The Investigation of Germanium Sample Properties

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(Dated: May 2, 2021)

The relationship between a p-type and n-type sample of germanium were investigated in order to determine how the effects of doping a sample contribute to its properties. The Hall constant, sample concentration, and the band gap energy was calculated by connecting a germanium crystal with a sample current flowing through the conductor that is perpendicular to a magnetic field. Due to the presence of the magnetic field, the transverse Hall voltage was determined. While varying the current or magnetic field, the sample properties and their relationships were calculated and compared with one another. The Hall constant for a p-type sample was calculated to be $(7.69 \pm 0.05) \times 10^{-3}$ m^3C^{-1} and the n-type sample to be $(7.29 \pm 0.18) \times 10^{-3}$ m^3C^{-1}. We can then calculate the carrier concentration for each sample. They were found to be $(8.610 \pm 0.002) \times 10^{-20}$ m^{-3} for the p-type and $(8.553 \pm 0.004) \times 10^{-20}$ m^{-3} for our n-type sample. The second section of our experiment contained heating up the sample and letting it cool in order to collect the voltage across the sample. The band gap for our p-type sample was found to be 0.76 ± 0.01 eV.

I. INTRODUCTION

The discovery of the Hall effect by Edwin Hall in 1879 has been used in a wide variety of equipment. The Hall effect was discovered when Hall had measured a voltage as a magnetic field was sent through a perpendicular conductor with a current running through. The principle suggested there was a build up of electrons at one end of the wire due to the magnetic field. The process of these electrons building up can be described with the Lorentz force, which is the force exerted on the electrons in the current as they pass through the magnetic field. After Hall's findings, his effect became very useful when integrated with semiconductors because of the build up of electrons created. Since then, scientists have been able to further understand the electromagnetic principles behind the Hall effect, such as the qualities of different semiconducting samples. By using the Hall effect, researches determine the band gap energy, the conductivity, the density of carries, and the hole concentration of different materials.

In our experiment, these different qualities for each of the germanium samples were determined with a PHYWE Hall effect module, as seen in Fig. 1. We identify how a p-type germanium sample differs from a n-type germanium sample.

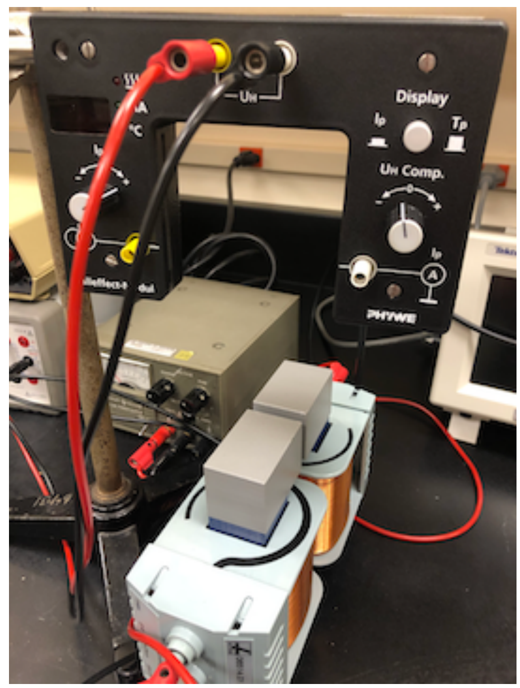

FIG. 1: Hall effect module used to collect the temperature, the sample voltage across the sample, and the strength of the current.

II. THEORY

In order to understand the conductivity of germanium, the Hall effect is measured with respect to the different semiconductor samples. It is important to understand how a current flowing through a conducting sample strip with a perpendicular magnetic field produces a voltage between the two sides of the strip. The produced voltage is known as the Hall voltage V_H.

