Testing Thermal Conductivity With Angstrom Method

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Angstrom method is a smart way to measure the thermal conductivity. We will measure the thermal conductivity of a brass rod by using this method. We will measure the temperature of two different positions on the rod. We plot the temperature as a function of time and then do the Fourier transform to it for further analysis. We picked three points in the temperature function to calculate the conductivity. Finally, the average value we get is 109 KM/m, while the actual value is 120 KM/m. The error is 9%.

I. INTRODUCTION

It is an interesting topic that different materials have different capability to conduct heat. The property of materials' capability to conduct heat is called thermal conductivity. There were researches about this topic hundred years ago.

In 1861, Anders Jonas Angstrom published a method to determiner the thermal conductivity of a rod. Generally, this method is to provide heat to one end of a rod periodically and monitor the the temperature as a function of time. Then, we can use a formula to calculate the conductivity. Later, a new refined method came out, it used Fourier analysis of a simple square wave heating function. For the new method, one end of the rod is at room temperature, and a heat pulse is applied to the other end. Two thermistors will be needed to record the temperatures of two different positions along the rod. We will record the temperature as a function of time by using a written program. Then, we need to do Fourier analysis to the two temperature functions. We are interested in the amplitude and phase of the harmonics present in the periodic temperature oscillations. [1]

II. THEORY

To determine the thermal conductivity of the rod, we will use a heat pulse to provide it a periodical heat on one end. There is a variable x, which is the distance of a point on the rod to the heated end. We can control the heat. We let $T[x,t]$ be the rod's temperature relative to the air. The heated end should have temperature as function $T[0,t]$ since the distance between the end and itself is zero, and

$$T[0,t] = T_0 cos[\omega t] \tag{1}$$

where T_0 is the amplitude and ω is the frequency. The heat from the heated end of the rod will diffuse alone the bar and finally be lost to the atmosphere. The relationship here is

$$\frac{dT}{dt} = D\frac{d^2T}{dt^2} - \epsilon T \tag{2}$$

where $D = \kappa/s\rho$, and $\epsilon = RC/s\rho A$. There are so many variables. D is the thermal diffusivity, κ is the thermal conductivity which is the thing we are measuring in this experiment. The specific heat s, the density ρ, the emission coefficient R, the circumference C and the cross sectional area A are all constants. We can guess that the solution for the temperature and position function is

$$T[x,t] = Ae^{-ax}cos[\omega t - bx] \tag{3}$$

As we put equation [3] into equation [2], the left part will be

$$\frac{dT}{dt} = -A\omega e^{-ax}sin(t\omega - bx) \tag{4}$$

We assume that

$$D = \frac{\omega}{2ab} \tag{5}$$

and

$$\epsilon = (a^2 - b^2)D \tag{6}$$

then, the right side of equation[2] will be consisted by two parts, one is

$$D\frac{d^2T}{dt^2} = A\omega e^{-ax}\frac{a^2-b^2}{2ab}cos(\omega t - bx) - Ae^{-ax}\frac{2ab\omega}{2ab}sin(t\omega - bx) \tag{7}$$

and the other one is

$$\epsilon T = -Ae^{-ax}cos(\omega t - bx)\frac{a^2-b^2}{2ab}\omega \tag{8}$$

To check our assumption, we use equation[7] subtracted by equation[8] and get $A\omega e^{-ax}sin(t\omega - bx)$. The result is equal to $\frac{dT}{dt}$, which means that our assumption about the value of D and ϵ is acceptable.

Since we will measure the temperature of two positions along the rod, we define the point near the heated end as x_R and the farther one as x_L. Also, their amplitudes are A_R and A_L respectively.

