Oh What a Drag: Determining Drag Force and Drag Coefficient Area for Five Paper Airplane Designs

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(Dated: May 2, 2021)

The flights of five paper airplane designs were analyzed to determine the drag force and the drag coefficient area for each design. Two HERO9 GoPro cameras and the programs Tracker and Igor Pro were used to determine the drag force F_D and the drag coefficient area C_DA for five paper airplane designs: Basic, Basic Dart, Stable, Cross Wing, and Navy Plane. The cameras captured front and side profile views of the flight paths, and this data was analyzed in Tracker to find the distance traveled and time of each flight. Mean velocities for each flight were calculated in Igor Pro and the uncertainties from those velocities were used to determine a negative acceleration due to the drag force for each airplane design. These values were then used to calculate the drag forces and drag coefficient areas. We found that the Basic design had the most drag force and the Stable design had the least. The drag coefficient areas were all within one standard deviation of each other, and therefore no significant conclusions could be made about the difference in drag force that each airplane design experienced. Qualitatively, we were able to determine that the Basic Dart was the most stable design and the Basic was the least stable.

I. INTRODUCTION

Flight has consumed the human mind for centuries, and with the advancements of the Wright brothers, commercial flight became an achievable concept. As improvements in technology are made, new concepts and complications are inevitable. One such complication in flight is drag, the aerodynamic force that opposes the motion of an aircraft through a fluid [1]. While drag can be applied to any fluid, the only relevant fluid for aircraft is air. Even before drag was applied to aircraft, the concept was discussed as early as the 17th century by Isaac Newton. Newton argued that drag force is proportional to the velocity of the object squared and is in the opposite direction as the velocity. However, this relationship cannot be accurately applied to every situation and is best implemented for situations with higher velocities and turbulent fluid flow, where turbulent flow is defined as fluid flow in which the fluid has irregular fluctuations or mixing with itself [2].

Sir George Gabriel Stokes published another experiment regarding drag in 1851 which studied how a falling sphere in a fluid, either liquid or gas, is affected by the drag force. His study found that the drag force is directly proportional to the velocity of the sphere, which contradicted Newton's findings. It was determined that Stokes' findings were best applied to objects moving linearly through fluids and that move with steady fluid flow [3]. Beyond the turbulence of the fluid, drag depends on the size and shape of the object [4]. Unlike Stokes' experiment, paper airplanes are not spherically shaped, which aids in the presence of turbulent flow during flight. In this experiment, we will determine the drag force and the drag coefficient area for five different paper airplane designs by using Newton's turbulent and high velocity assumptions for flight.

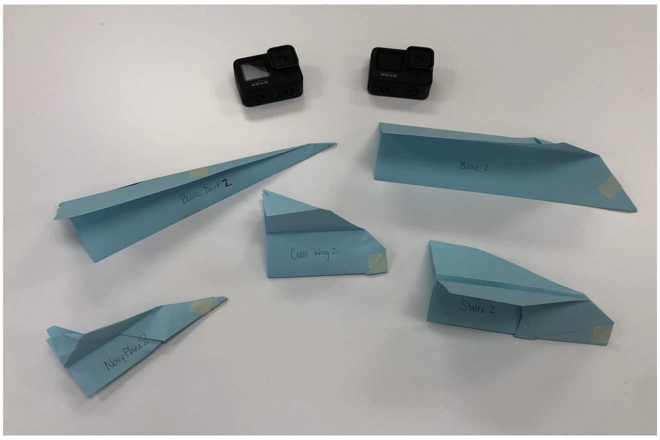


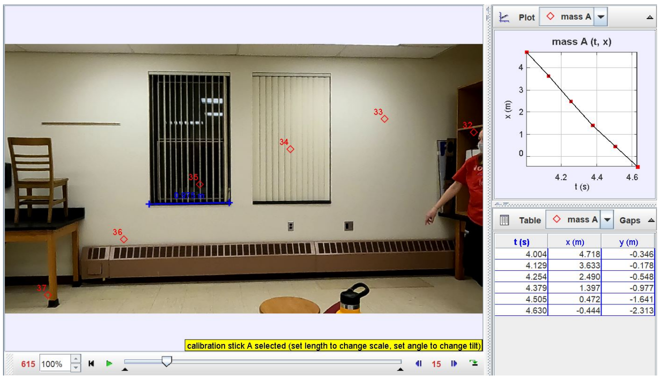
FIG. 1: The two HERO9 GoPro cameras used to record the flights and the five different paper airplane designs. Top: Basic Dart and Basic. Bottom: Navy Plane, Cross Wing, and Stable.

