation{Physics Department, The College of Wooster, Wooster, Ohio 44691, USA}

\date{\today}
```

```
\begin{abstract}
The abstract is a short summary of the article. This is filler text. This is filler text. This is
filler text. This is filler text. This is filler text. This is filler text. This is filler text.
\end{abstract}
```

```
\maketitle % generate title, including abstract
```

```
% %%%%%%%%%%
\section{Introduction}
This is text. This is \textbf{bold} text. This is text with \emph{emphasis}. This is ``double
quotes".
```

One or more blank lines separate paragraphs.

```
% %%%%%%%%%%
\section{Math & Citations} % "&" is a special character
Examples of inline math are  $\alpha = \sqrt{\gamma^2 + \Gamma^2}$  and  $\vec{v} = 7$ 
 $\hat{x} - 5 \hat{y}$  and  $\vec{u} \times \vec{v}$  and  $c = (2.99 \pm 0.01) \times$ 
 $10^8 \text{ m/s}$ . One example of block (display) math is
```

```
%
\begin{equation}
\int_0^1 x^2 dx = \frac{1}{3},
\label{myIntegral}
\end{equation}
```

```
%
and a second example is
%
```

```
\begin{equation}
\xi = \alpha \left( \frac{1}{\omega_0^2} + \omega^2 \right)
\label{signal}
\end{equation}
%
```

Note how block math is punctuated like words in a sentence! The block math equations

are automatically numbered. We can reference Eq.~\ref{myIntegral} or Eq.~\ref{signal} by inserting labels in the block, but then we must compile `\LaTeX` twice.

We can readily cite both articles `\cite{Duke2003}` and books `\cite{Loecher2002}` in our bibliography, but again we must compile `\LaTeX` twice.

```
% %%%%%%%%%%
```

```
\section{Figures & Tables}
```

We can also include figures but need to use package `"graphicx"` under document class. We can reference Fig.~\ref{Schematic} like equations. All figures should have captions.

```
\begin{figure}[ht] % "ht" = here or top
  \includegraphics[width=0.8\linewidth]{CoWFigure} % PDFs or PNGs
  \caption{Figure captions go on bottom.}
  \label{Schematic}
\end{figure}
```

Finally, we can also include tables, such as Table~\ref{demoTable}. Like figures, we can also `\emph{attempt}` to force their positions. In the document class line, we can easily convert from `"preprint"` one-column, double-spacing for rough drafts to `"twocolumn"` single spacing for final drafts!

```
\begin{table}[h] % indenting is optional
  \caption{Table captions go on top.}
  \label{demoTable}
  \begin{ruledtabular}
    \begin{tabular}{cc} % "cc" = center each column
      abscissa & ordinate\\
      \hline
      1.0 s & 5.6 m\\
      3.0 s & 9.9 m
    \end{tabular}
  \end{ruledtabular}
\end{table}
```

```
% %%%%%%%%%%
```

```
\begin{thebibliography}{99} % place widest label in final brackets
```

```
  \bibitem{Duke2003} T. Duke and F. J\"{u}licher,
  \textit{Phys. Rev. Lett.}, \textbf{90}, 158101 (2003).
```

```
  \bibitem{Loecher2002} M. Loecher,
  \textit{Noise Sustained Patterns} (World Scientific Publishing Company, 2002).
```