The amplitude ratio is

$$\frac{A_L}{A_R} = e^{-a(x_L - x_R)} \tag{9}$$

and the phase difference is

$$\varphi_L - \varphi_R = b(x_L - x_R) \tag{10}$$

Then, we want to define a and b with the equations above. We invert equation[9] and equation[10] and get

$$a = \frac{log[A_L/A_R]}{x_R - x_L} \tag{11}$$

and

$$b = \frac{\varphi_L - \varphi_R}{x_L - x_R} \tag{12}$$

Within some algebra, we can get the equation to calculate our goal, the thermal conductivity, κ

$$\kappa = \frac{s\rho\omega(x_R - x_L)^2}{2(\varphi_L - \varphi_R)log[A_L/A_R]} \tag{13}$$

We know that the rod in this experiment is made of brass, with $\rho = 8470\text{kg/m}^3$ and s=368J/kg•K. We will find the rest of variables in our experiment.

III. PROCEDURE

As explained in the theory part, we want to find $x_R - x_L$, ω, $\varphi_L - \varphi_R$, andA_L/A_R in the experiment. The overview of the apparatus is shown as Figure [1]
The cylindrical thing at the front is a brass rod, with 1 cm in diameter. The machine on the left is called Kepco power supply, It can provide power to a thermofoil heater attached to one end of the rod. The top one is a Tektronix function generator, which could provide square wave signal to the Kepco power supply machine. There's a insulation bubble wrap surround the entire rod to control the loss of heat from the rod. To measure the temperature on two points, two YSI44004 Precision thermistors are used. They are 15.1 cm apart from each other. The thermistors are wired into a series circuit with a reference resistance of 15 kΩ and a 1.5 V battery. The thermistors have resistance which varies along temperature. The thermistors are connected to the Hewlett Pachard 3421A Data Acquisition Unit, which can measure the voltage on the resistors. We used LabVIEW to record the datas automatically.
The apparatus are prepared well. We can turn on the machines and go back to sleep.
After 15 hours, the data we collected is shown as figure [2]. Then we will do the Fourier Transform to a part of the row data.

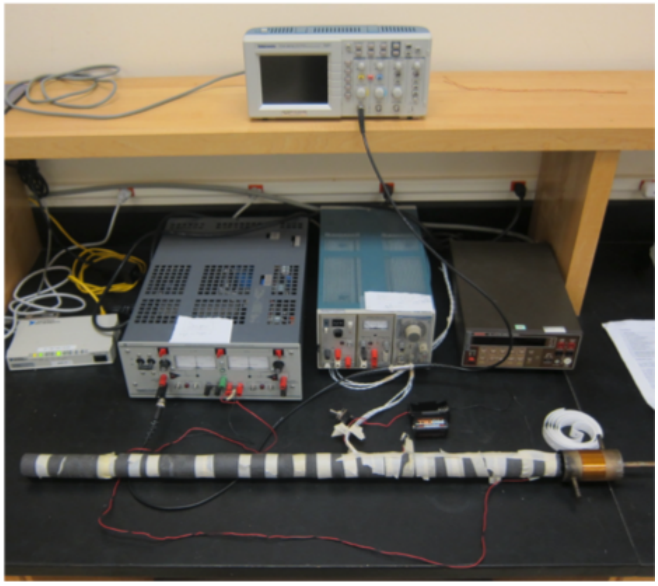

FIG. 1: Top view of instrument sets for Angstrom experiment (This figure is borrowed from ref[1])

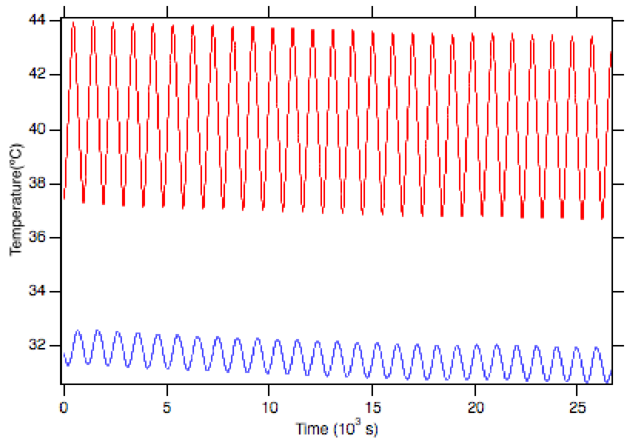

FIG. 2: The temperature as function of time. The red line refers to the temperature at x_L and the blue line refers to the temperature at x_R

