The Chaotic Candle Seesaw

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The goal of this experiment was to gain a greater understanding of the candle seesaw and its chaotic oscillations. The candle seesaw seems to be a very chaotic system with many variables determining its motions, some of which are difficult to account for in a model like the length of exposed wick or distribution of unmelted wax around the flame. The stable points observed in the system where at $-180°$ and $180°$ the vertical points on the system.

I. INTRODUCTION

The candle seesaw has been written about since as early as the 1890's with pictures like the one shown in Fig. 1. Even though the seesaw has been known about for over a century for most of that time including now it is just treated as a magic trick. Although it is wrong to say that no one has been trying. There have been many questions about this system even if it is as simple as a candle burning at both ends. The unique thing about the candle seesaw is that the oscillation grows over time as the candles burn and even recent papers like [1] from 1997 couldn't identify a clear reason for this increase in oscillation over time.
Finally in 2019 a paper looking into the non chaotic oscillation of the candle seesaw determined/confirmed that the source of the increase in oscillations over time was due to the droplets of wax falling off the candle. [2] It was not the combustion of the wax acting like a propellant like some where thinking it was more like a waterwheel. The wax drops lowering gravitational potential energy and then the other side is heavier and it oscillates back and forth and the reason the oscillations always increase is the fact that the droplets can only fall off if that side is below horizontal and these oscillation can get so strong that the system can transition from linear to chaotic motion.

FIG. 1: Portrete of the the candle seesaw with paper riders from the book La Science Amusante (from [3]).

II. THEORY

There have not been many papers looking into understanding the motion of the candle seesaw and describing them with equations. One published paper on the subject is a paper from 2009 titled *Osculations of a candle burning at both ends*. [4]. They set up some equations that can describe the simple non chaotic oscillation of the seesaw. There main consideration is determining how the candles orientation can effect the rate at which the wax is burned and this requires a good understanding as to how candles burn [5]. The simple approximation is that the candle burns more wax the further below the horizon it is because the flame more directly melts the wax. This can be modeled as

$$\dot{m}_{\mathrm{R}}(\theta) = \alpha - \beta \cdot \sec(\theta_0 - \theta) \tag{1}$$

$$\dot{m}_{\mathrm{L}}(\theta) = \alpha - \beta \cdot \sec(\theta_0 + \theta), \tag{2}$$

where $\beta > 0$, m_{R} and m_{L} are the masses of the Left and right candles. The variable α would be a constant based on the type of wax used. The dots denote a time derivative. These angles can become more clear from pictures shown in Fig. 2, θ is the angle of the candle relative to horizontal and θ_0 is the angle of the wax as the candle burns . Sadly these equations are only built to work accuracy for small angles and have not been tested/compared to large angles.
The effective cross-section of the candle will be $A \sec(\theta_0 + \theta)$, provided $\theta < \theta_0$, if $\theta > \theta_0$ the cone would not meet the wax and no drops will fall. Sadly this assumption will not always be true for the chaotic system however as after a flip the cone will be in the wax even at $\theta > \theta_0$. This makes comparing this model to data collected from chaotic oscillation impossible as θ_0 for the oscillations is in no way a constant as generally assumed for the equations above and is also very difficult to measure in anyway accurately for the system used and some assumptions about when wax can drip fall apart.