```
\end{thebibliography}
```

```
\end{document}
```

Bibliography of Useful Books

Data and Error Analysis

| | | |
|--------------|----------------|---|
| QC33.L9 1991 | Louis Lyons | <i>Practical Guide to Data Analysis for Physical Science Students</i> |
| QC39.H284 | Carl W. Hall | <i>Errors in Experimentation</i> |
| QC39.T4 1997 | John R. Taylor | <i>An Introduction to Error Analysis</i> |
| QC39.B17 | D. C. Baird | <i>Experimentation</i> |

Computer Simulation

| | | |
|--------------------|-------------------------|---|
| QC21.2 .G67 | H. Gould & J. Tobochnik | <i>An Introduction to Computer Simulation Methods</i> |
| QC20.7.E4 G56 1997 | N. Giordano | <i>Computational Physics</i> |

Scientific Techniques & Lab Manuals

| | | |
|------------------|---------------------------|--|
| Q180.A1W57 | E. Bright Wilson, Jr. | <i>An Introduction to Scientific Research</i> |
| Q185.M66 2002 | John H. Moore | <i>Building Scientific Apparatus</i> |
| QC33.M52 2003 | Adrian C. Melissinos | <i>Experiments in Modern Physics</i> |
| QC385.E913 | M. Francon, <i>et al.</i> | <i>Experiments in Physical Optics</i> |
| QC33.T74 | George L. Trigg | <i>Landmark Experiments in Twentieth Century Physics</i> |
| QC33.C.5 1997 | D. Edmonds | <i>Cioffari's Experiments in College Physics</i> |
| QC37W7 | Worsnop & Flint | <i>Advanced Practical Physics</i> |
| QC33.D86 1988 | R.A. Dunlap | <i>Experimental Physics</i> |
| QC33.P74 1991 | D.A. Preston & E.R. Dietz | <i>The Art of Experimental Physics</i> |
| Q183.A1 L63 1992 | Lobban & Schefter | <i>Successful Lab Reports</i> |
| T11 .Y68 2002 | Matt Young | <i>The Technical Writer's Handbook: Writing with Style and Clarity</i> |

Mechanics

| | | |
|---------------|---------------------------|---------------------------------|
| QC235.F74 | A. P. French | <i>Vibrations & Waves</i> |
| QC125.2.F74 | A. P. French | <i>Newtonian Mechanics</i> |
| QC136.P56 | Pippard | <i>The Physics of Vibration</i> |
| QA862.P4B35 | G.L. Baker & J. P. Gollub | <i>Chaotic Dynamics</i> |
| Q172.5.C45W55 | G. Williams | <i>Chaos Theory Tamed</i> |

Electricity and Magnetism

| | | |
|----------------|-------------------|---|
| QC41.E8 | Estermann | <i>Methods of Experimental Physics, Vol. 1: Classical Methods</i> |
| QC522.B57 1989 | Bleaney & Bleaney | <i>Electricity and Magnetism</i> |
| QC534.T4 | Terry | <i>Advanced Laboratory Practice in Electricity and Magnetism</i> |

Optics

| | | |
|-------------------|--------------------|--|
| QC355.2 J46 1976 | Jenkins & White | <i>Fundamentals of Optics</i> |
| QC365H5 | Wallace Hilton | <i>Experiments in Optical Physics</i> |
| QC371.L48 | L. Levi | <i>Applied Optics</i> |
| QC688.C67 | Alan Corney | <i>Atomic and Laser Spectroscopy</i> |
| QC449.D4 | DeVelis & Reynolds | <i>Theory and Applications of Holography</i> |
| QC449.09 | Outwater | <i>Practical Holography</i> |
| QC449.565 | Smith | <i>Principles of Holography</i> |
| QC365.W74 | G. Wright | <i>Elementary Experiments with Lasers</i> |
| QC355.2.B67 1999 | Born & Wolf | <i>Principles of Optics</i> |
