Rolling Bottle Experiment

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This experiment investigated the motion of a bottle with liquid in it. A liquid is different from a solid since it has viscosity. The viscosity of liquid influences the rolling motion of a bottle. In this experiment, we investigated the relationship between the energy dissipated by the liquid and its viscosity. By calculating the acceleration of the bottle, we found the energy dissipated by the liquid. We assumed that the bottle was a solid cylinder without friction or sliding. Then, we took a video to analyse its acceleration. By comparing the acceleration, we found that the bottle with higher viscosity liquid had a lower acceleration, and the bottle full of liquid had a higher acceleration than the bottle half-full of liquid.

I. INTRODUCTION

Liquid has an interesting characteristic called viscosity. This factor describes how the liquid resists itself when it is flowing. When a bottle is filled with liquid, it is different from a solid cylinder with the same mass.[1] The liquid molecules will hit each other when the liquid is flowing, and that consumes energy and generates some resistance. A simpler description is that viscosity is the internal friction in the liquid. The ideal fluid does not have viscosity, but it is hard to find in real life. Different liquids have different viscosities. The investigation of viscosity can help us understand more about how fluid flows.

II. THEORY

In a bottle with liquid that does not move, all of the energy of the system is potential energy

$$E = E_p = mgh, \tag{1}$$

where E is the total energy of the bottle and the liquid, E_p is the potential energy of them, m is the mass of the bottle plus the liquid, g is the gravity constant, and h is the height of the starting point from the ground.

For the kinetic energy of the system, we can use

$$E_k = \frac{1}{2}mv^2, \tag{2}$$

where E_k is the kinetic energy, and v is the final velocity of the bottle when it hits the ground.

If we assume the liquid in the bottle is a solid, we can use the moment of inertia to calculate the energy. For both the bottle full of liquid and half-full of liquid, the equation is

$$I = \frac{1}{2}mr^2, \tag{3}$$

where I is the moment of inertia, and r is the radius of the bottle. The energy dissipated by the moment of inertia during the motion E_r is

$$E_r = \frac{1}{2}I\omega^2, \tag{4}$$

where ω is the angular speed. The angular speed can be written as v/r.

When the bottle rolls down to the ground, the total energy should be

$$E = E_k + E_r + E_v, \tag{5}$$

where E_v is the energy dissipated by the viscosity.

Since the energy dissipated by the viscosity relates to the mass of the liquid, we can expand E_v to be the product of a factor γ and the mass of the liquid. The mass of the bottle is so light that we can ignore the moment of inertia of the bottle. The moment of inertia of the liquid also can be ignored because it always stays at the bottom when the bottle is rotating. Then, if we expand Eqn. 5, we can get the equation

$$mgh = \frac{1}{2}mv^2 + 0 + m\gamma, \tag{6}$$

and the mass m can be cancelled out. The final equation is

$$gh = \frac{1}{2}v^2 + 0 + \gamma. \tag{7}$$

For the constant value gh, if the final velocity v is larger, the factor γ is smaller. That means the energy dissipated by the viscosity is smaller. Since the distance from the starting point to the end point is constant, the larger acceleration gives a faster final velocity. For this reason, we can compare the energy dissipated by the viscosity by comparing the acceleration of each bottle.

FIG. 1: The set up picture of the equipment. The bottles are full water, half alcohol, half water, and half cooking oil from left to right. The wood broad and the books are at the behind of the bottles.

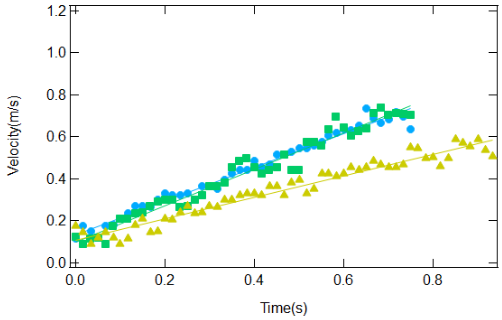

FIG. 2: The velocity of the bottles versus the time is shown in this graph. All the bottles are on the incline with 5.5 degrees slope. The blue one is the bottle with half full of alcohol, the green one is the bottle with half full of water, and the yellow one is the bottle with half full of cooking oil.