The sample is first introduced to a current, which will cause the electrons to flow straight across as normal. We know from the right hand rule that a small magnetic field will then be produced as a result of the current. When an exterior magnetic field is brought perpendicular to the current, the electrons traveling within the conducting sample will want to move to one side of the sample. The force produced on the electron with charge $-e$ is known as the Lorentz force $\vec{F}$. The force is given as

$$\vec{F} = -e(\vec{v} \times \vec{B}), \quad (1)$$

where v is the charges velocity and B is the applied magnetic field. As seen in Fig. 2, the force on the positive

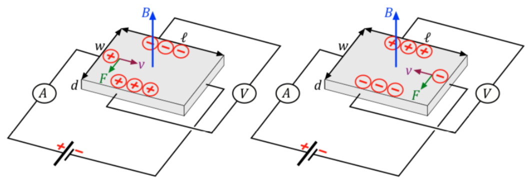

FIG. 2: Representation of the Lorentz force as a function of velocity v, and magnetic field B for a positive and neagtive charge carrier. Diagram taken from Junior Independent Study Manual. [2]

charge is in the same direction since it is originally heading in the opposite direction of the electron. [1] As you can see, the voltage and sample current are in the same plane, but are still perpendicular to one another and to the magnetic field. The perpendicular layout is important because the Hall voltage is created from the Lorenz force on the charges as they pass by. The Hall voltage is determined based on the set up of the direction of flow for the current and magnetic field. A linear relationship has been determined between the sample current and the perpendicular magnetic field. The Hall voltage V_H, equation is shown to be linear with the current I, and magnetic field B, in two different situations. The first situation occurs when there is constant magnetic field, but a varying sample current. The linear relationship with the Hall voltage is given by

$$V_H = \alpha \cdot I, \tag{2}$$

where α is the proportionality factor. The second situation occurs when there is a constant current I, and a varying magnetic field. This linear relationship with magnetic field B, is given by

$$V_H = V_0 + b \cdot B, \tag{3}$$

where V_0 represents the initial voltage and b represents the slope of the graph of Hall voltage vs varying magnetic field.

The linear relation for both the current and the magnetic field with respect to the Hall voltage can be applied to calculate the Hall constant R_H. Not only is the constant dependent on the Hall voltage, current, and magnetic field, but also the thickness d. Our relationship for the Hall constant first arises from the Hall voltage. Due to the Lorenz force equation and the perpendicular layout of the system, there are two force equations. The first equation is in terms of the current and magnetic field being perpendicular with one another. The second equation is written in terms of the potential difference V_H and the magnetic field since they are also perpendicular. The Lorenz force can be written in two forms as

$$F = \frac{I \cdot B}{n \cdot A} = \frac{V_H \cdot e}{\omega}, \tag{4}$$

where n is the hole concentration, A is cross-sectional area, and ω is a component that is perpendicular to both the magnetic filed and current, which will simplify with A to become known as the thickness d. Then by solving for the Hall voltage, it is found to be

$$V_H = \frac{I \cdot B}{nde}, \tag{5}$$

which can be used to find the Hall constant R_H as

$$R_H = \frac{V_H d}{BI}. \tag{6}$$

The Hall constant allows us to calculate the hole concentration for a p-type sample or the electron concentration for a n-type sample by solving for n using Eq. 5 and Eq. 6. This defines n to be given as

$$n = \frac{1}{e \cdot R_H}, \tag{7}$$

where e is the electron charge.

In order to calculate the band gap energy E_g, we measure the specific resistivity going across the germanium conductor as a function of temperature. For the case of this experiment, resistivity will be different for the p-type and n-type samples due to their impurities. A p-type semiconductor is intentionally made with holes due to their being less electrons in the outer shell. The band gap energy provides the amount of energy needed for an electron to move between the valance band and the conduction band. Finding the band gap energy becomes important when wanting to use a material who has a high or low conductivity. The conductivity of the material σ can be determined as a ratio. The conductivity is given as a relationship with the voltage across the sample V_p, in the direction of current as

$$\sigma = \frac{I \cdot l}{A \cdot V_p}, \tag{8}$$

where l is the length of the metal.

The relationship between intrinsic conductivity and absolute temperature T, is given by

$$\sigma = \sigma_0 \cdot \exp\left[-\frac{E_g}{2kT}\right], \tag{9}$$

where k is Boltzmann's constant. From here, the equation can be turned into a linear relationship of $y = mx + b$ in order to find the band gap energy. By taking the natural logarithm of Eq. 9, we get

$$\ln \sigma = \ln \sigma_0 - \frac{E_g}{2kT}. \tag{10}$$

This equation allows us to graph and find the slope b, between $\ln \sigma$ and $\frac{1}{T}$. The band gap energy is then determined to be

$$E_g = b \cdot 2k. \tag{11}$$

The theory provided allows for an experimental analysis to find the electromagnetic properties of a germanium sample.