| QC355.3 .H43 2002 | Hecht | <i>Optics</i> |
| TA1520.M37 2002 | Mansuripur | <i>Classical Optics and its Applications</i> |
| TA1675.S52 2004 | G. Wright | <i>Elementary Experiments with Lasers</i> |

Other

| | | |
|-----------------|------------------|--|
| QC21.B4445 | Berkeley series: | <i>Vol. 1-3: Mechanics, Waves, E & M</i> |
| QA403.5B7 | Bracewell | <i>The Fourier Transform & its Application</i> |
| QC311.B293 1993 | Ralph Baierlein | <i>Thermal Physics</i> |

References

[1] Photograph by Andrew Dunn (2004 November 5). Published under the Creative Commons Attribution-Share Alike 2.0 Generic license.

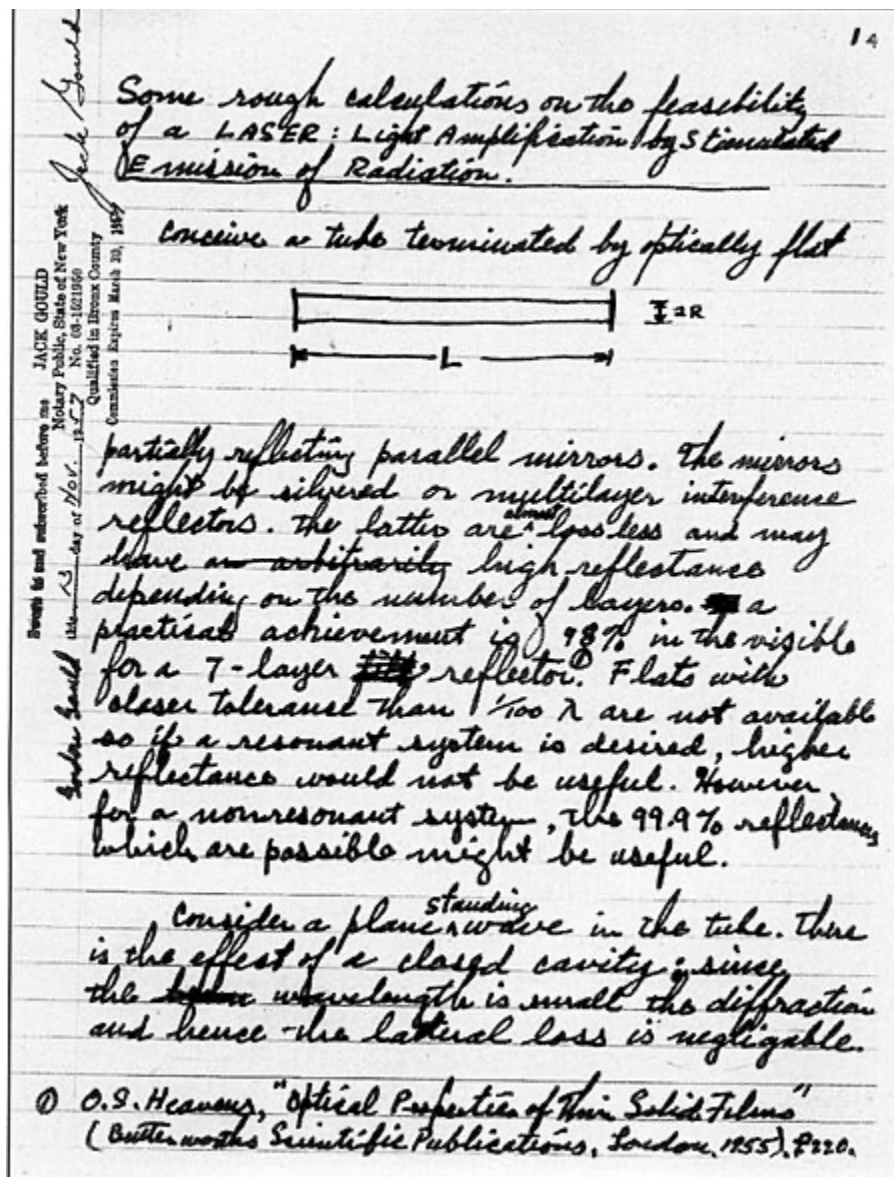


FIG. 20.1. A page of Gordon Gould's famous laboratory notebook, in which he coined the acronym *LASER* and described how to build one. This notebook was the focus of a thirty-year court battle for the patent rights to the laser.

20. Jr. I.S. Syllabus

Last updated 2012 January 10

Purpose of Course

- To provide students with an experience which exposes them to some of the phenomena and principles of physics and to integrate the fields of physics through laboratory work.
- To acquaint students with some of the tools and techniques of modern experimental physics, especially those that are available in this department.
- To allow students the opportunity to utilize computers as research tools. This includes data collection, analysis, simulations, and literature searching.
- To introduce students to the methods of literature searching for both journal articles and texts.
- To develop the ability to take and analyze data, and to draw warranted conclusions from the data.
- To improve the ability to write clear and informative reports using a thesis style of writing.

Format of Course

We will take one hour per week to discuss experimental techniques, literature searching, error analysis, and other general topics. The two 3-hour lab periods are Tuesday and Thursday afternoons; **you are expected to be in the lab during these times.**

The format of this course is quite different from a regular course. In other courses with a lab, the course is 1 credit and the lab is 0.25 credits, so the lab is a small portion of the coursework. In Junior I.S., **the lab is the course** and you will have to spend numerous hours outside of lab analyzing data, reading, and writing.

You will typically work on three experiments at once – analyzing and writing the last experiment, taking data on the current one, and planning your next one. Plan your time accordingly. If you fall behind, it can be very difficult to catch up. Keys for the Jr. I.S. room are available to all physics majors.

Requirements

Overview

- Attendance at the lecture period, the two weekly lab sessions, and library instruction sessions
- Total of five projects
 - One self-designed experiment with a serious literature review; a proposal is required.
 - At least one project from each of the four experiment categories.
 - At least one experiment must use computer data acquisition.
 - The self-designed project may be a computer simulation.
