Measuring Resistance Through Temperature

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This experiment aimed to test whether a resistor's resistance can be derived using only the current across it and its temperature. The experiment tested four resistors with a range of (2200-22 000) Ω. It was discovered that for each, a dissipation constant could be derived that would allow its resistance to be accurately calculated as a function of current and temperature.

I. INTRODUCTION

In 2022, during the Physics 401 course at the College of Wooster, an experiment was carried out relating the power radiated by a light bulb to the difference in its temperature compared to its surroundings. [1] This experiment was built upon prior discoveries by J. Stefan [2] and L. Boltzmann [3] and the Stefan-Boltzmann law, an equation which applies only to blackbody surfaces.

Other experiments have used the Stefan-Boltzmann law, Ohm's law, and the formula for the resistance of a material to relate the temperature of a filament to the electric power across it. [4] In 2004, A. Kesin and T. Yanar derived a further equation relating the temperature of a thermistor to the ambient temperature, the power across it, and its heat dissipation constant. [5]

Using the discoveries and theories in these prior experiments, it is possible to derive a solution for the resistance of a resistor based solely on its temperature, a pre-measured constant, and either its voltage or current. This would provide an alternate method to the common practice of calculating resistance from both current and voltage.

II. THEORY

The rate of energy loss over time of an object $\mathrm{d}E_{\mathrm{loss}}/\mathrm{d}t$ is equal to

$$\frac{\mathrm{d}E_{\mathrm{loss}}}{\mathrm{d}t} = \alpha(T_{\mathrm{R}} - T_{\mathrm{A}}) \ , \tag{1}$$

where α is the dissipation constant of the object, T_{R} is the temperature of the object, and T_{A} is the ambient temperature. The dissipation constant is defined as the amount of energy required to heat the object 1 °C hotter than its environment.

This change in energy can also be expressed as power P, which for a resistor is equal to

$$\frac{\mathrm{d}E}{\mathrm{d}t} = P = VI = I^2R \ , \tag{2}$$

where V is the voltage drop across the resistor, I is the current across the resistor, and R is the resistance of the resistor.

Following Ohm's law, the resistance can also be written in terms of current and voltage as

$$R = \frac{V}{I} \ . \tag{3}$$

Substituting the current-resistance expression of Eq. (2) into Eq. (1), we can calculate the resistance of a resistor from just its current and temperature via the equation

$$R = \frac{\alpha}{I^2}(T_{\mathrm{R}} - T_{\mathrm{A}}) \ . \tag{4}$$

In order to use this equation, we would first need to know the dissipation constant of the resistor. This constant can be measured by solving Eq. (4) for α, with a result of

$$\alpha = \frac{I^2R}{T_{\mathrm{R}} - T_{\mathrm{A}}} \ . \tag{5}$$

But as the goal is to calculate resistance as an end result, we can substitute the numerator for an equivalent expression also in Eq. (2), with a result of

$$\alpha = \frac{VI}{T_{\mathrm{R}} - T_{\mathrm{A}}} \ . \tag{6}$$

III. PROCEDURE

A variable voltage source was connected across a resistor, along with an ammeter connected in series and a voltmeter connected in parallel. The voltage source was then set to provide a variety of voltages in its approximately (1-15) V range, and the circuit was allowed to reach an equilibrium state where the power functions in Eq. (1) and Eq. (2) were equal. This was indicated by the current and resistor temperature becoming stable after their initial change due to the changing voltage.

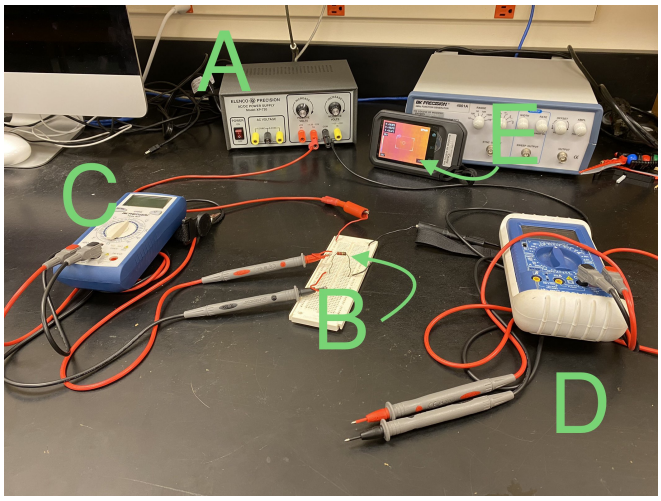

FIG. 1: The equipment for the experiment, including variable voltage source [A], resistor [B], ammeter [C], voltmeter [D], and thermal camera [E].