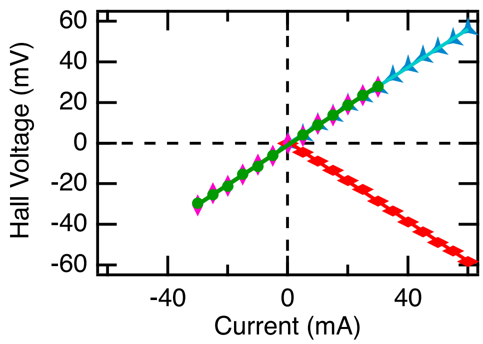

FIG. 3: Hall voltage vs current for a p-type germanium sample. Three runs (turquoise triangles, pink diamonds, and green circles) were collected separately with constant magnetic field. An additional run (red diamonds) was collected with a constant negative magnetic field to check for systematic error.

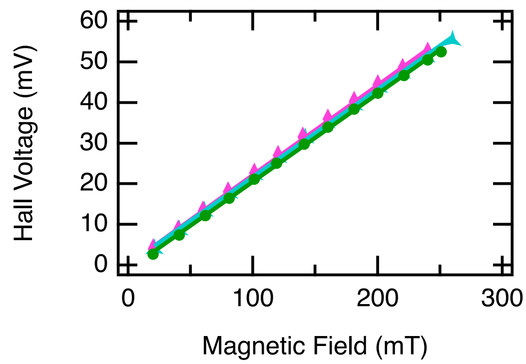

FIG. 4: Hall voltage vs magnetic field for a p-type germanium sample. Three runs (turquoise triangles, pink diamonds, and green circles) were collected separately with constant current.

III. PROCEDURE

In order for us to experimentally find the properties of a germanium sample, a PHYE Hall effect module was used. A p-type and an n-type sample of germanium were measured to determine their properties. What distinguished the two samples of germanium are an addition of impurities in order to change the majority charge carrier. The p-type samples are being doped with small traces of aluminum or boron, where the n-type sample can have small traces of phosphorous which help the electrons move into the conducting band. [3]

A. Hall Coefficient & Sample Concentration

Each germanium sample went through two sets of similar processes for evaluation. The first set of our measurements was collected when the germanium sample was attached to the Hall effect module with a constant perpendicular magnetic field. The samples were introduced to a magnetic field by placing them in between two electromagnets. We measured the Hall voltage V_H, as the sample current varies through our germanium sample. We were able to retrieve our Hall voltage measurement from a multimeter by placing the leads on the top and bottom of the sample, as show in Fig. 2. This process was also done for a constant sample current with a varying magnetic field. This was done to show the linear relationship with the Hall voltage. Measurements were collected when there was a constant magnetic field of 0.125 T present while increments of 5 mA from -30 to +30 mA for the sample current. Then by doing the vice versa of magnetic field and sample current, there was a constant 30.0 mA current with a varying magnetic field in increments of 20 mT from 0 to 250 mT.

B. Band Gap

In the second set of our data, we wanted to find the band gap energy for germanium. We had the p-type germanium sample hooked into the Hall effect module. We then heated up the sample to 170 °C and recorder the voltage across as the sample as it cooled back down to room temperature, 20 °C, by recording the measurements using a phone camera. For these specific runs, there was no magnetic field present since the band gap energy is not dependent on a magnetic field, as seen from Eq. 8 to Eq. 11.

IV. RESULTS & ANALYSIS

A. p-type Sample

In the first section of our experiment, we plotted the Hall voltage as a function of varying current, as seen in Fig. 3. As discussed in the theory section, the slope of the graph for the Hall voltage vs current is the proportionality factor α. The slope of the lines from Fig. 3 show a reliable relationship for α. They allow us to determine a more precise Hall constant. The collected measurements from our four runs was calculated to have an average proportionally factor of 0.969 ± 0.007 A^{-1}.