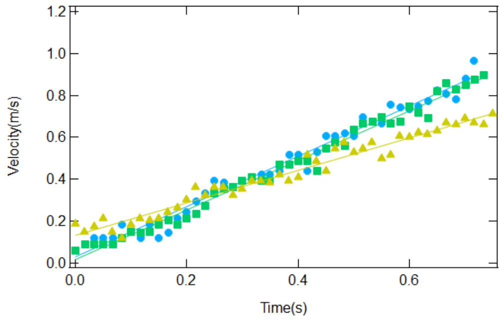

FIG. 3: The velocity of the bottles versus the time is shown in this graph. All the bottles are on the incline with 8.0 degrees slope. The blue one is the bottle with half full of alcohol, the green one is the bottle with half full of water, and the yellow one is the bottle with half full of cooking oil.

III. PROCEDURE

The equipment was set up as shown in FIG. 1. One end of the wood board was placed on a stack of books. In this experiment, the incline is the wood board. By adding books, the angle of the incline was increased. We used 3, 6, and 9 books to make 5.5, 8.0, and 12.0 degrees angles. The liquids in four bottles were full of water, half-full of alcohol, half-full of water, and half-full of cooking oil, and were arranged from left to right. A video was recorded by a mobile phone camera in 60 frames per second. The phone was lifted straight above the ground side of board. The tape marker on the board indicated the starting point of the bottles.

First, the angle of the incline was set at 5.5 degrees. The bottle full of water was put on the starting point. The bottle was released after the video began recording. When the bottle reached the end point, another bottle was selected to repeat the experiment again. After four bottles were used, the angle of the incline was adjusted to 8.0 and 12.0 degrees to repeat the whole trial.

The videos were imported to the software Tracker to analyse the velocity. The axis of the system was set at the top right corner of the wood board, and the short side of the board was set to be 0.5 meters. By adding points on the bottle in each frame in Tracker, the distance and the time data were exported in tables. The data was exported to Igor Pro to calculate the acceleration. By differentiating the distance data, the velocity data can be calculated. Then, the velocity versus time data was plotted in a graph. Since acceleration is velocity divided by time, the slope of velocity versus time graph is the acceleration of that bottle.

IV. RESULTS & ANALYSIS

A. The influence of different liquids

The graphs of velocity versus time of the bottles at 5.5 degree are shown in FIG. 2. The graphs of velocity versus time of the bottles at 8.0 degree are shown in FIG. 3. The graphs of velocity versus time of the bottles at 12.0 degree are shown in FIG. 4. The acceleration of each trial is shown in Table 1. According to previous studies, the viscosity of water at 20 degrees Celsius is 0.89 mPa·s, the viscosity of alcohol is 1.09 mPa·s, and the viscosity of oil is 33.10 mPa·s.[2] By comparing the slope of the blue, yellow, and green lines, we found little difference in the acceleration of half-full bottles of water and alcohol. The half-full bottle of cooking oil had a much lower acceleration. As shown in FIG. 5, in the 5.5 degree trial, the half-full bottle water had the highest acceleration and the cooking oil had the lowest acceleration. In the 8.0 degree trial, the alcohol had the highest

TABLE I: The acceleration of each type of liquid

Type	α 5.5deg(m/s^2)	α 8.0deg(m/s^2)	α 12.0deg(m/s^2)
Full Water	0.93±0.02	1.35±0.03	2.02±0.05
Half Alcohol	0.80±0.02	1.19±0.03	1.83±0.06
Half Water	0.86±0.02	1.18±0.02	1.79±0.07
Half Oil	0.51±0.02	0.77±0.02	1.28±0.03

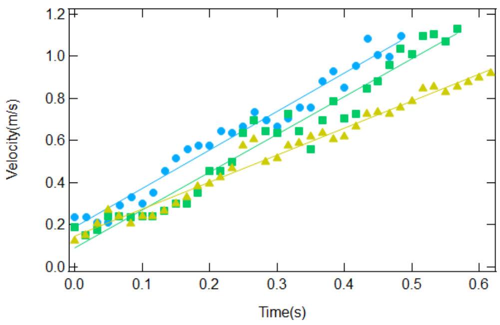

FIG. 4: The velocity of the bottles versus the time is shown in this graph. All the bottles are on the incline with 12.0 degrees slope. The blue one is the bottle with half full of alcohol, the green one is the bottle with half full of water, and the yellow one is the bottle with half full of cooking oil.

