Confirming Newton's and Stokes' model of Viscous Torque

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Stokes and Newton proposed different models for viscous drag on a body moving through a fluid. Stokes stated drag on an object flowing through a fluid is proportional to its velocity. Newton argued that drag on an object flowing through a fluid is proportional to the square of its velocity. We confirmed both models by investigating their rotational counterparts. By spinning a rotor ball and letting it come to a halt, measuring its frequency, and converting the frequencies to angular velocities, we were able to look at the relationship between drag and its angular velocity. On calculating the power via the slope of angular velocity versus time in a semi-log plot for motion without additional resistance in form of attached flags, we were able to get approximately 1.20, which is close to 1, suggesting it follows Stoke's model for laminar flow more closely than Newton's model. On adding additional resistance with flags onto the rotor ball, the power was calculated to be approximately 1.95, which is close to 2, thus suggesting that it follows Newton's model for turbulent flow more closely than Stokes' model. The amount of turbulence is not always constant throughout rotor's motion and thus experimental values of power varies from theoretical values. The flow of the body can also be described as a combination of the two models, which is also possible when it is transitioning from one type of flow to the other.

I. INTRODUCTION

If you move your hand through air, you can feel some resistive force acting on your hand in the opposite direction. The faster you move your hand, the harder it is to pass through the air. Now, if you tilt your hand sideways, you will feel less resistance, because of the decreased surface area that the air interacts with. Now if we sift our hand in a bucket of water , the resistive force we feel is even higher. This phenomenon was formally summarized by Sir George Gabriel Stokes and Sir Isaac Newton for all objects travelling in different fluids.

Whenever an object moves through a fluid, there is a resistive drag force acted upon it by the fluid. Sir George Gabriel Stokes said that the viscous drag force of a sphere travelling or rotating in a fluid is directly proportional to the velocity of the object, which is the case when the flow of the object is laminar [1]. Laminar flow of an object occurs when there is no lateral movement of the fluid while the object is flowing through the fluid. Laminar flow usually occurs when the velocity of the object is relatively low [2].

However, Sir Isaac Newton had said that the drag force is directly proportional to the square of the velocity of the object, which best describes turbulent flow [1]. A motion or flow can be called turbulent if the fluid is moving irregularly laterally. Turbulent flow is usually tied to a relatively higher velocity of the object [3].

We conduct this experiment to confirm these models by using a sphere and setting it in rotational motion. This experiment investigates the relationship between frictional torque and angular velocity to see which model fits the best for different angular velocities. This experiment also investigates adding extra air resistance in the form of flags on the sphere. Most of the times, it is a combination of laminar and turbulent flow, and thus, we try to find the power of velocity of the object, which may be between 1 and 2. A power closer to 1 tells us that it is mainly laminar and power closer to two tells us that it is mainly turbulent. This could be because the speed may vary at different times in the run, which affects the type of flow.

II. THEORY

Flow of every object moving through any fluid depends on multiple factors such as the object's surface area interacting with the fluid, speed of the object travelling through the fluid, and density of the fluid itself.

Drag force increases with increase in the surface area, becuase larger the area in contact with the liquid, the object has more opposition while passing through the fluid. Secondly, drag increases with increasing speed, as we touched on it earlier in the introduction. A stationary object will not experience any drag. However, a slowly moving object will experience some drag and a fast moving object will experience an even higher drag. In real life, the speed of the object is not always constant throughout, and thus the drag also varies as a result. Lastly, drag also depends on the density of the fluid that the object is travelling through. Denser the fluid, particles are closer and the object requires more force to get through the fluid. n this experiment, we vary the surface area of our object and compare resultant drag forces.

The above relationships can be modeled into an equation using the following models, depending on the type of flow. All the above factors, except for the velocity of the object are combined together to give a constant depending on density of fluid, surface area of fluid, and other factors such as temperature of the fluid, which is not very significant to our experiment. Stokes' model states that for a laminar flow, that is at a lower velocities, the resistive drag force $\vec{F_d}$ that acts upon a moving object is directly proportional to the object's velocity $\vec{v}$ and can be written as

$$\vec{F_d} = -c_1\vec{v}. \tag{1}$$

Newton's model states that for a turbulent flow, the resistive drag force acting upon an object moving in that fluid is directly proportional to the square of the object's velocity. Specifically

$$\vec{F_d} = -c_2 v^2 \hat{v}. \tag{2}$$

In most cases, the type of flow is a combination of laminar and turbulent flow and thus, on combining these two models, we get an equation,

$$\vec{F_d} = -c_n v^n \hat{v}. \tag{3}$$

Here c_1, c_2 and c_n are constants proportional to the radius of the object and the fluid that it is passing through.
Experimentally, n is usually calculated between 1 and 2, as a combination of two types of flows. For a purely laminar flow, n would be 1 and for a purely turbulent flow, n would be 2.