We then took three runs for measuring a varying magnetic field with a constant current of 0.030 A, as seen in Fig. 4. The graph allows us to find the Hall constant a second way, due to the linear relationship the constant has with both the magnetic field and sample current. This relationship is proven from Eq. 5. From here we can calculate the Hall constant R_H, for both instance of linear relationships using Eq. 6. This means we need to multiply the proportionally factor by the thickness over the constant magnetic field when using the results from Fig. 3. This produces a Hall constant of $(7.69 \pm 0.05) \times 10^{-3}$ m^{3}C^{-1}. The calculation for the hole

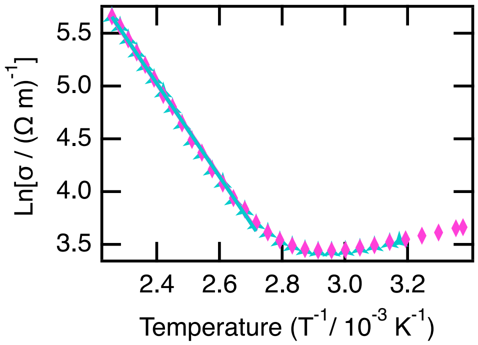

FIG. 5: Natural logarithm of conductivity vs the inverse of absolute temperature. Two runs (turquoise triangles, and pink diamonds) were collected separately for the p-type sample.

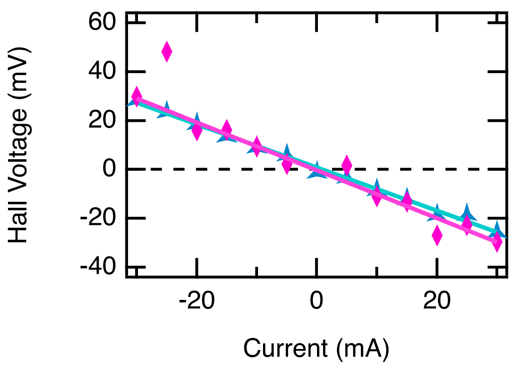

FIG. 6: Hall voltage vs current for a n-type germanium sample. Two runs (turquoise triangles, and pink diamonds) were collected separately with constant positive magnetic field. The -25 mA current measurement was excluded from the pink diamonds best fit.

concentration n can be determined for the p-type sample by taking the inverse of the Hall constant and the inverse of an electron charge, as seen in Eq. 7. We have experimentally calculated a hole concentration average of $(8.610 \pm 0.002) \times 10^{-20}$ m^{-3}, which is relatively close to previous measurements from students and the PHYWE manual.

We then found the band gap energy for the p-type sample. This was determined by first calculating the conductivity from the measured sample voltage going across the germanium crystal and then taking the natural logarithm of it. We then had to plot the natural logarithm vs the inverted absolute temperature multiplied by 1000 to help with the dimensions so that we could use the slope for our final calculation. As seen in Fig. 5, we found an average slope of $(-4.41 \pm 0.06) \times 10^3$, which is given by using Eq. 10. The best fit line was applied to the left half of the graph due to the relationship following better at higher temperatures. After the two runs, we found an average band gap energy of 0.76 ± 0.01 eV. The theoretical band gap energy of Germanium is 0.67 eV. [4] The band gap energy could have been closer to the value if an undoped sample of germanium was used.

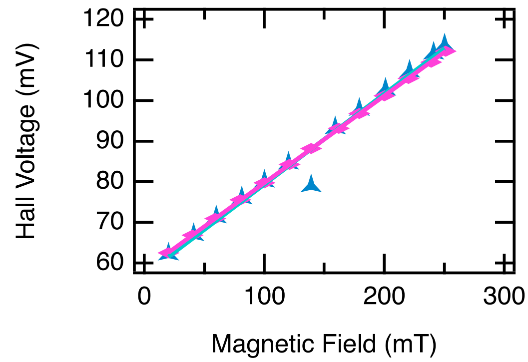

FIG. 7: Hall voltage vs magnetic field for a n-type germanium sample. Run 1 (turquoise triangles) and Run 2 (pink diamonds) were collected with a constant current of 30.0 mA. The 150 mT measurement was excluded from run 1 when a best fit was applied.