II. THEORY

A paper airplane in flight acts as a gliding aircraft. From this model, the plane has three forces acting on it, the force of gravity F_g, the lift force F_L, and the drag force F_D. F_g and F_L are acting on the plane in the vertical direction, while F_D is acting in the horizontal direction. For this experiment, we will only be looking at the horizontal direction velocities and therefore only F_D. We can use Newton's second law to analyze the drag force and get

$$F_D = ma, \tag{1}$$

where m is the mass of the paper airplane and a is the negative acceleration of the plane due to the drag force. We can further evaluate the drag force with the two types of drag discovered by Newton and Stokes, but because



calibration stick A selected (set length to change scale, set angle to change tilt)

t (s)	x (m)	y (m)
4.004	4.718	-0.346
4.129	3.633	-0.178
4.254	2.490	-0.548
4.379	1.397	-0.977
4.505	0.472	-1.641
4.630	-0.444	-2.313

FIG. 2: Image of the Tracker program and the data for the side profile of a tracked flight path.

this research project most accurately fits with Newton's evaluation of drag, we will only apply his theory. From [6], we know that

$$F_{quad} = -cv^2, \tag{2}$$

where v is the velocity and c is a constant that depends on the object's cross-sectional area and the fluid density. Ignoring Stokes' linear analysis of drag and applying Newton's quadratic analysis, we find that the drag force is

$$F_D = -cv^2. \tag{3}$$

We can now define c by applying the fluid density ρ and object cross-sectional area A to find

$$c = -\frac{1}{2}C_D\rho A, \tag{4}$$

where C_D is the drag constant. Finally, we can insert this new definition for c into Eq. (3) to get

$$F_D = \frac{1}{2}C_D\rho v^2 A. \tag{5}$$

We can then rearrange Eq. (5) to get

$$C_D A = \frac{2F_D}{\rho v^2}, \tag{6}$$

where $C_D A$ is the drag coefficient area. With this equation, we can determine the drag coefficient area by measuring the drag force, velocity, and the fluid density. For this study, the fluid density is the density of air.

III. PROCEDURE

This experiment has a relatively simple set up consisting of two HERO9 GoPro cameras, five paper airplanes of different designs, and a room large enough for the airplanes to fly without interruption. One camera was placed to capture the side profile of the planes in flight and the other was oriented to view the front profile of the planes. The GoPro application Quik was downloaded on an iPhone and used to start, stop, and magnify the videos on both cameras.

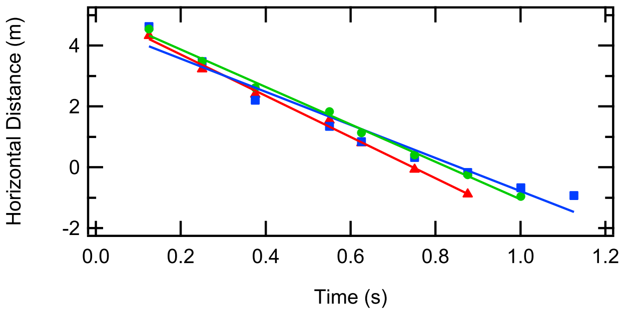


FIG. 3: A plot of the horizontal coordinate versus time for the three Stable paper airplane design flights. The position data is fitted to a line to determine the absolute value of the velocity. For the first flight (red triangles) the velocity was 6.8 ± 0.2 m/s. The second flight (blue squares) gave a velocity of 5.4 ± 0.4 m/s. Lastly, the third flight (green circles) had a velocity of 6.2 ± 0.2 m/s. This data results in a mean velocity of 6.1 ± 0.3 m/s for the Basic airplane design.

The five paper airplane designs, seen in Fig. 1, were the Basic, Basic Dart, Stable, Cross Wing, and Navy Plane [5]. Each plane was folded very carefully according to the instructions in order to minimize any folding mistakes that could have led to a less aerodynamic aircraft. Additionally, careful folding provides the best chance for symmetry on each side of the plane, which again will produce the most aerodynamic version of each plane. To ensure that each plane had the same mass, every plane was made of the same type of standard letter paper and had one 4 cm long piece of masking tape added to keep the designs together, resulting in each plane having a mass of 4.66 g.

Once the paper airplanes were created, each was thrown three times and the front and side profiles of the flight were recorded. The videos were saved on the SD card of each camera and were then transferred to the computer via the computer's SD port.

IV. RESULTS AND ANALYSIS

The videos collected by the GoPro cameras were uploaded to the program Tracker, seen in Fig. 2, where the data was then analyzed. Only the side profile videos were tracked because the view provided the best quantitative data to determine the horizontal velocity, and thus the drag force. With Tracker, we were able to mark where the planes were located throughout their flights and trace their flight paths. By measuring the actual size of an object in the camera view and designating that length in the program, Tracker was then able to produce accurate distances that the planes travelled during their flights.