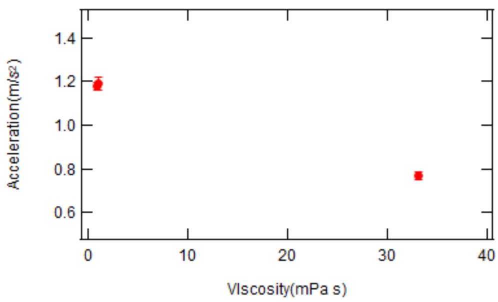

FIG. 5: The acceleration of the bottles versus the viscosity of the liquid in the bottle is shown in this graph.

acceleration and the cooking oil had the lowest acceleration. In the 12.0 degree trial, the alcohol had the highest acceleration again and the cooking oil had the lowest acceleration. For this reason, we can infer that cooking oil dissipates energy most by its viscosity, and that the viscosity of a liquid has a positive linear relationship with

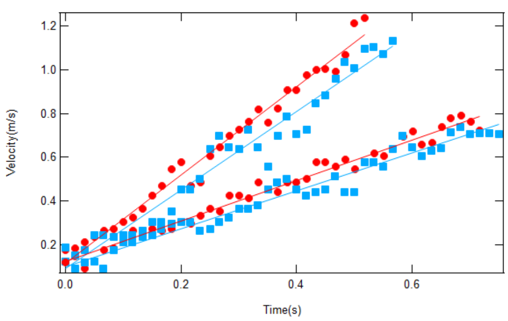

FIG. 6: The velocity of the bottles versus the time is shown in this graph. The lower red line and dots are the data of full water bottle at 5.5 degree. The upper red line and dots are the data of full water bottle at 12.0 degree. The lower blue line and dots are the data of half-full water bottle at 5.5 degree. The upper blue line and dots are the data of half-full water bottle at 12.0 degree.

the energy it dissipates.

B. The influence of the amount of liquid

The velocity for the full water bottle and the half-full water bottle of the steepest angle(12.0 degree) and the shallowest angle(5.5 degree) are shown in FIG. 6. By comparing the red dots and lines with the blue dots and lines in the figure, the difference between the acceleration of the bottles full of water and the bottle half-full of water can be calculated. In all three trials, the acceleration of the bottle full of water was higher than the acceleration of the bottle half-full of water. The reason of this is that during the rotation of the bottle, the surface of the liquid vibrated. This allowed more water molecules to create friction with each other. This friction required more energy from potential energy.[3] Also, we can see the velocity was not a straight line in this experiment because the upper surface of the liquid vibrated. This applied an opposite force when it vibrated to the starting point direction, and this applied a force on moving direction when it vibrated to the moving direction.

V. CONCLUSION

We found that the liquids with different viscosities dissipated different amounts of energy during the motion. We found that the energy dissipated by the liquid in a rotating motion had a positive relationship with the viscosity of that liquid. The energy dissipated by the liquid had a negative relationship with the volume of the liquid in the bottle. However, some limitations did not let us calculate the accurate value of the energy dissipated. In the theory part, the equations ignored the moment of inertia of the system and the friction between the bottle and the incline. Also, the rotating motion in real life exhibited a sliding motion. That also dissipated some energy. The shaking of the camera caused the inaccurate measurement of the distance when the bottles rolled down through the incline, and the Tracker was not a perfect software to measure the velocity. In future research, the laser rangefinder would be a better method to measure the distance and the velocity.

VI. ACKNOWLEDGEMENT

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