B. n-type Sample

Now by repeating the similar steps from above, the properties of a p-type and n-type sample can be compared. Measurements were made for the Hall voltage as a function of varying current, as seen in Fig. 6. The slopes from this graph will be used to calculate the proportionality factor α. We collected three runs of measurements and found an average proportionally factor of -0.932 ± 0.004 A^{-1}. When applying the best fit line for the second run, the -25 mA current measurement was excluded due to how far off the point is from the proposed linear relationship. The measurements around -25 mA seem to go back to following the linear fit.

We can begin to switch the magnetic field and current and find our linear relationship in order to find the Hall constant the second way. These measurements were found with a constant sample current of 0.030 A with a varying magnetic field, as seen in Fig. 7. From our graph, the average linear fit of the slope was determined to be 0.219 ± 0.005 VT^{-1}. This gives rise to allowing the calculation for the Hall constant for the n-type sample to be determined. By calculating the Hall constant for both sets of runs from the variation of one component from Eq. 6, we found an average of $(7.29 \pm 0.18) \times 10^{-3}$ m^{3}C^{-1}. The calculation for the electron concentration for our n-type sample can be determined from the graphs of Fig. 6 and Fig. 7. By doing the same calculation that we did for our p-type sample, we calculated an electron concentration of $(8.553 \pm 0.004) \times 10^{-20}$ m^{-3}.

TABLE I: Properties of p-type and n-type germanium sample.

Sample Type	Hall Constant $[\mathrm{m^3C^{-1}}]$	# of current carriers per unit volume $[\mathrm{m^{-3}}]$
p-type	$(7.69 \pm 0.05) \times 10^{-3}$	$(8.610 \pm 0.002) \times 10^{-20}$
n-type	$(7.29 \pm 0.18) \times 10^{-3}$	$(8.553 \pm 0.004) \times 10^{-20}$

V. CONCLUSIONS

From our analysis of the Hall experiment, important properties of each semiconducting sample have been determined. The Hall effect in p-type and n-type germanium was measured by varying the magnetic field while keeping the current constant, and also by varying the current while keeping the magnetic field constant. The system set up has allowed for the appropriate calculations to find their concentration, Hall constant, and the band gap energy for the p-type sample. The Hall constant for the p-type sample was found to be $(7.69 \pm 0.05) \times 10^{-3}$ $\mathrm{m^3C^{-1}}$ and the n-type sample to be $(7.29 \pm 0.18) \times 10^{-3}$ $\mathrm{m^3C^{-1}}$. From here we calculated the carrier concentration for each sample. They were found to be $(8.610 \pm 0.002) \times 10^{-20}$ $\mathrm{m^{-3}}$ for the p-type and $(8.553 \pm 0.004) \times 10^{-20}$ $\mathrm{m^{-3}}$ for our n-type sample. The concentrations for the different sample do not vary by a lot. This is relatively good to see since they have the same base element, germanium. The band gap for our p-type sample was found to be 0.76 ± 0.01 eV. There is no standard value for carrier concentration to compare to our results, however, we were able to compare our calculations for the p-type sample with Nash's paper, *Characterizing Germanium with the Hall Effect*. [3] Nash found a band gap energy for the undoped germanium sample to be (0.727 ± 0.001) eV. He also found the sample concentration for the n-type sample to be 8.42×10^{-20} $\mathrm{m^{-3}}$. When comparing our data from Table I we can see our findings are not too far off with our n-type data run average. Previous papers on the Hall effect from the College of Wooster could not be compared with our results because the previous papers were done before the purchase of the current p-type and n-type samples.

[1] Ratna, *Hall Effect Principle – History, Theory Explanation, Mathematical Expressions and Applications*, https://electricalfundablog.com/hall-effect-principle-history-theory-explanation-mathematical-expressions-applications (2018), Last accessed February 27, 2021.

[2] *Junior IS Mechanics Physics Lab Manual*, Department of Physics, The College of Wooster, Wooster, OH (2019).

[3] S. Nash, *Characterizing Germanium with the Hall Effect*, http://physics.wooster.edu/JrIS/Files/Web_Article_Nash.pdf (2018), Last accessed February 30, 2021.

[4] *Germanium*, https://en.wikipedia.org/wiki/Germanium (2021), Last accessed March 3, 2021.