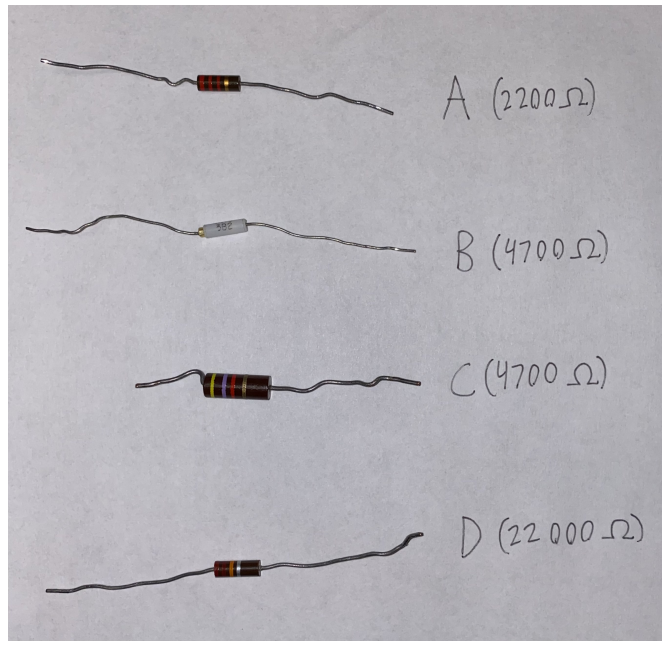

FIG. 2: The four resistors used during the experiment, alongside their letter designations and labelled resistances.

For each applied voltage, the current and voltage were recorded from the ammeter and voltmeter, and the difference between the resistor's temperature and the ambient temperature was recorded using an FLIR C5 handheld thermal camera. The setup is depicted in Fig. 1.

The product of the voltages and currents was then plotted against the temperature difference according to Eq. (6), and the slope of the line of best fit was recorded as α.

This process was performed on a total of four resistors, shown in Fig. 2. Resistor A had a labelled resistance of $2200\,\Omega$, with a tolerance of $\pm 5\%$. Resistors B and C had labelled resistances of $4700\,\Omega$ with tolerances of $\pm 5\%$, but Resistor B was a different type of resistor. Resistor D had a labelled resistance of $22\,000\,\Omega$, with a tolerance of $\pm 10\%$.

For resistor B, the resistance and tolerance were written on the resistor, though this is not depicted in its photo. For resistors A, C, and D, the resistance and tolerance were obtained through standard resistor bar color coding.

IV. RESULTS AND ANALYSIS

A. Data Results

For each of the four resistors tested, I was able to calculate the dissipation constant. In each case, a line of best fit using the dissipation constant described the behavior of the resistor fairly accurately, in accordance with the expectations from Eq. (6).

Fig. 3 presents the result for the $2200\,\Omega$ resistor A, with a dissipation constant of $(9.70 \pm 0.42) \times 10^{-3}$ VA/°C.

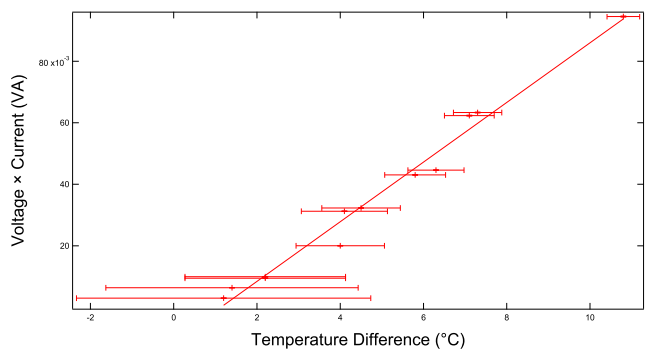

FIG. 3: Graph of the VI vs. ΔT function of Eq. (6), for the $2200\,\Omega$ resistor A. The slope of the line of best fit is $(9.70 \pm 0.42) \times 10^{-3}$ VA/°C.

Fig. 4 presents the result for the $4700\,\Omega$ resistor B, with a dissipation constant of $(6.49 \pm 0.28) \times 10^{-3}$ VA/°C.

Fig. 4 presents the result for the $4700\,\Omega$ resistor C, with a dissipation constant of $(12.30 \pm 0.56) \times 10^{-3}$ VA/°C.

Fig. 5 presents the result for the $22\,000\,\Omega$ resistor D, with a dissipation constant of $(12.24 \pm 0.27) \times 10^{-3}$ VA/°C.

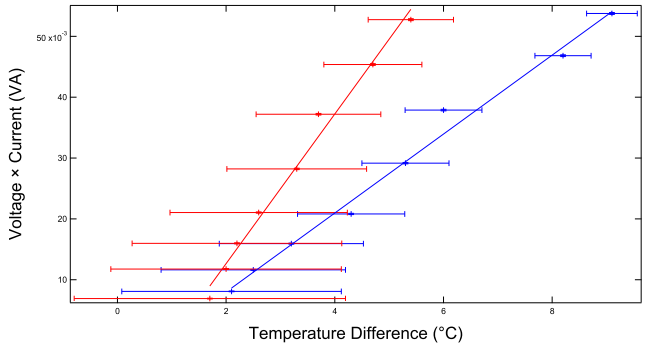

FIG. 4: Graph of the VI vs. ΔT function of Eq. (6), for the 4700 Ω resistors B (blue) and C (red). The slope of the blue line of best fit is $(6.49 \pm 0.28) \times 10^{-3}$ VA/°C, and the slope of the red line of best fit is $(12.30 \pm 0.56) \times 10^{-3}$ VA/°C.