- Completed lab notebook
- Reports
 - Five thesis style reports. One is the self-designed project.
 - One brief report (condensed from one of the original five reports) to be posted online.
 - One oral presentation.
- Meeting submission deadlines

Experiments

You must choose four two-week experiments from this manual. Some of the experiments are straightforward while others are more difficult and may still have bugs. There are instructions for all the experiments.

You should discuss with your instructor the experiment you expect to do *a week in advance* so that equipment and supplies can be prepared for you. It is also a good idea to keep an alternate experiment in the back of your mind for emergencies.

Course Grade

- **10 % of course grade**
 - Classroom and laboratory participation (attendance, handling of apparatus, discussions, and so on)
 - Laboratory notebook (completeness, neatness, care, and so on)
 - Other class assignments
- **90 % of course grade**
 - Laboratory reports (evaluated for clarity, conciseness, completeness, accuracy of data, development of theory, analysis of data, level of difficulty, and so on).
 - Rewrites are required for all reports.

Details

- You must complete at least one experiment in **each** of the four categories: **Mechanics, Thermal Physics, Electricity & Magnetism, Optics & Quantum Mechanics.**
- One of your experiments must be a *self-designed* experiment that is researched, assembled and investigated by you over the last 7 weeks of the semester. This experiment cannot be one of those in the lab manual without permission of the instructor. This experiment must include a literature search including an INSPEC search. The self-designed experiment may satisfy one of the four experiment categories.
- A “grant” proposal where you think through and plan for your self-designed experiment is required. It will include an introduction, list of equipment (preferably items available in the department), procedure and an annotated bibliography of at least **ten** sources (some books, but mostly journal articles). You should conduct a literature search, including an **INSPEC search**. This proposal leads directly to the self-designed experiment (see above).
- At least one of your experiments must involve computer data acquisition.
- Your **lab notebook** should contain the essential background information, sketches, references, data, discussion, calculations, and so on, for all experiments and the outline of the oral report. Your complete lab notebook will be turned in on the last day of class. The instructor will ask to see it routinely throughout the semester. If you want a copy of it, you should use one with duplicating pages.
- Results from your self-designed project will be presented to the physics community as an **oral report** in addition to the normal written report. The reports are scheduled for the last Tuesday of classes. You should prepare a PowerPoint or Keynote presentation, which you should practice aloud to get the timing correct, that introduces the topic as well as presents the results. Limit your oral presentation to 12 minutes and allow for a brief question period.
