Using a Sensitive Torsion Pendulum to find Newton's Gravitational Constant

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(Dated: May 30, 2021)

The purpose of this experiment was to find out the gravitational constant G of Newton's law of gravity. To prove that this concept existed, and to find the specific value, we replicated Henry Cavendish's torsion pendulum experiment. Using a laser and a torsion balance, we managed to find out the the equilibrium positions of two separate configurations of the torsion pendulum. With that, we were able to calculate $G = (6.40 \pm 0.05)x10^{-11}m^3kg^{-1}s^{-2}$. This is in comparison to the accepted value $G = 6.67x10^{-11}m^3kg^{-1}s^{-2}$. This relative accuracy can be attributed to the large amounts of data points we collected during the experiment. Not only that, but the specific time in the day in which we took the data led to little to no disturbance in the pendulum.

I. INTRODUCTION

In 1797, Henry Cavendish used a torsion pendulum in order to find the Newton's gravitational constant G. The purpose of this experiment is to try and also find G with modern technology while still using Cavendish's methods. In this case, we used a sensitive torsion pendulum connected to a computer in order to see if Cavendish's method for finding G was valid.

II. THEORY

When two objects of any mass are in proximity to each other, the fundamental theory of gravity states that

$$F_g = \frac{(m_1)\,(m_2)\,(G)}{r^2} \tag{1}$$

where r is the distance between the two objects,m_1 and m_2, and G is Newton's gravitational constant. To find this gravitational constant, one uses a torsion balance. This torsion balance is a sensitive device used to measure the force created by objects in proximity to each other. Inside the enclosed structure is a pendulum arm containing two small lead spheres at opposite ends to each other. To counteract the force of gravity, a torsion wire is suspended above the device, cancelling the effect gravity has on the pendulum arm. Outside the enclosed structure is a rotary arm. This holds two large lead spheres of much greater mass than the two in the pendulum arm.

This rotary arm is used to adjust the position of the two large masses. The gravitational force of attraction between the large masses m_1 and small masses m_2 creates a torque on the pendulum arm. This torque causes the pendulum arm to wobble in a sinusoidal motion.

To measure the magnitude of this torque over time, the external parts of the torsion balance are used. A laser fires at a downward angle to a mirror that is inside the enclosed portion of the balance. This light from the reflected laser, when equilibrium is disturbed by either position S_1 or S_2, would proceed to oscillate. These oscillations could be used to find G.

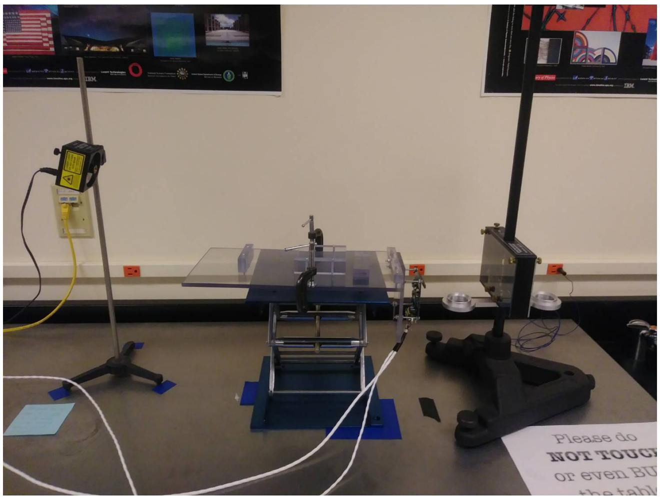

FIG. 1: Figure displaying the laser and the torsion balance. The laser is on the left, and the torsion balance is to the right of the image. The device in the middle of the image is irrelevant to the experiment, except for holding up the electronic device that collects data. The electronic device is connected to the bottom of the table. Figure borrowed from Ref.[1].

As the masses interact with each other, a torque is created. This is the torque due to gravity, τ_{grav}, which is given by

$$\tau_{grav} = 2F_g d \tag{2}$$

where d is the length of the lever arm, and F_g is the gravitational force. We know, however, that the system is in equilibrium. This means that there must be an equal and opposite torque, not due to gravity, but due to the torsion band being twisted. This torque $\tau_{band}/$ is given by

