

Coulomb's Law and Leakage of Charges

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We used two charged spheres and a Coulomb balance to determine how the Coulomb force was dependent on the charge of two spheres and their separation. Leakage of charge was observed and measured during the experiment. During the experiment, we held either the charge or the distance between spheres constant while varying the other. The Coulomb force was measured and plotted into several graphs. It was shown that the force between the two charges was directly proportional to their charges and inversely proportional to the distance squared, which obeyed Coulomb's Law. Specifically, the force was measured to be inversely proportional to the distance to the power of 2.42 ± 0.03 , which was 21 % different from the expected value, and proportional to the charge to the power of 0.9 ± 0.1 , in which the predicted value was within our uncertainty. We also measured the leakage of charge and concluded that the charge decayed exponentially with a decay rate $\lambda = 0.018 \pm 0.001$ 1/s and mean lifetime $\tau = 55.7 \pm 1.8$ s.

I. INTRODUCTION

Long before any knowledge of electricity existed, people were aware of some observations on static electricity. Ancient cultures around the Mediterranean found that rods of amber could be rubbed with fur to attract light objects like feathers and papers. However, people did not understand the static electricity produced by rubbing amber until the 1600s. More work on electrostatics was conducted in the 17th and early 18th centuries. In the early 1770s, Henry Cavendish discovered the dependence of the force between charged bodies upon both distance and charge, but he did not publish this discovery[1].

Finally, in 1785, the French physicist Charles-Augustin de Coulomb published his first three reports of electricity and magnetism where he stated his law. This publication was essential to the development of the theory of electromagnetism. Coulomb used a torsion balance to study the repulsion and attraction forces of charged particles, and determined that the magnitude of the electric force between two point charges is directly proportional to the product of the charges, and inversely proportional to the square of the distance between them[2].

Because of the inverse relationship between the electrostatic force and the distance squared, the force produced by very small amounts of charges can be enormous. One gram of protons placed one meter away from one gram of electrons can result in a force large enough to lift an object whose mass is about one fifth of the moon. However, in our experiment, we used a delicate torsion balance to measure the force because the electrons are so mobile that it is hard to keep them separated. Since charges can also leak into the air, the force decreases with time exponentially. Hence, only a tiny part of the force can be sustained in our lab, which can be measured by our torsion balance.

In this lab, we used a Coulomb balance to investigate the force between two charged spheres. In the setup, a sphere was suspended from a thin torsion wire and an identical sphere was mounted on a slide assembly. We used a charging probe to charge the spheres and the charging

probe was connected to a power supply. By changing the output voltage of the power supply, we changed the charge on the spheres. The distance between charges was changed by moving the sliding sphere. The force between the charges were measured by the torsion balance. Using a similar experiment setup to Coulomb's, we examined the inverse square relationship and the charge dependence that were stated in Coulomb's Law. Since the leakage was not negligible during a typical time of our measurements, we also measured the leakage of charges.

II. THEORY

In order to investigate the force between two charged spheres, we charged the spheres and brought them close. Since one of the spheres was suspended from a thin torsion wire, the electrostatic force between spheres would cause the wire to twist. A torsion knob allowed us to twist the wire back to its equilibrium position. Therefore, with the degree scale around the knob we could measure the electrostatic force by measuring the angle θ through which the torsion wire was twisted. It was directly proportional to the force such that

$$F = K\theta, \quad (1)$$

where K was the torsion constant.

The separation between the center of the two spheres was r and the charges on spheres were q_1 and q_2 . According to Coulomb's Law, the force between them was directly proportional to the product of charges and inversely proportional to the distance squared, which was represented as

$$F = k \frac{q_1 q_2}{r^2}, \quad (2)$$

where F was the force and k was the Coulomb's constant.

A. Force vs. Distance

In order to verify the inverse square relationship between force and distance, we changed Eq. (2) to a linear equation by taking natural logarithm on both sides. It followed that

$$\ln F = -2 \ln r + \ln(kq_1q_2). \quad (3)$$

Since we held the charge constant for this part of experiment, the product of charges q_1q_2 should be constant and therefore the term $\ln(kq_1q_2)$ was a constant. With Eq. (1), Eq. (3) could be rewritten as

$$\ln(K\theta) = -2 \ln r + \ln(kq_1q_2) \quad (4)$$

$$\ln \theta = -2 \ln r + \ln(kq_1q_2) - \ln K, \quad (5)$$

where the last two terms were constants.