- Except for the self-designed experiment that will be going on concurrently with other experiments, you have two weeks per experiment. The **written reports** are due as noted in the table and schedule.
- **Rewrites are required for all of the written reports.** Part of the grade is based on your revisions and the attention paid to comments, **but most of the grade is determined by the quality of the initial report.** The grade for the revised report may improve (or be lowered) by as much as a whole letter grade from the original.
- Not handing in a mandatory revision will **lower** the final grade from that on the original report.
- Late reports will be heavily penalized.

Jr. I.S. Etiquette

Please cooperate with each other in the care of our equipment. If an instrument is damaged through abuse or neglect, or if a computer is stolen because a door was jammed open, it cannot be replaced. We all lose!

- Do not unlock or prop open lab doors. **Do not open windows in the lab.** You may open the blinds but you must then close them before you leave.
- Do not prop open hallway doors. This is not only dangerous but is a fire code violation!
- *You* are responsible for guests allowed into the lab. Remember, the computers and printer are there for physics only. The lab contains delicate, dangerous, and expensive equipment. *Never* leave guests unattended! A few years ago, \$90 000 worth of microscopes were stolen from a lab in Scovel Hall, and we do not want that to happen in Taylor!
- Computers in the Senior I.S. lab (Taylor 07) or the Research Lab (Taylor 04 and 05) are mainly for research and are not available for Junior I.S..
- Computers in the Junior I.S. lab (Taylor 06) and the Electronics Lab (Taylor 09) can be used for this course. Computers should not be shut down since others may be running programs in the background.
- The color laser printer **taysec-109-hp.wooster.edu** located in Taylor 06 and the laser printer, **Physics Lab Printer** located in the general physics lab Taylor 101 are available for your printing. Having an easily accessible free color printer is a privilege. Conserve paper and the printer by only printing necessary **physics** material. There are other color printers available all over campus (including Jackie's office) for your non-physics needs.
- Don't hog hard disk space! In past years our Macs suffocated under a weight of backed-up files, multiple versions of term papers for other classes and digital music and movies. **We will regularly discard files if our hard disks get choked up.** Save important files elsewhere.
- Use caution when deleting files—you may wreck someone's I.S.!
- Having keys to lab rooms is a luxury that few students enjoy on this campus. You will lose your key privilege by
 - playing games or misusing the department's computers in the labs,
 - loaning your key to anyone,
 - leaving a guest unattended,