For the purpose of our experiment, we can convert these to rotational kinematics in order to study torque of a smooth rotating sphere using the equation,

$$\tau = I\alpha = I\frac{d\omega}{dt} \tag{4}$$

where τ is the rotational torque or rotational force causing the body to rotate around its axis. The moment of inertia I is the body's tendency to resist angular acceleration [4].It is proportional to the object's mass. Angular acceleration α can also be written as derivative of angular acceleration with respect to time. On solving for dω/dt, we get

$$\frac{d\omega}{dt} = \frac{-C_n}{I}\omega^n. \tag{5}$$

Eqn. (5) can also be written as

$$\frac{d\omega}{dt} = -k\omega^n, \tag{6}$$

where $k = C_n/I$,
If n=1, we can solve differential Eqn. (6), and get

$$\omega = \omega_o e^{-kt}. \tag{7}$$

This means that if any data represents Stokes' model of laminar flow, then on plotting a semi-log plot of ω versus time, the linear fit will be well fit. The slopes of this semi-log plot gives the proportionality constant k. If n=2, we can solve differential Eqn. (6) and get

$$\frac{-1}{\omega} = kt + \frac{1}{\omega_o} \tag{8}$$

On plotting $1/\omega$ verus time and fitting it, if a straight line fits the data well, it represents Newton's model for turbulent flow where drag is proportional to the square of body's velocity.

III. PROCEDURE

In order to experimentally confirm the two models and determine the type of flow, we spin a rotor ball suspended with a supply of nitrogen from under the ball making it float without any resistance. As it can be seen in Fig. 1, the shiny rotor ball is covered with vertical strips of black tape. This is done to cut off laser supply periodically and measure the frequency. Above the rotor ball, there is a stabilizer to prevent the ball from processing when rotating. This was built by Adam Deeley at the College of Wooster as a part of his Junior Independent Study. At the focal point of the laser, there is a photodiode to collect the light. A laser is placed across the shiny rotor ball, it reflects off of the ball and goes into the photodiode. Between the ball and the photodiode, there is a converging lens through which the light passes. The photodiode is connected to a Schmitt trigger, which either gives high or low values of voltages. The trigger is connected to an oscilloscope which displays the voltage readings, which is connected to the LabView computer program. This program records frequency at an interval of every 10 seconds.

To align and focus the laser, I observed its pattern by placing a plain notecard in its path before and after the converging lens. The rough sphere pattern it formed became smaller and focused after passing through the lens. To get most of the light in, I placed the photodiode at the focal length of the laser. After this, the voltage supply created by the laser was checked. Since Schmitt trigget only gives clean square signals, I connected the circuit such that it would give continuous reading of voltage. Once enough voltage was seen on the oscilloscope, the circuit was connected such that the Schmitt trigger was included in it again.

The rotor is given an initial spin and data are recorded until it comes to a rest. Once the rotor starts rotating, the laser continuously shines onto the photodiode, except when passing through the black strips and records the frequency. The frequency is recorded at an interval of every 10 seconds. I took multiple datasets in this configuration.

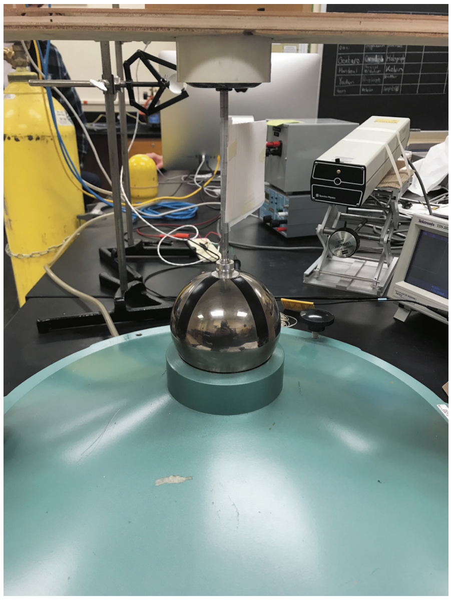

Figure 1: Ealing Air Gyroscope with black strips, compressed nitrogen supply from under, and a laser beam across. On top of it, is a stabilizer to prevent the Gyroscope from processing while runs are being taken.