However, we could not simply treat the spheres as point charge since the radius was not small enough compared to their separation and the charges would redistribute themselves on the spheres to minimize the electrostatic energy. Therefore we applied a correction term B to correct the deviation. B is calculated by

$$B = 1 - 4 \frac{a^3}{r^3}, \quad (6)$$

where a was the radius of the sphere. We divided θ by B and labeled the corrected value of θ to be θ_c . Therefore, we now had

$$\ln \theta_c = -2 \ln r + (\ln(kq_1q_2) - \ln K), \quad (7)$$

Based on this equation, we expected a linear relationship between $\ln \theta_c$ and $\ln r$.

B. Force vs. charge

In order to verify the dependence of electrostatic force on charge, we started by rewriting Eq. (3), which gave

$$\ln F = -2 \ln r + \ln k + \ln q_1 + \ln q_2. \quad (8)$$

In this part, we held the distance r and charge on one sphere q_2 constant. We substituted F with Eq. (1) to get

$$\ln \theta = \ln q_1 + (\ln q_2 - 2 \ln r + \ln k). \quad (9)$$

Hence we also expected a linear relationship between $\ln \theta$ and $\ln q_1$.

C. Charge Leakage

In the experiment, we expected charge from the spheres to gradually dissipate into the air. According to Coulomb's Law, the electrostatic force would decrease as the charge decreased. Since θ is directly proportional to force, it should also decrease with time as charges leaked out. Seaver [3] has proposed a theory in which we expected the charge to decay exponentially with time.

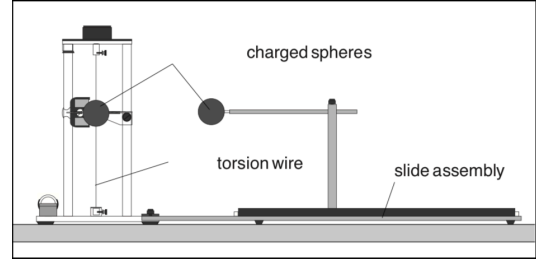


FIG. 1: Schematic of torsion balance. A conductive sphere is mounted on a rod, counterbalanced and suspended from a thin torsion wire. An identical sphere is mounted on a slide assembly so it can be positioned at various distances from the suspended sphere. The wire can be twisted by using a knob, and the displacement is indicated by the index marks on the pendulum. This schematic is taken from PASCO Instruction Manual [4].

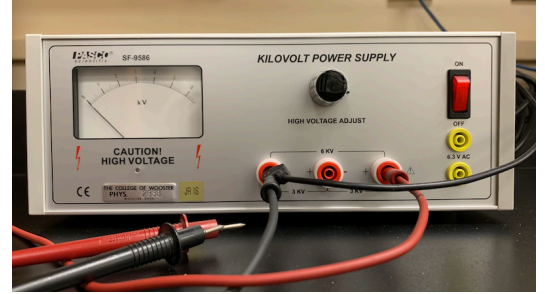


FIG. 2: Image of power supply. The black probe is grounded and the red probe is the charging probe. The voltage can be adjusted from 0 kV to 6 kV but the voltage is not reliable under three kilovolt.

III. PROCEDURE

The experimental setup is illustrated in Fig. 1. A conductive sphere was mounted on a rod, counterbalanced and suspended from a thin torsion wire. An identical sphere was mounted on a slide assembly so it could be positioned at various distances from the suspended sphere. We also had an adjustable kilovolt power supply as shown in Fig. 2 so that the spheres could be charged to different potentials.

A torsion dial was placed around the torsion knob in order to read the angle of the torsion. When the spheres were fully discharged, I set the torsion dial to 0 and rotated the torsion wire so that the pendulum assembly was at the zero displacement position as shown in Fig. 3.

A. Force vs. Distance

The voltage was fixed to 6 kV and both spheres were charged at their maximum separation. Immediately after charging the spheres, I positioned the sliding sphere to 20 cm. Then, the torsion knob was adjusted to balance

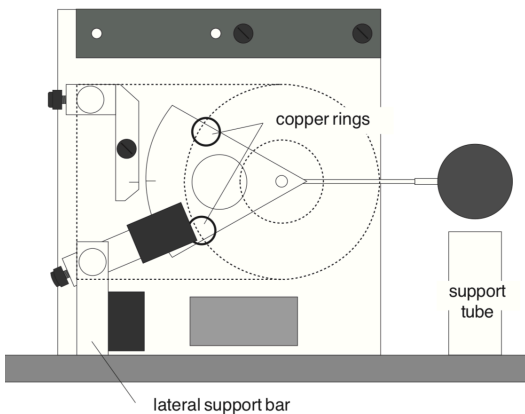


FIG. 3: Schematic of the pendulum and the suspended sphere. When the pendulum is at zero displacement (the equilibrium position), the marks on the pendulum align with marks on the tube. This schematic is taken from PASCO Instruction Manual [4].

the force until the pendulum was back to its equilibrium position. I recorded the distance r and the angle θ on the dial.