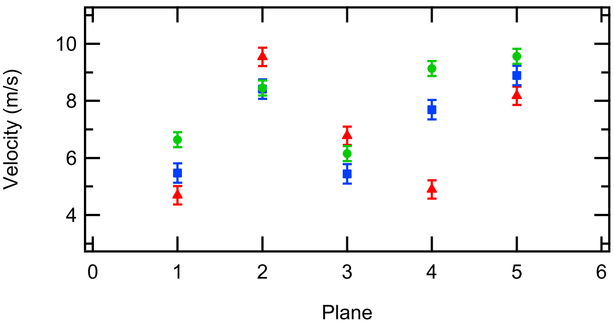


FIG. 4: The velocities of each of the three flight for each paper airplane design. For each plane, the red marker indicates the first flight, the blue marker indicates the second flight, and the green marker indicates the third flight. The error bars are derived from the uncertainties within the velocities. Reference Table I for the plane numbers.

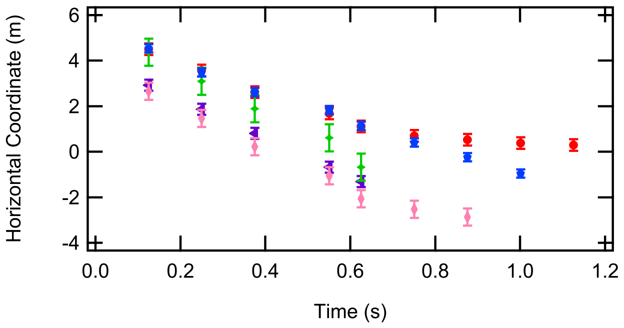


FIG. 5: The horizontal coordinate of the third flight for each paper airplane design, Basic (red circles), Basic Dart (purple triangles), Stable (blue pentagon), Cross Wing (pink diamond), and Navy Plane (green star), graphed versus time. The error bars are derived from the uncertainties within the velocities.

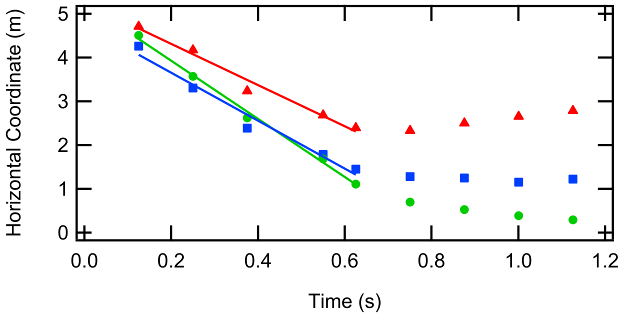


FIG. 6: A plot of the horizontal coordinate versus time for the three flights completed by the Basic paper airplane design. For the first flight (red triangles) the velocity was 4.7 ± 0.4 m/s. The second flight (blue squares) gave a velocity of 5.5 ± 0.6 m/s. Lastly, the third flight (green circles) had a velocity of 6.6 ± 0.3 m/s. This data results in a mean velocity of 5.6 ± 0.4 m/s for the Basic airplane design.

The position of the origin for these points was not significant to the research because only the difference between the initial and final distances was needed to calculate velocity. From the tracked data points, corresponding distances and times were produced. This data was moved from Tracker to Igor Pro for better analysis.

In Igor Pro, the horizontal distance the plane traveled, denoted as the horizontal coordinate, was plotted versus time and then a line of best fit was created for each of the three runs for all five plane designs. Figure 3 shows a sample plot of the flights for the Stable paper airplane design flights which was used to determine the velocity of the three flights for each design. Similar graphs were produced for the remaining designs to further calculate the velocities. The velocities for each individual run are displayed in Table I and are plotted in Fig. 4. The table allows for the values of the velocities to be analyzed across the flights within each design and the figure allows comparison across all of the designs. Figure 5 also gives a comparison across all five of the paper airplane designs for each design's third flight.

We used the uncertainty value from the mean velocity of each of the five designs to estimate a high and low velocity for the duration of the flights. Then we applied the definition of acceleration, $a = \Delta v/\Delta t$, to determine the upper bound of the negative acceleration due to the drag force. These values of negative acceleration were then used in Eq. (1) to find the maximum drag force F_D due to these negative accelerations. We found that the design that experienced the highest maximum drag force was the Basic, while the Stable design felt the lowest maximum drag force. Because the negative accelerations due to drag were significantly small, we were able to assume that velocity was constant throughout the flights. From this assumption, we used the mean velocities, drag force and Eq. (6) to calculate the drag coefficient area $C_D A$ for each design. Table II lists all of the calculated negative accelerations, drag forces, and drag coefficient areas.