IV. ANALYSIS

As we have the temperature data, we can start to look for the variables we need to calculate the thermal conductivity. As listed above, the variables we are interested in are $x_R - x_L$, ω, $\varphi_L - \varphi_R$, andA_L/A_R. Since the two thermistors are 15 cm between each other, which means $x_R - x_L$=15 cm. Then, we will need to do Fourier transform to find the amplitude and phase for each position. And also, we need to find the frequency to calculate ω.
As we use Igor to do the Fourier transform, we want to first know the interval of time we will use, since Igor will not do that automatically. We plot the time function and find the time interval as 5.328 s. Then, we can do the Fourier transform and change the wave scaling into

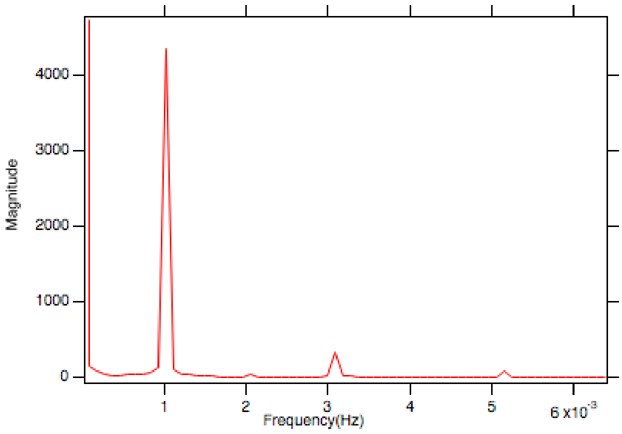

FIG. 3: The magnitude versus frequency from Fourier transforms at x_L. The peaks are the first, third, fifth harmonics. The frequencies for each of them are 1.03mHz, 3.1mHz, and 5.15mHz. and the magnitudes are 4330, 322, and 81.

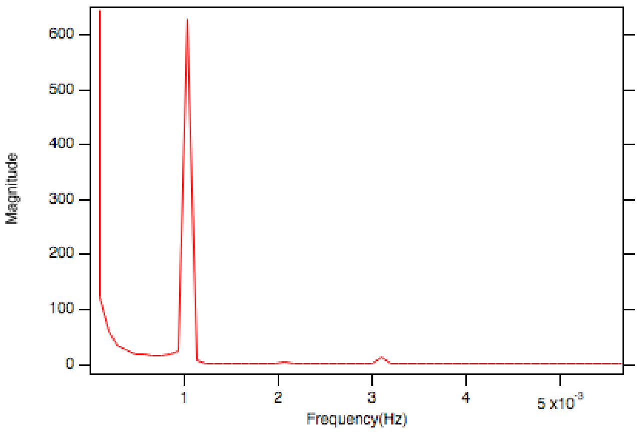

FIG. 4: The magnitude versus frequency from Fourier transforms at x_R. The peaks are the first, third, fifth harmonics. The frequencies for each of them are 1.03mHz, 3.1mHz, and 5.15mHz. and the magnitudes are 629, 14, and 1.5.

