The Stefan-Boltzmann Law

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A tungsten filament was used as an approximate blackbody to verify the Stefan-Boltzmann Law. By changing the voltage and the current of the tungsten filament, we were able to measure its resistance which varied at different temperatures and power inputs in the electric current. The resistance of the tungsten filament at room temperature was measured to be $0.370 \pm 0.004\ \Omega$. We applied two analysis methods to confirm that the power radiated by a body in thermal equilibrium is proportional to the fourth power of its temperature. The first method included plotting power versus temperature to fit four parameters and the fifth-order polynomial function while keeping the second- and third-order values at zero to compare to their parameters, $\epsilon\sigma A$ and κl. The results are $(1.03 \pm 0.01) \times 10^{-12}$ W/K^4 and $(0.9 \pm 0.3) \times 10^{-3}$ W/K. The second method included using a log-log plot of the input power versus temperature. The slope of these two factors is 3.368 ± 0.05 compared to 4.

INTRODUCTION

Imagine there is a cup of hot coffee on a table. Wait a while, and the coffee will cool down and the table's temperature will increase. This is because the net flow of the heat energy between the table and the coffee is nonzero. When their temperatures become the same, which means that there is no net flow of thermal energy between them, we say they reach the thermal equilibrium. However, if we imagine that the coffee is hovering over the table without touching it, meaning that the coffee cup and the table don't conduct heat, they will still reach the same temperatures after enough time. This occurs because both the table and the coffee emit and absorb energy via electromagnetic radiations. If an object is able to absorb or emit all incident electromagnetic radiation regardless of the frequency, we call it a black body. For example, a black hole can be regarded as a black body. If light gets close enough to the black hole, all light with different wavelengths will be absorbed with no reflection making it almost an ideal black body. A black body is an idealized physical model, and at thermal equilibrium, it emits electromagnetic radiation [1].

In 1879, Josef Stefan through experimentation discovered that the power of a black body is proportional to the fourth power of the temperature in thermal equilibrium [2] and Ludwig Boltzmann theoretically derived this law. Therefore, this law is called Stefan-Boltzmann's Law.

In our experiment, we used a simple tungsten filament and confirmed the relationship between the power and the temperature of a black body at thermal equilibrium. The tungsten filament is a high temperature source of thermal radiation which can be considered an idealized black body [3]. As we change the voltage and current, the tungsten filament's temperature will change. By analyzing the voltage and current, we can confirm Stefan-Boltzmann's Law.

THEORY

Stefan-Boltzmann Law

As mentioned before, a black body is able to emit or absorb electromagnetic radiation in various ranges. The Planck Law gives a function of the energy density,

$$u_\lambda(\Omega) = \frac{2h}{c^3}\frac{\lambda^3}{e^{h\lambda/(kT)} - 1}, \quad (1)$$

where u_λ is the energy density, h is the Planck constant, c is the speed of light, k is the Boltzmann constant, and T is absolute temperature [4].

Thus we are able to find the total energy density, U, by integrating the energy density over all wavelengths.

$$U = \int_0^\infty \frac{2h}{c^3}\frac{\lambda^3}{e^{h\lambda/(kT)} - 1} d\lambda, \quad (2)$$

$$U = \frac{2\pi^5 k^4}{15h^3c^2}T^4. \quad (3)$$

Josef Stefan found this result that, for a black body, the radiation per unit area is proportional to the fourth power of the temperature:

$$U = \sigma T^4, \quad (4)$$

where σ is called the Stefan-Boltzmann constant equivalent to 5.67×10^8 W/m^{2}K^4.

However, the radiation per unit area depends on the surface materials such as its color and composition. So we need to introduce a new factor ϵ, the emissivity factor. Then Eq. 4 becomes

$$U = \frac{P}{A} = \epsilon\sigma T^4. \quad (5)$$

For an ideal black body, the emissivity factor ϵ is 1. So we have

$$U = \frac{P}{A} = \sigma T^4, \quad (6)$$

$$P = \sigma A T^4, \tag{7}$$

where P is the power radiated by a black body in thermal equilibrium.