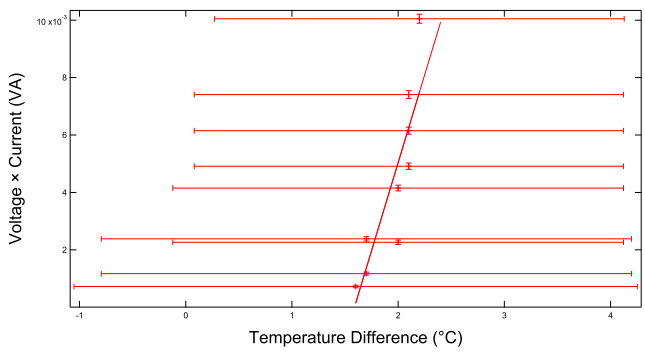

FIG. 5: Graph of the VI vs. ΔT function of Eq. (6), for the 22 000 Ω resistor D. The slope of the line of best fit is $(12.24 \pm 0.27) \times 10^{-3}$ VA/°C.

B. Error Discussion

The measurement error for the ammeter and voltmeter were relatively small, at ± 0.01 V and ± 0.01 mA.

As shown in Figs.3 to 5, there is notably large error in the temperature difference measurements. This expected error is much larger than the proximity between the line of best fit and the measurement points should suggest. The large error bars are due to the fact that the thermal camera has an error of ± 3 °C. However, the thermal camera is capable of taking multiple simultaneous measurements, a function which was used when recording the temperature difference.

Therefore, for each temperature difference measurement, the error within the ± 3 °C range was approximately equal for the resistor temperature and ambient temperature, causing the error to almost cancel itself out. From my own observations while conducting the experiment, I would estimate the error in temperature difference at closer to ± 0.3 °C.

V. CONCLUSION

For a range of resistors, the connection between voltage, current, and temperature was measured in order to derive a constant that could be used to calculate resistance from only the current and temperature. This constant, also known as the resistor's dissipation constant was successfully determined for four resistors across the (2200-22 000) Ω range.

As a result, it is clear that for a resistor in a circuit, it is possible to calculate its resistance using its temperature and current, or from its temperature and the voltage across it. While this does require initial measurements to calculate the dissipation constant of the resistor, the usefulness of this information comes in the convenience of measuring resistance later.

Normally to measure resistance, both current and voltage are needed, and adding an ammeter to a circuit usually requires modifying the circuit itself. However, using a voltmeter and a thermal camera requires no change to the circuit whatsoever, allowing for more convenient resistance measurements.

VI. ACKNOWLEDGEMENT

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Appendix A: Appendix: Error Propagation

The vertical error δVI in Figs.(3 to 5) is defined as

$$\delta VI = \sqrt{\left(\frac{\partial(VI)}{\partial V}\delta V\right)^2 + \left(\frac{\partial(VI)}{\partial I}\delta I\right)^2}\ , \tag{A1}$$

Which can be simplified to

$$\delta VI = VI\sqrt{\left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2}\ . \tag{A2}$$

Due to the nature of the temperature error as discussed in Section IV B, Error Discussion, the following substitution is being used for error propagation:

$$T_{\mathrm{R}} - T_{\mathrm{A}} = \Delta T\ . \tag{A3}$$

The horizontal error $\delta\Delta T$ is defined as

$$\delta\Delta T = \sqrt{\left(\frac{\partial\Delta T}{\partial\Delta T_{\mathrm{R}}}\delta\Delta T_{\mathrm{R}}\right)^2 + \left(\frac{\partial\Delta T}{\partial\Delta T_{\mathrm{A}}}\delta\Delta T_{\mathrm{A}}\right)^2}\ . \tag{A4}$$

Remembering that

$$\delta T_{\mathrm{R}} = \delta T_{\mathrm{A}} \tag{A5}$$

from Section IV B, we can simplify this to

$$\delta\Delta T = \frac{\sqrt{2}\ \delta\Delta T_{\mathrm{R}}}{\Delta T}\ . \tag{A6}$$

Using the same method as above final error for the dissipation constant $\delta\alpha$ is defined as

$$\delta\alpha = \frac{VI}{\Delta T}\sqrt{\left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2 + \frac{\sqrt{2}\ \delta\Delta T_{\mathrm{R}}}{\Delta T}}\ . \tag{A7}$$