 - using tools, computers or equipment inappropriately,
 - propping or leaving windows or doors open or unlocked, or
 - inappropriate behavior (like chair races, sabotaging other students' work, and so).

Safety Notes

- The beam from a laser can punch a hole in your retina and permanently damage your eye. Use the provided goggles when aligning any laser beam or when using a high intensity laser (for example, the He-Ne with the holography equipment).
- Do not move a compressed gas cylinder without direct faculty supervision. A cylinder can become a powerful rocket if damaged.
- Stand to one side when opening a regulator on a compressed gas cylinder.
- When using liquid nitrogen (LN₂) remember that it can cause severe burns when contacting your skin. **Do not touch the metal transfer tube.** After transferring the LN₂, do not close the knob too tightly. (It will close more as it warms due to thermal expansion.)
- Never work in the shop alone nor without being cleared on the machines.

Some experiments have dangerous voltages or currents accessible. Use caution before turning on any power supply capable of more than 20 V.

- Special precautions and eyewear are required when working with organic solvents or acids/bases, or when using an open flame.
- An eyewash station is located in the Senior I.S. lab. Know how to use it.
- Broken glass must be placed in special containers and cannot be placed in the trash can. Please ask the instructor, the TA, or the lab technician for assistance.

Academic Integrity

Cheating in any of your academic work is a serious breach of the Code of Academic Integrity and is grounds for an F for the entire course. Such violations include turning in another person's work as your own, copying from any source without proper citation, crossing the boundary of what is allowed in a group project, and lying in connection with your academic work. You will be held responsible for your actions. **If you are unsure as to what is permissible, please contact your course instructor and ask!**

Allowed:

- Discussing electrical connections or procedures in the lab with other students and how procedures might be improved
- Understanding the theory and how to apply it to the experiment by working with another student
- Discussing general features of graded reports
- Talking with the professor or the teaching assistant

Not Allowed

Penalty of F for the course or the assignment for all students involved.

- Showing another student your graded lab report (exception – if the other student is a student who has already done the lab for a grade)
- Copying any part of another student's lab reports
- Taking ideas, suggestions, or text from another source without reference

Curricular and Extra-curricular Conflicts

The College of Wooster is an academic institution and its fundamental purpose is to stimulate its students to reach the highest standard of intellectual achievement. As an academic institution with this purpose, the College expects students to give the highest priority to their academic responsibilities. When conflicts arise between academic commitments and complementary programs (including athletic, cultural, educational, and volunteer activities), students, faculty, staff, and administrators all share the responsibility of minimizing and resolving them.