In our experiment, a tungsten filament which was connected in a circuit was regarded approximately as an ideal black body. At thermal equilibrium, the input power equals the output power. When an electrical current which is driven by voltage goes through the tungsten filament, the filament gains power from the circuit, it also absorbs radiation from the environment (temperature of the room). Power into the tungsten filament includes the power from the electrical current running through the resistor which changed via the temperature (P_{elec}), and the radiation that is released from the room and absorbed by the tungsten filament (P_{absorb}). The tungsten filament also radiated away some power and there is a small amount lost on the conduction. Power going out of the tungsten filament includes the power that is radiated away to the room ($P_{radiated}$) and lost by the conduction ($P_{conducted}$),

$$P_{in} = P_{put}, \tag{8}$$

$$P_{elec} + P_{absorbed} = P_{radiated} + P_{conducted}. \tag{9}$$

P_{elec} can calculated by multiplying the current of the circuit by the potential difference (voltage) across the tungsten filament. $P_{absorbed}$ and $P_{radiated}$ can be calculated by using Eq. 7 adding the emissivity factor ϵ which does not equal 1.

$$\begin{aligned} P_{radiated} &= \epsilon\sigma A {T_f}^4, \\ P_{absorbed} &= \epsilon\sigma A {T_0}^4, \end{aligned} \tag{10}$$

where T_f is the temperature of the tungsten filament and T_0 is the room temperature.

The power that is lost by the conduction can be expressed as

$$P_{conducted} = \kappa l (T_f - T_0), \tag{11}$$

where κ is the conductivity and l is a characteristic length scale [3]

Inserting Eq. 10 and Eq. 11 into Eq. 9 and slightly rearranging them, P_{elec} can be solved as

$$P_{elec} = VI = \epsilon\sigma A {T_f}^4 - \epsilon\sigma A {T_0}^4 + \kappa l (T_f - T_0). \tag{12}$$

In our experiment, the room temperature T_0 and surface area A are constant, so the power of the filament absorbed from the environment $P_{absorbed}$, T_0 term is a constant. The power that is lost in the conduction $P_{conducted}$, $T_f - T_0$ term is negligible. Rearranging Eq. 12, we can find the relationship

$$P_{elec} \propto {T_f}^4. \tag{13}$$

Taking the natural logarithm of both sides, we have

$$\ln(P_{elec}) \propto 4\ln(T_f). \tag{14}$$

Power and Resistance

By measuring the current in the circuit and the voltage across the filament, we are able to calculate the applied power P_{elec} and filament's resistance. The applied power can be calculated by

$$P_{elec} = VI, \tag{15}$$

and the resistance can be calculated by

$$R = \frac{V}{I}. \tag{16}$$

However, the resistance of the filament relates to the resistivity ρ and the volume of the tungsten which can be expressed as

$$R = \rho\frac{L}{A}, \tag{17}$$

where L is the length of the filament, and A is the cross sectional area. Because of the resistivity changes via the temperature, the resistance of the filament also changes. In order to find the temperature, we need to find the ratio between the resistance at time t and the resistance at the room temperature.

$$\frac{R_t}{R_{temp}} = \frac{\rho(t)L/A}{\rho(0)L/A} = \frac{\rho(t)}{\rho(0)}, \tag{18}$$

where $\rho(0)$ is resistivity at room temperature, and $\rho(t)$ is resistivity at time t. By checking the table in the *Stefan-Boltzmann Lamp's Manual*, we are able to find the corresponding temperature [5].

PROCEDURE

In this experiment, a TD-8555 Stefan-Boltzmann Lamp was connected with an auto-ranging voltmeter and an ammeter, and a power supply as shown in Fig. 1. The lamp is a light bulb with a tungsten filament which has an upper limit of 13 V and 3 A.

First, we measured the resistance of the tungsten filament at room temperature by applying small current in the circuit. The current ranged from -1.95 mA to 2.537 mA, and the voltage ranged from -0.723 mV to 0.941mV. The value of voltages and current was read on the voltmeter and ammeter. The data were recorded and analyzed in Igor Pro to calculate the tungsten filament's resistance at room temperature.