5.328 s. There comes a magnitude versus frequency plot for each position, shown as figure [3] and figure [4]
Therefore, now, we know A_L, A_R, and frequency. The next step is to find the phase. We can use Igor to plot a phase versus frequency graph to get the value.
The phases are shown as figure [5] and figure [6].
As we have all the information we need, we can start to calculate the thermal conductivity with equation [13], I'll show an example of my calculation for the first harmonic.

$$\kappa = \frac{s\rho\omega(x_R - x_L)^2}{2(\varphi_L - \varphi_R)log[A_L/A_R]} \tag{14}$$

$$= \frac{368\text{KJ/kg} \times 8470\text{kg/m}^3 \times 0.007\text{rad/s} \times (0.15\text{m})^2}{2(3.02\text{rad} - 1.8\text{rad}))\log[4330/629]} \tag{15}$$

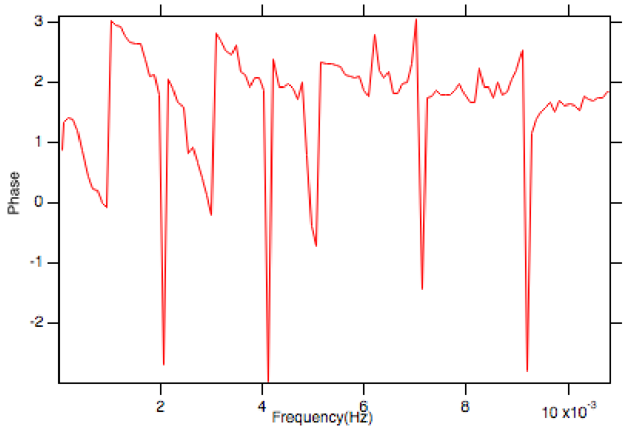

FIG. 5: The phase versus frequency from Fourier transforms at x_L. The peaks are the first, third, fifth harmonics. The frequencies for each of them are 1.03mHz, 3.1mHz, and 5.15mHz. and the phases are 3.02, 2.83, and 2.33

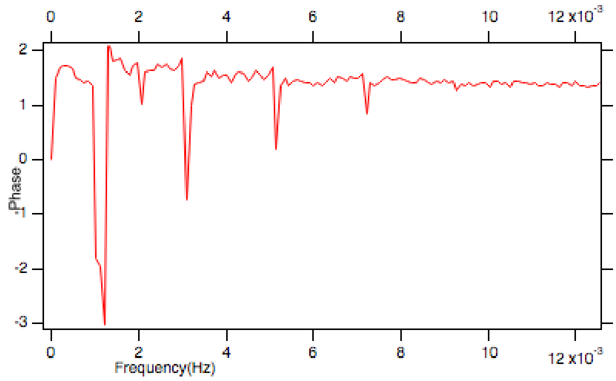

FIG. 6: The phase versus frequency from Fourier transforms at x_R. The peaks are the first, third, fifth harmonics. The frequencies for each of them are 1.03mHz, 3.1mHz, and 5.15mHz. and the phases are -1.8, -0.74, and 0.18

$$= 104\text{KW/m} \tag{16}$$

To show the data in a better way, the important datas mentioned above and the results are all shown in table [1].

The average value for κ in this experiment is 109 KW/m, while the actual thermal conductivity of brass is 120 KW/m. So, the error in this experiment is

$$\frac{120 - 109}{120} = 9\% \tag{17}$$

TABLE I: Summary of important datas and results

n	1	3	5
$\log(A_L/A_R)$	1.93	3.13	3.98
$\delta\varphi$(rad)	1.22	2.09	2.51
ω(rad/s)	0.007	0.02	0.033
κ(KW/m)	104	107	116

V. CONCLUSION

We used the Angstrom method to measure the thermal conductivity of a brass rod. The error is 9%. This method is really complicated theoretically. But when we understand the theory part, making measurements on different materials becomes really easy. This is a really good method to test thermal conductivities.

VI. REFERENCE

[1] "Angstrom: Thermal Conductivity" ,Physics 401 Junior Independent Study, *Physics Department, The College of Wooster, Wooster, Ohio 44691, USA*, Jan 13, 2019

[2] James McElroy, "Phase Shifts due to Thermal Conductivity ", *Physics Department, The College of Wooster, Wooster, Ohio 44691, USA*, May 16, 2018.