As a student you have the responsibility to inform me of potential conflicts as soon as you are aware of them, and to discuss and work with me to identify alternative ways to fulfill your academic commitments.

Accommodations for learning disabilities

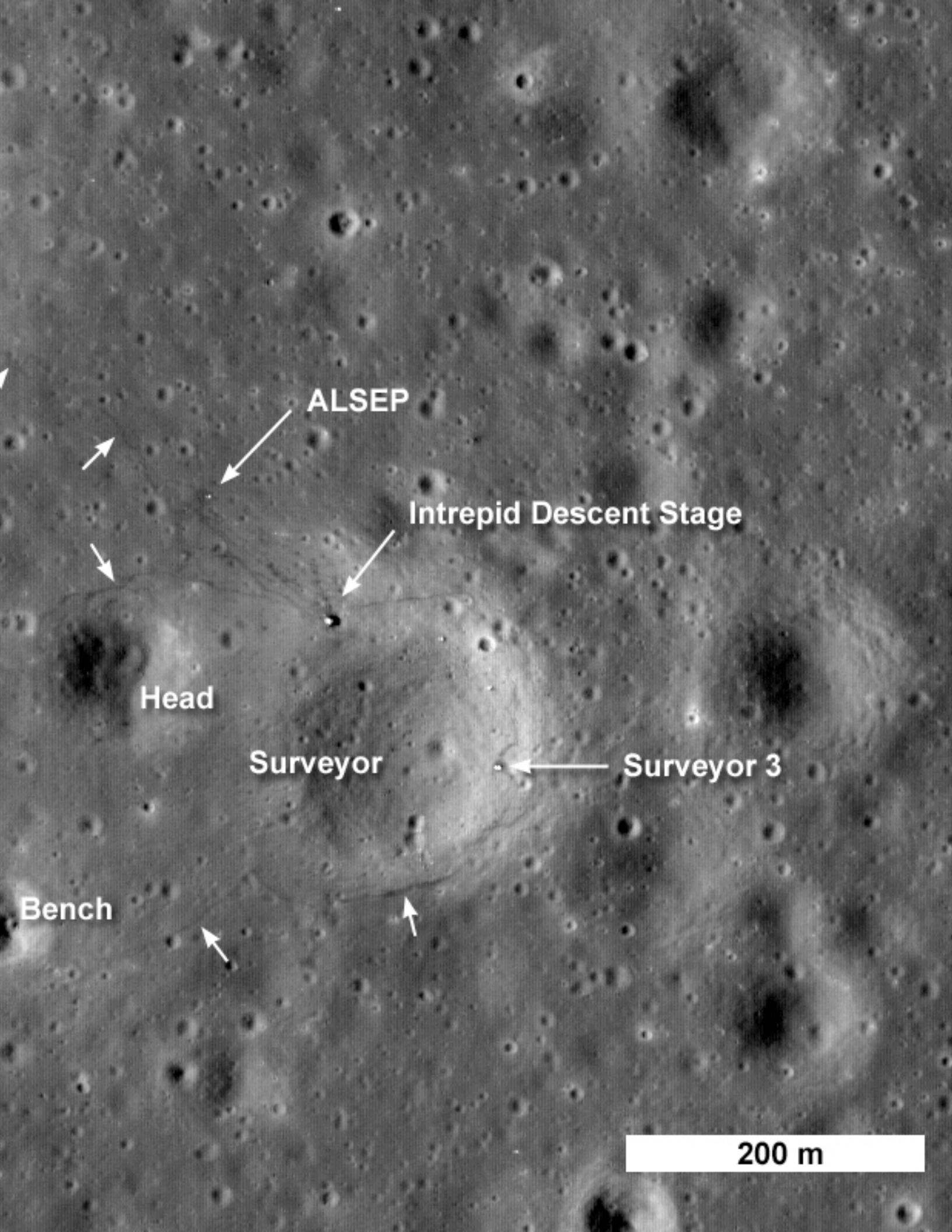
The Learning Center offers services designed to help students improve their overall academic performance. Sessions are structured to promote principles of effective learning and academic management. Any student on campus may schedule sessions at the Learning Center.

Any student with a documented learning disability needing academic accommodations is requested to speak with Pam Rose, Director of the Learning Center (ext. 2595), and the instructor, as early in the semester as possible. All discussions will remain confidential.

Spring 2012 Jr. I.S. Calendar

All submissions are electronic uploads via Woodle of PDFs generated from LaTeX with the American Physical Society's REVTeX macros in preprint or galley style, as indicated. All submissions are due just before midnight at 11:55 PM.

| | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|----|--|------------------------|----------------------------|---|----------------------------------|--------|--------|
| 1 | Jan 16 | Jan 17 Exp 1 | Jan 18 | Jan 19 Exp 1 | Jan 20 | Jan 21 | Jan 22 |
| 2 | Jan 23 | Jan 24 Exp 1 | Jan 25 | Jan 26 Exp 1 | Jan 27 Preprint 1 | Jan 28 | Jan 29 |
| 3 | Jan 30 | Jan 31 Exp 2 | Feb 1 | Feb 2 Exp 2 | Feb 3 Galley 1 | Feb 4 | Feb 5 |
| 4 | Feb 6 | Feb 7 Exp 2 | Feb 8 | Feb 9 Exp 2 | Feb 10 Preprint 2 | Feb 11 | Feb 12 |
| 5 | Feb 13 | Feb 14 Exp 3 | Feb 15 | Feb 16 Exp 3 | Feb 17 Galley 2 | Feb 18 | Feb 19 |
| 6 | Feb 20 | Feb 21 Exp 3 | Feb 22 | Feb 23 Exp 3 | Feb 24 Preprint 3 | Feb 25 | Feb 26 |
| 7 | Feb 27 | Feb 28 Exp 4 | Feb 29 Self Idea | Mar 1 Exp 4 | Mar 2 Galley 3 | Mar 3 | Mar 4 |
| 8 | Mar 5 | Mar 6 Exp 4 | Mar 7 | Mar 8 Exp 4 | Mar 9 Preprint 4 | Mar 10 | Mar 11 |
| | Mar 12 | Mar 13 | Mar 14 | Mar 15 | Mar 16 | Mar 17 | Mar 18 |
| | Mar 19 | Mar 20 | Mar 21 | Mar 22 | Mar 23 | Mar 24 | Mar 25 |
| 9 | Mar 26 | Mar 27 Self | Mar 28 Self Proposal | Mar 29 Self | Mar 30 Galley 4 | Mar 31 | Apr 1 |
| 10 | Apr 2 | Apr 3 Self | Apr 4 | Apr 5 Self | Apr 6 | Apr 7 | Apr 8 |
| 11 | Apr 9 | Apr 10 Self | Apr 11 | Apr 12 Self | Apr 13 | Apr 14 | Apr 15 |
| 12 | Apr 16 | Apr 17 Self | Apr 18 | Apr 19 Self | Apr 20 | Apr 21 | Apr 22 |
| 13 | Apr 23 | Apr 24 Self | Apr 25 | Apr 26 Self | Apr 27 Preprint Self | Apr 28 | Apr 29 |
| 14 | Apr 30 | May 1 Orals Self | May 2 | May 3 Orals Self | May 4 Galley Self Lab book | May 5 | May 6 |
| | May 7 Web Galley 1 st draft | May 8 | May 9 | May 10 Web Galley 2 nd draft | May 11 | May 12 | May 13 |



ALSEP

Intrepid Descent Stage

Head

Surveyor

Surveyor 3

Bench

200 m