Then, we increased the range of the voltage and the current (ranged from 1.704 V to 11.65 V and 1.068 A to 2.65 A) through the filament. The value of voltages and current was read on the voltmeter and ammeter and was recorded quickly. To minimize the systematic error, we read and recorded quickly to avoid the filament's temperature changing too much. After each recoding, we

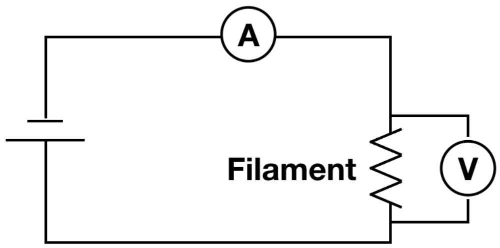

FIG. 1: A sketch of the circuit that was set up in this experiment. The resistor is a tungsten filament in a TD-8555 Stefan-Boltzmann Lamp. The ammeter is connected in the circuit, and the voltmeter is connected to measure the lamp's voltage.

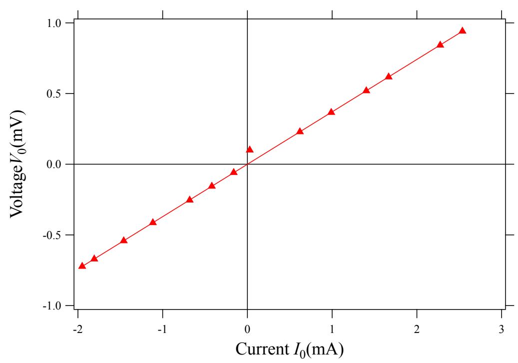

FIG. 2: Resistance of the filament at room temperature. We applied small current to minimize temperature's changing. The slope of the function is the resistance of the filament at room temperature that has value of $0.370 \pm 0.004\Omega$. The intercept was held at zero.

turned off the power supply for about 1 minute to let the lamp cool down, so a systematic error caused by the conduction would not have influence on this experiment.

RESULTS & ANALYSIS

Measuring Filament's Temperature

We measured the tungsten filament's resistance at room temperature ($R_{300\mathrm{K}}$) by plotting the voltage versus small current in the Igor Pro. Because the temperature change was really small, we can simply apply Eq. 16. Plotting the voltage V_0 versus current I_0, the slope is the resistance of the filament at room temperature as shown in Fig. 2. We hold the intercept as zero and the resistance has a value of $0.370 \pm 0.004\ \Omega$, this value did not change appreciably when we did not hold the intercept as zero.

The resistance of the filament at time t (R_T) with variable temperature was calculated using the same equation

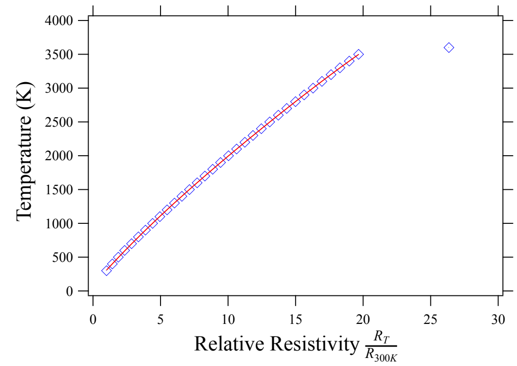

FIG. 3: The plot of temperature versus relative resistance was fitted to a polynomial function. The data came from the *Stefan-Boltzmann Lamp Manual* [5]. The coefficients were used to calculate the temperature of the filament in our experiment with data we recorded.

above, but in order to check the table to find the temperature, we calculated the ratio of $R_{300\mathrm{K}}$ and R_T. The calculated ratio could not perfectly fit the table because it only shows the ratio for temperature varied every 100 K. But our experimental data shows the ratio for temperature varied unevenly. So we entered the table data into Igor Pro to build a function to find the temperature in our experiment. We plot temperature versus resistance ratio to fit four parameters and fourth-order polynomial function as shown in Fig. 3. This function has the form

$$y = K_0 + K_1 x + K_2 x^2 + K_3 x^3 + K_4 x^4, \qquad (19)$$

and the result we had is

$$y = 85.618 + 228.62x - 5.71x^2 + 0.2398x^3 - 0.00465x^4, \qquad (20)$$

where y represented temperature and x represented resistance ratio.

Applying Eq. 20 into our experiment data, we found the temperature of the tungsten filament.

Stefan-Boltzmann Law

We used two methods to verify the Stefan-Boltzmann Law.