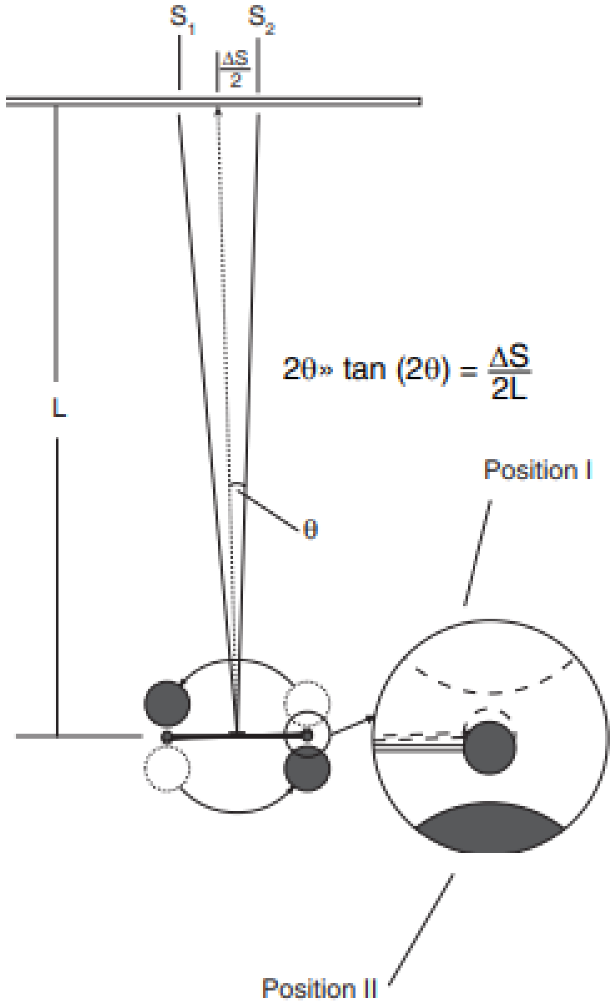

FIG. 2: Figure showing how the one can find θ by the positions of S_1 and S_2 Figure borrowed from Ref.[1].

$$\tau_{band} = -\kappa\theta = -\frac{2dGm_1m_2}{b^2} \tag{3}$$

Where κ is the torsion constant, and θ is the angle of rotation and b is the distance between the two masses. With S_1 and S_2, it is possible to find θ using trigonometry. As the masses m_1 and m_2 are shifted, the angle between the emitter and the emitted laser θ changes in accordance $\tan 2\theta$. Since θ is so small, however, we can treat is as simply $\tan\theta = \theta$. Using the principles of trigonometry, we can set 2θ equal to $2\theta = (S_2 - S_1)/2L$. We can find then θ as

$$\theta = \frac{S_2 - S_1}{4L} \tag{4}$$

when it comes to the angle. To find κ, one must know the period between oscillations. These oscillations are from the lever arm's interactions with the gravitational force created by both masses. In this case,

$$T = \sqrt{\frac{4\pi^2 I}{\kappa}} \tag{5}$$

where I is the moment of inertia of the small masses m_2 inside the box. I can be found as

$$I = 2m_2(d^2 + \frac{2}{5}r^2) \tag{6}$$

Where m_2 is the small mass inside the enclosed structure. Plugging this into the Eq.(5), and rearranging it for κ, we get:

$$\kappa = \frac{8\pi^2 m_2(d^2 + \frac{2}{5}r^2)}{T^2} \tag{7}$$

With both θ and κ found, we can plug in both Eq.(4) and Eq.(5) into Eq.(1), giving us

$$G = \pi^2 \Delta S b^2 \frac{d^2 + \frac{2}{5}r^2}{T^2 m_1 L d} \tag{8}$$

III. PROCEDURE

As stated in the theory, the device used was a torsion balance. The laser of this balance was turned on, and it was made sure that the device was properly calibrated. The first step was to measure the length from the mirror to receiver L. This was measured by taking a small ruler to the beginning of the enclosed mirror and the measured screen, resulting in a measurement of L=121.4 $\pm$ 0.5 mm. The uncertainty for L was given because of the tool of measurement would give the result to the nearest millimeter. We measured d to be 50 mm, r to be 9.55 mm, b to be 46.5 mm, and m_1 to be 1.5 kg. The large masses were placed on the lever arm, with the arm initially positioned perpendicular to the enclosed portion of the device. The LABVIEW data collection software on the computer was turned on. The lever arm was then set to position S_1 for 2 hours and 30 minutes. With the LABVIEW software still running, the lever arm was then set to position S_2 for another 2 hours and 30 minutes. The long running time was necessary in order to minimize uncertainty in the results, especially because of the extreme sensitivity of the device. We also made sure that the experiment would run at a time in which very little people would be in the building where the device was located. The extreme sensitivity of the device necessitated that there be no one around, since even the disturbances in the air caused by someone walking near the device could potentially skew the data. The software was then stopped, and the trial concluded. The data was transferred over to IGOR Pro for analysis. The resulting graph was able to let us find ΔS, and G.