Table I: Constant of proportionality (k) values for three datasets on calculating slopes for equations for laminar and turbulent flow

Data set	slope = k
no flag run 1(n=1)	$(19.18 \pm 0.2) \times 10^{-4}$
no flag run 2(n=1)	$(18.06 \pm 0.2) \times 10^{-4}$
flag run 1(n=2)	$(12.48 \pm 0.4) \times 10^{-4}$

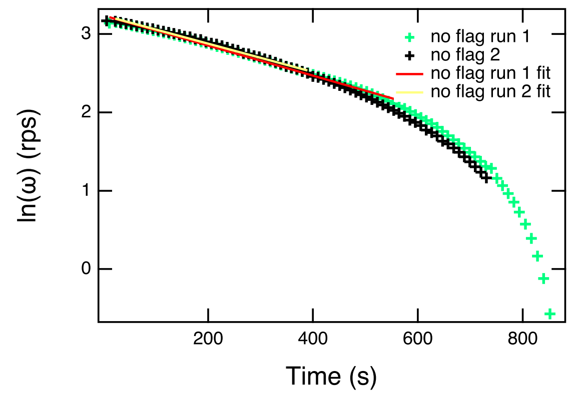

Figure 2: A semi-log plot of ω versus time with a linear fit for no flag run 1 and 2. The slope of 1 and 2 give k.

To increase the surface area and resistance, a 2.5 $\times$ 3 inch notecard was attached to the rod above the sphere. More data is collected in the same manner as previously.

IV. DATA AND RESULTS

We measured the angular velocity of the rotor ball by setting it in motion, multiple runs were taken in all orientation and selected the best ones. First, we recorded data by putting the rotor ball in motion and recording its angular velocity. A flag was then attached on top of the rotor ball and its angular velocity was recorded again. The data obtained for a rotating sphere without any flags attached was plotted as a semi-log plot of ω versus time. It was then fitted using a linear fit. From Fig. 2, we are able to see that the no flag-run 1 and 2 have a good linear result and thus is modeled well by Eqn. (7), which states that if n=1, angular frequnecy of the object will be linearly proportional to constant k. The values of the slopes of all three datasets are listed in Table I. We can see that for n=1, k is smaller, because there is no extra surface area of the notecard and the air around it remains comparatively stationary. For n=2, on solving Eqn. (8) for k, we get a higher value of k. This is because the flow becomes turbulent due to the added notecard. By these values of k, we are able to differentiate between laminar and turbulent flow.

To confirm Newton's model for turbulent flow, which can be done using Eqn. (8), we plotted a linear plot of $1/\omega$ verus time for flag run 1. It is the data of the rotating sphere with a flag attached above the rotor ball, as it can be seen in Fig. 1 causing the turbulence. A linear result for this graph would tell us if the data is modeled well by Eqn. (8) and follows Newton's model. On plotting $1/\omega$ verus time for flag run 1 and doing a linear fit on the first half of the points, we can see in Fig. 3, there is a well linear fit for $\approx$ 200 seconds. This means that Newton's model of turbulent flow fits well and it is in turbulent motion for approximately the first half. At lower velocities, there could be multiple factors affecting the rotating sphere, causing the speed of ball to decrease.

To further confirm the results of both Newton's and Stokes' models, we plotted Eqn. (6), which states that on differentiating the angular velocity, the slope will gave us the power n, and its intercept gave us the constant of proportionality k. A slope of 1.95 ± 0.1 in Fig. 4 tells us that it does follow Newton's model, because n is approximately equal to 2.

Unfortunately, we were not able to confirm the results by this method for no flag run 2, becuase its velocity was changing very rapidly with time. It is unusual for this pattern to occur, but I am unsure of why this could happen. In Fig. 5, the power law fit of log-log plot of dω /dt versus ω gives us the slope n, which is 1.210 ± 2.5. This data did not have any flag attached to it and thus it represents Stoke's model of laminar flow where n=1.

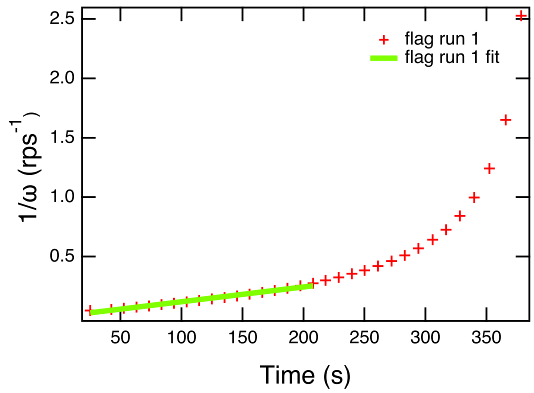

Figure 3: A linear plot $1/\omega$ versus time for flag run 1 with a linear fit .