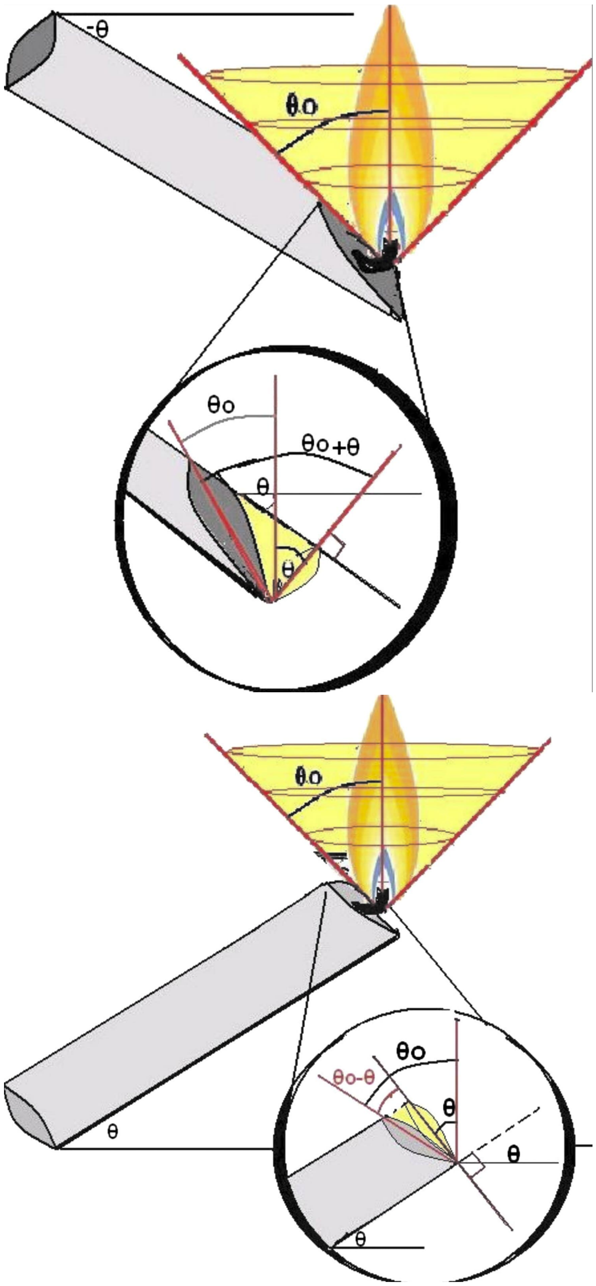

FIG. 2: Diagram of the angles used in Eqs. (1) and (2) (from [4]).

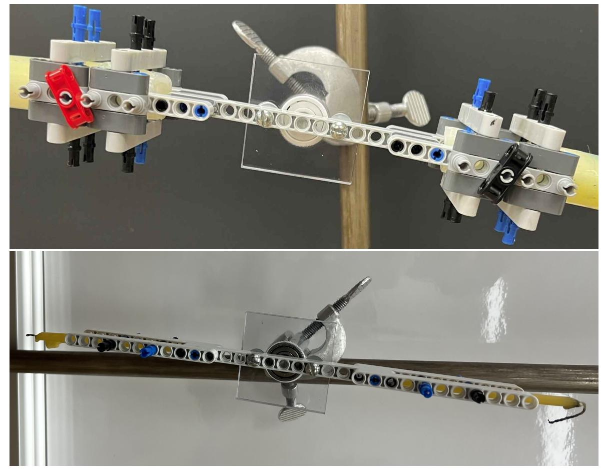

FIG. 3: Photo of the setup used to hold the candles as they burned. The photo on top was used to hold candle with diameters around 1.3 cm to 3.0 cm, and the photo on the bottom is the holder used for the birthday candles with 0.5 cm to 1.0 cm diameters.

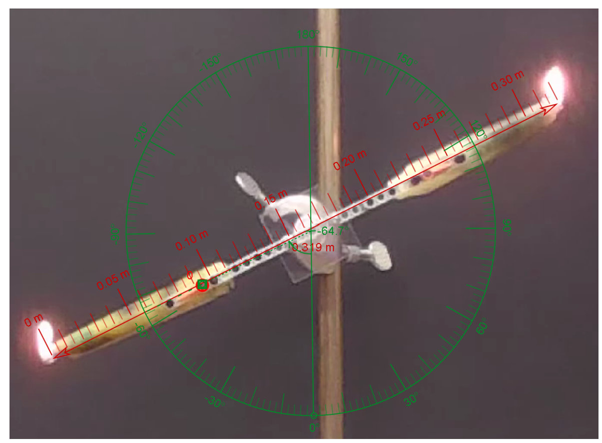

FIG. 4: Photo demonstrating how the angles were defined for the Tracker software. The angles were not actually collected based on the protractor but rather a point mass tracking that could track a wider range than $-180°$ to $180°$.

III. PROCEDURE

720p video was collected from a laptop webcam and the points of interest were cut out using the free Adobe Premiere Rush video editing software [6]. Data was then collected using the Physlets free tracking software [7]. The setup used was fairly simple, the setup was a simple LEGO candle holder strapped to a VXB brand full ceramic ball bearing as shown in Fig. 3. The ceramic bearing was required as the friction of other pivot points tested was to high, although decreasing the friction even more would likely provide longer more chaotic oscillations and cleaner data in future works. The angles where collected as shown in Fig. 4 where clock wise rotations decreased the angle and counter clockwise increased it.

IV. RESULTS & ANALYSIS

The angle was recorded as a function of time as shown in Fig.5. With this data next angular velocity's where calculated and plotted as a function of angle, a phase plot as shown in Fig. 6. This data provided an interesting incite into the setup used. It showed that the setup is surprisingly most stable at a $-180°$ and $180°$ angle (vertical). This run was done with two tapered candles with a top diameters of 1.2 cm and a bottom diameter of 2.3 cm.

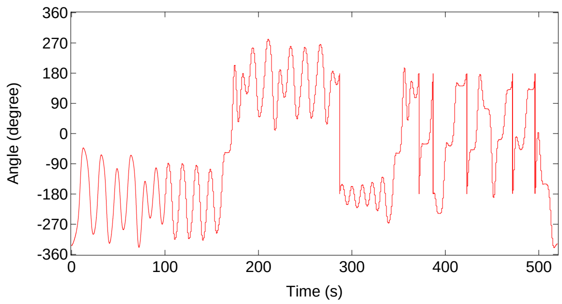

FIG. 5: Exemplary oscillations seen in the chaotic motion of the candle seesaw. The error available to the tracking software was ±0.1 degrees but a majority of the error is likely tracking errors that can not be shown in the graph. This data has also been folded in on itself to keep the range between $-360°$ to $360°$ even when it flipped twice in the same direction.