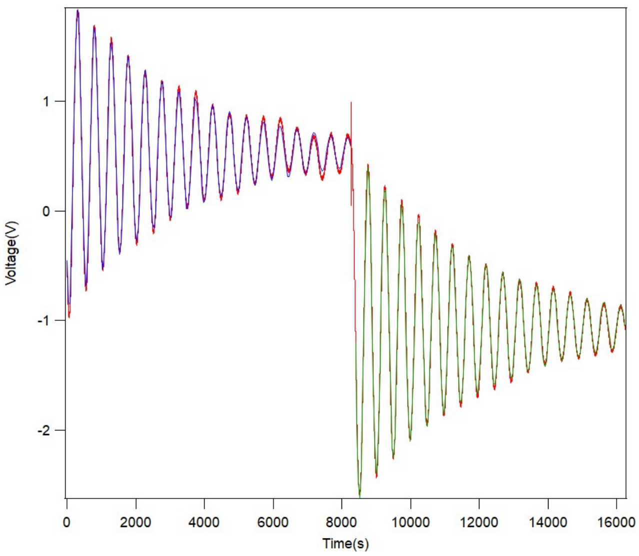

FIG. 3: Voltage vs. Time plot of the Torsion Balance plotted on IGOR. The red line represents the data collected. The blue line is the fit line of S_2. The green line represents the fit line of S_1. The middle of the fit lines give both the positions S_1 and S_2, and the distance from one peak to another to find T. Since the LABVIEW Software gave a Voltage vs. Time Graph, rather than Position vs. Time, it was necessary to use Eq.(11) to convert the values properly.

IV. RESULTS & ANALYSIS

Figure 3 shows the graph of the oscillations. We see that over time, the magnitude of the oscillations lessen. Theoretically, it would eventually show a straight line. The value of these two straight lines, S_1 and S_2, is what we needed to find G. To find the value of S_1 and S_2, we had to create a custom fit line on IGOR. The fit line was fitted to the equation

$$x = e^{-\gamma t}\alpha cos(\omega t - \alpha) + P \tag{9}$$

What is most important is αS and t. Using a fit line for both portions, we saw that ω=0.0128±0.0003.The uncertainty for ω was found with IGOR. In this case we used the differences in the value for ω for S_1 and S_2 to find the uncertainty. We used this value to find T with the equation

$$T = \frac{2\pi}{\omega} \tag{10}$$

Plugging in for ω, we see that period T=523.60 ± 1 s. The uncertainty for T was calculated by looking at the significant figures of the Voltage vs. Time graph, and seeing that it only went to the nearest whole second. To find ΔS, we use the value of the offset P. However, the LABVIEW software gives a voltage vs time graph, so we must convert the voltage we have to the actual displacement S_1 and S_2. To do this, we simply use the equation

$$x = (V_0)\left(\frac{37}{20}\right) \tag{11}$$

With this, we can find that S_1=0.995±0.0002 mm and S_2=-1.958±0.0001 mm , so ΔS=2.953 ±0.0003 mm. The uncertainty for ΔS was made by simply adding the uncertainties for S_1 and S_2 together. With both T and ΔS found, we can find G. Plugging in all the values into Eq.(8), we got

$$G = (6.40 \pm 0.05)x10^{-11}m^3kg^{-1}s^{-2} \tag{12}$$

Thus the gravitational constant G was found. Compared to the accepted value, G=6.67x10$^{-11}m^3kg^{-1}s^{-2}$, we obtained a value 4.2 percent away from the accepted value.

V. CONCLUSION

This experiment showed the validity of Cavendish's methods when it came to measuring G. Because of the numerous procedures which were used to mitigate uncertainty, we got a value that was very close to the actual value for G, only 4.2 percent away from the accepted value. The extreme sensitivity of the balance was actually a boon here. While it would be a problem if it was being used in a busy room where people walking would create influencing air currents, the data was collected late in the afternoon, when there was nobody in the room. This made it so that the air would be completely still, and that nobody would accidentally bump into it. We also mitigated uncertainty with the many data points that were collected, therefore making the fit line more accurate. One way to experiment could be improved, however, could be measuring L more accurately, as that had the highest uncertainty compared to any of the other values. The high accuracy and low uncertainty of this experiment shows that Cavendish's method to find G was reliable for both his time and today.

VI. ACKNOWLEDGMENT

This project would not be possible without Dr.Lehman for having the torsion balance pre-calibrated. Not only that, but a special thanks to the physics department for making this experiment possible.

VII. BIBLIOGRAPHY

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