TABLE I: Velocities of all three flights for each paper airplane design.

Design/Plane#	Flt.1 v (m/s)	Flt.2 v (m/s)	Flt.3 v (m/s)
Basic/1	4.7 ± 0.4	5.5 ± 0.6	6.6 ± 0.3
Basic Dart/2	9.5 ± 0.3	8.4 ± 0.1	8.5 ± 0.2
Stable/3	6.8 ± 0.2	5.4 ± 0.4	6.2 ± 0.2
Cross Wing/4	4.9 ± 0.4	7.7 ± 0.3	9.1 ± 0.4
Navy Plane/5	8.2 ± 0.3	8.9 ± 0.4	9.6 ± 0.6

For a more qualitative analysis, the front profile videos were studied to see the stability of the plane designs. The design that experienced the most stability was the Basic Dart, which experienced minimal spin and some tilt relative to its starting position. The Stable, Cross Wing, and Navy Plane designs all spiralled around the axis parallel to the ground. The Navy Plane and Stable, however, had tighter spirals while the Cross Wing had a much looser spiral. The plane with the least stability

TABLE II: Calculated negative accelerations, drag forces, and drag coefficient areas for the five paper airplane designs.

Design	a (m/s^2)	F_D (N)	C_DA ($\times 10^{-4}$ m^2)
Basic	1.4	6.3×10^{-3}	3.3 ± 0.2
Basic Dart	0.6	2.6×10^{-3}	0.6 ± 0.1
Stable	0.5	2.6×10^{-3}	1.1 ± 0.1
Cross Wing	0.9	4.3×10^{-3}	1.4 ± 0.1
Navy Plane	1.1	5.0×10^{-3}	1.1 ± 0.2

was the Basic design, which initially flew straight and then abruptly turned to the left and flew up towards the ceiling for each flight. This flight path variation can be seen in Fig. 6 where the horizontal coordinate flattens out and stays consistent while the plane flew directly towards the side profile camera. The remaining planes had a much more linear total flight path, as illustrated with the Stable design's flight seen in Fig. 3. In this figure, there is no flattening out of the data points which would indicate movement towards the side profile camera.

V. CONCLUSION

The goal of this experiment was to determine the drag force F_D and the drag coefficient area C_DA for five paper airplane designs, Basic, Basic Dart, Stable, Cross Wing, and Navy Plane. To find these values, each plane design was thrown three times and the front and side profiles were recorded with two HERO9 GoPro cameras. These videos were then transferred to the program Tracker where the flight paths were traced, and distance traveled and time of each flight were found. From that data, average velocities for each design were calculated in Igor Pro and the uncertainties of those velocities were used to determine a negative acceleration due to the drag force. Equation (1) applied the negative accelerations to find the drag force which was ultimately used to calculate the value of the drag coefficient area for each plane design. We discovered that the Basic design experienced the most drag force while the Stable design experienced the least. The drag coefficient areas were all within one standard deviation of each other, indicating that no significant conclusions can be made about how drag force affects the different paper airplane designs. However, there was qualitative data from the front profile video that showed significant stability differences between the designs. The Basic Dart design had the most stability for its flight duration. The Stable, Cross Wing, and Navy Plane all had a spiralling component to their motion, with the Navy Plane having the tightest spiral and the Cross Wing having the loosest. Lastly, the Basic design had the least stability, causing the plane to turn to the left and fly upwards.

There were multiple sources of error throughout this experiment. Since each paper airplane was flown by hand, the force used to throw each plane likely differed which would affect the initial velocity of each flight and alter the calculations for the drag force and drag coefficient area. The folding of the paper airplanes could also cause error if they were not symmetric on each side, as any lack of symmetry could create instability in flight. Additionally, it was difficult to measure the precise length of the known object when calibrating the distance in Tracker which could have affected the distance values. In regards to future work, dropping the planes, with the noses of the planes facing the ground, would minimize the error from the inconsistent force of throwing while still determining the drag force. Furthermore, other aspects of the planes, like the cross-sectional area size, could be tested to see if the drag force equation, Eq. (5), stands.

VI. ACKNOWLEDGMENTS

I would like to thank Dr. Susan Lehman for her guidance and assistance, the laboratory assistants Katie Shideler and Andrew Kunkel for their help with the material, and the College of Wooster Physics Department for providing the materials and data necessary to complete this experiment.

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