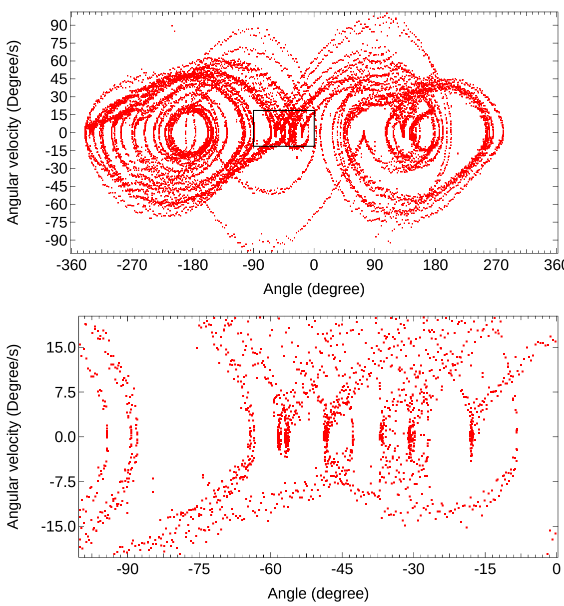

FIG. 6: Phase plot for the chaotic motion of the candle seesaw with a zoomed graph below. The two stable points seems to be the $-180°$ and $180°$ degree angles the vertical points in this system, and the unstable equilibrium point is surprisingly not centered around $0°$ but more around $-45°$ as shown in the bottom graph.

Both of these candles weighed 36 g with a length of 20 cm and the holder for that run weighed 30 g. The candles were lite from the horizontal. Interestingly the candles did not start to osculate until about 2 cm of the candle had burned and the diameter had increased to 1.5 cm. The candle then oscillated chaotically for 9 minutes until the candle had burned to the point that the diameter reached 1.9 cm. At that point the candle stopped oscillating even though the candle still had about 6 cm left to burn and weighed 19 g.

V. CONCLUSION

This candle seesaw was a very finicky system to get to osculate chaotically. The system's behavior depends on the friction of the pivot point, thickness of the candles used, and weight of the the candles relative to the holder, and distribution of the weight along the system. Other harder to determine variables are the length of exposed wick and the distribution of wax around the flame. The phase plot showed that the stable points where $-180°$ and $180°$. This showed that the system osculated about the vertical, but it was still asymmetric about $0°$, the main flipping point was generally around $-45°$.
I believe it my have preferred a vertical equilibrium because of the friction in the system, the candles where only allowed to burn quickly when vertical so if it was horizontally stable it would just stop oscillating. I do not know if the system was asymmetric because of differences in the candles or an asymmetry in the holder. I believe, based on the footage, that it is just a consequence of how the candles burn when they are more then about $45°$ below the horizontal. When the candle is in that position the falling wax will run down the wick of the candle causing it to burn slower most of the time. So, the system likely can only burn fast enough in that position to have enough of a weight difference to complete a full rotation. Another interesting aspect shown from this one data set is the different chaotic regimes in Fig. 5. It only flipped once before $t = 300$ s and then it oscillated non chaotically for about 50 s before hitting the strongly chaotic regime after $t = 350$ s.
All of these variable together seems to indicate that the system may be difficult to fully understand with a system of equations because the candles burning is a chaotic system of its own sometimes. As for some future works would be running the system with candles of constant diameters could be a good next step as it would allow calculations of the moment of inertia $I(t)$ as a function of time far easier.

VI. ACKNOWLEDGMENT

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Appendix A: Error Propagation

The only real error to propagate was the error in angular velocity and this equation was very simple and end up giving very small errors.

$$\omega = \frac{\theta_\mathrm{i} - \theta_\mathrm{f}}{t_\mathrm{i} - t_\mathrm{f}}, \tag{A1}$$

where θ_i and θ_f are the initial and final angles, the denominator is the change in time. The video was taken at 30 frames a second and angles where collected at each frame making the times measured very accurate. The variable ω is the angular velocity. Making the error just

$$\delta\omega = \omega\sqrt{\left(\frac{\delta\theta_\mathrm{i}}{\theta_\mathrm{i}}\right)^2 + \left(\frac{\delta\theta_\mathrm{f}}{\theta_\mathrm{f}}\right)^2}, \tag{A2}$$

These values $\delta\omega$ the error in angular velocity all end up being very small because the relative errors of the angles are very small.

Appendix B: Video Recording's

A link to the full 9 min osculation can be found at `https://www.youtube.com/watch?v=dHUTYe-J2mQ`

[1] Ehrlich, R. *Why Toast Lands Jelly-Side down Zen and the Art of Physics Demonstrations* (Princeton University Press, Princeton, N.J, 1997).

[2] Tjossem, P., Case, W. & Bass, R. The candle seesaw. *American Journal of Physics* **87**, 370–376 (2019).

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[5] Faraday, M. & Crookes, W. *The Chemical History of a Candle* (London : Chatto and Windus, 1908).

[6] Video editing app | Mobile video editing | Adobe Premiere Rush. https://www.adobe.com/products/premiere-rush.html (2022). Accessed on 5 May 2022.

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