First, Eq. 12 was tested by plotting the power (P_{elect}) as a function of temperature. We let the function to fit four parameters and the fourth-order polynomial function while keeping the second- and third-order values at zero to compare to their parameters, $\epsilon\sigma A$ and κl. So the function has the form

$$y = K_0 + K_1 x + K_4 x^4. \qquad (21)$$

The result is

$$P_{elec} = -0.045 + (0.912 \times 10^{-3})T + (1.03 \times 10^{-12})T^4. \qquad (22)$$

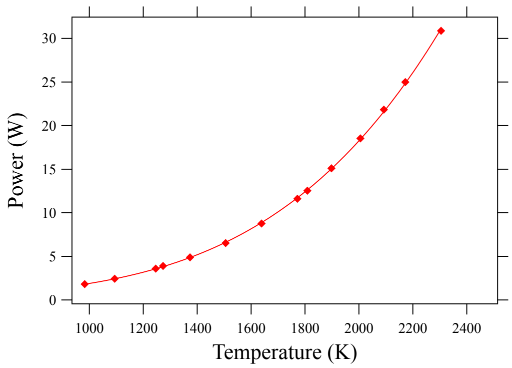

FIG. 4: The plot of power versus temperature was fitted to a four parameters and the fourth order polynomial function while holding the second- and third- order values to be zero. The coefficients represent the constants in Eq. 12.

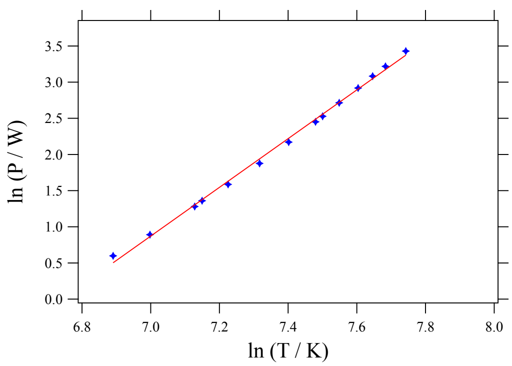

FIG. 5: The log-log plot of nature log of power versus nature log of temperature. This plot shows the linear relationship between these two factors. The slope is 3.368 ± 0.05 compared to 4, which confirms Eq. 14

The value of the coefficients was used to compare with the constants' values in Eq. 12, $\kappa l = (0.9 \pm 0.3) \times 10^{-3}$ W/K^4 and $\epsilon\sigma A = (1.03 \pm 0.01) \times 10^{-12}$ W/K as shown in Fig. 4

Then, Eq. 14 was tested by plotting the log-log plot of the power versus temperature as shown in Fig. 5. As shown in the equation, the natural logarithm of the power should be a linear function of the natural logrithm of temperature with slope of four. The result in our experiment shows that, these there is a linear relationship between these two factors and the slope is 3.368 ± 0.05 compared to 4 and the intercept point is -22.7 ± 0.3 within the accepted value.

CONCLUSION

In this experiment, we used a simple tungsten filament as an approximate idealized black body to verify the Stefan-Boltzmann law. The tungsten filament's resistance changes while the temperature changes. When we increase the current through the circuit, the filaments gain radiated power and the temperature changes. By analyzing the current and voltage of the lamp, we are able to calculate its relative resistance at different temperatures. In order to find the temperature, the data of temperature and relative resistance in the Stefan-Boltzmann Lamp manual was plotted and fit a polynomial function. When we fit the experiment data into this function, the temperature was known. We applied two analysis methods to confirm that the power radiated by a black body in thermal equilibrium is proportional to the fourth power of its temperature. The first method included plotting power versus temperature to fit four parameters and the fifth-order polynomial function while keeping the second- and third-order values at zero to compare to the constants $\epsilon\sigma A$ and κl. The results are $(1.03 \pm 0.01) \times 10^{-12}$ W/K^4 and $(0.912 \pm 0.318) \times 10^{-3}$ W/K. The second method included uses a log-log plot of the input power versus temperature. The slope of these two factors is 3.368 ± 0.05 compared to 4. This experiment confirmed the Stefan-Boltzmann Law: the power radiated by a black body is proportional to the fourth power of the temperature. It also confirms that the tungsten filament can be considered as an approximate idealized black body.

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