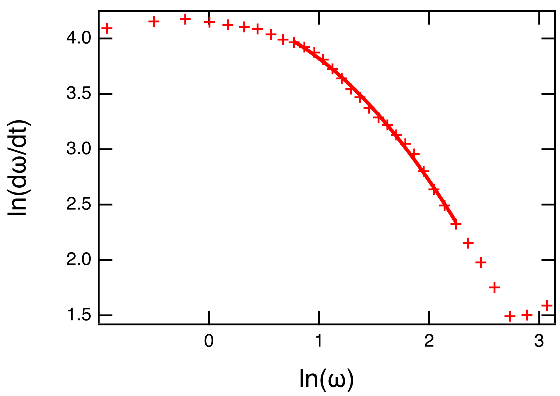

Figure 4: A log-log plot of dω /dt versus ω for flag run 1 with a power-law fit. The slope gives n. Slope calculated for this flag run 1 $= 1.95 \pm 0.10$

V. CONCLUSION

This experiment allowed us to learn two different models - Stokes' and Newton's for laminar and turbulent motion of fluids. We were able to study the rotational analogs of these models. After looking at the data and plotting equations of both models, we were able to see that if the flow is laminar, the drag force on the body moving through the fluid is proportional to its velocity. This was suggested by solving Eqn. (6) assuming n=1, for laminar flow. This was done by doing a linear fit on two datasets without any flag on it, it was well fit. If the fluid flow is turbulent, the drag force on the body moving through the fluid is proportional to the square of its velocity. This was seen in a similar manner to the first one, by solving Eqn. (6) assuming n=2. This was done by fitting a linear fit on the third dataset with the flag attached to rotor causing turbulence and increasing the surface area of the object.

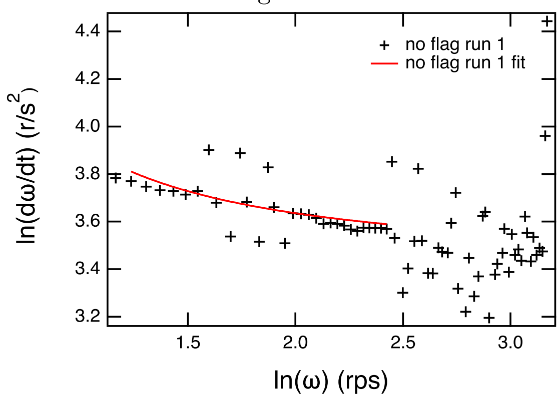

Figure 5: A log-log plot of dω /dt versus ω for no flag run 1 with a power-law fit. The slope gives n. Slope calculated for this no flag run 1 $= 1.210 \pm 2.5$

We were also able to calculate n for first dataset with flags (flag run 1) which was ≈ 1.95. This means that in this case, the drag on the object moving through the fluid is equal to the square of its velocity in case of a turbulent flow, which was caused by the flag, suggesting that it follows Newton's model closely. On plotting second dataset without the flag (no flag run 1), n was approximately 1.20, which is close to 1. This means that in this case, the drag on the object moving in this fluid is equal to its velocity due to a laminar flow, suggesting that it follows Stokes' model. Using Eqn. (7) and (8), we were able to calculate k, the constant of proportionality, which is dependent on the surface area of the object, and the fluid it is passing through. From Table I, we can see that for n=1, k is smaller, because there is no extra surface area of the notecard, and for n=2, it increases, as discussed earlier. Therefore, we were able to observe the effects of both Stokes' and Newton's models of viscous drag for translational mechanics to a certain extent.

[1] Department of Physics, The College of Wooster, Wooster, OH. *Junior Independent Study Lab Manual*, 2020.

[2] Laminar flow fluid review - engineer's edge. `https://www.engineersedge.com/fluid_flow/laminar_flow.htm`.

[3] What is turbulent flow? - simscale. `https://www.simscale.com/docs/content/simwiki/cfd/what-is-turbulent-flow.html`.

[4] Manon E. Grugel. Modeling the viscous torque acting on a rotating object, College of Wooster, 1998.