After the spheres were recharged to 6 kV, I repeated the process for different distances of 14 cm, 10 cm, 9 cm, 8 cm, 7 cm, and 6 cm. The angles were recorded and were corrected by the correction factor.

B. Force vs. Charge

In the second part, the separation between spheres was held at a value of 10 cm. With the spheres at maximum separation, I charged both spheres to 6 kV then moved the sliding sphere to 10 cm. After balancing the repulsion with torsion, the angle θ was recorded. I repeated the process with the sliding spheres charged to 5 kV, 4 kV, and 3 kV while the charge of the suspended sphere was fixed to 6 kV. The angle was measured for each voltage.

C. Charge Leakage

I observed charge leakage while performing the last two parts of the experiment, which motivated me to measure the decaying of the charges. In order to measure the dissipation of the charges, both spheres were charged to 6 kV when they were at their maximum separation. The sliding sphere was positioned at 10 cm, and the torsion wire was twisted 56° for the pendulum to move back to its equilibrium position.

I started a stop watch when the pendulum was back to its equilibrium position. Then, I adjusted the knob to decrease the angle by 10° , which caused the pendulum to move away from its equilibrium position, since the balance force was decreased. Fig. 4 showed how the pen-

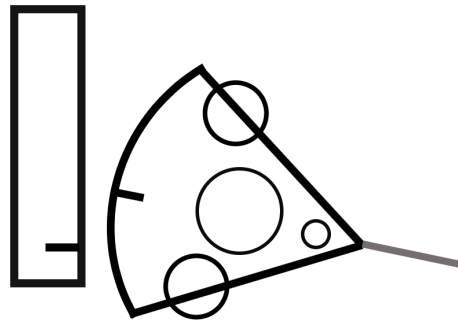


FIG. 4: Schematic of the pendulum after decreasing of the angle. The pendulum moves away from the equilibrium position due to the decreasing of torsion force. However, as charges leak into the air, the electrostatic force decreases, and the pendulum keeps moving towards the equilibrium position. The equilibrium is reestablished when the torsion force balances the electrostatic force again.

dulum moved away from its zero displacement after the angle was decreased. As time progressed, the charges leaked into the air, which resulted in the decrease of the electrostatic force between the spheres, as established in part C of the Theory section. Therefore, the pendulum kept moving towards its equilibrium position. The equilibrium was reestablished when the torsion force balanced the electrostatic force again. I recorded the time taken for the equilibrium to be reestablished and I adjusted the knob again, so that the angle was 10° smaller. After a few seconds, the pendulum was back to its zero position again and the time it took was recorded. The same processes were repeated until the angle was smaller than 10° . Everytime the angle was decreased, the time taken for the pendulum to move back to its equilibrium position was recorded.

IV. RESULTS AND ANALYSIS

The angle of the torsion was measured with holding either the separation or the voltage constant while varying the other, as discussed in previous section. To analyze the data, the correction term B was applied to the angle of the torsion θ . The natural log of the corrected angle $\ln \theta_c$ was plotted with respect to the natural log of the separation $\ln r$. Fig. 5 was the plot obtained. According to Eq. (7), we expected a linear relationship between $\ln \theta_c$ and $\ln r$ with a slope of -2 . In Fig. 5, the plot was well fitted by a linear function. The slope of the fitted line was -2.42 ± 0.03 , which was 21 % different from our predicted slope. The uncertainty of $\ln \theta$ became bigger as the angle got smaller.

We also expected a linear relationship between $\ln \theta$ and $\ln V$ with a slope of 1 from Eq. (9). In Fig. 6, $\ln \theta$ was plotted versus $\ln V$. The linear relationship was clearly shown through the graph and the slope was 0.9 ± 0.1 ,

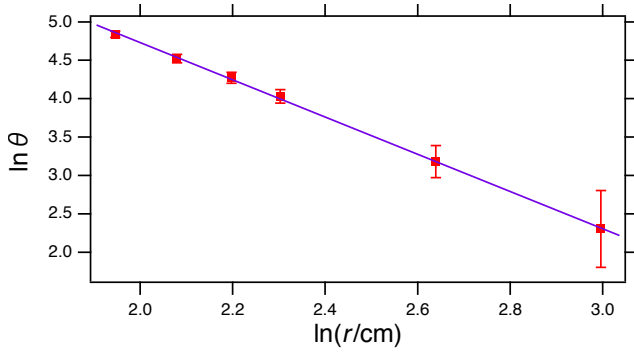


FIG. 5: The logarithm of the torsion angle $\ln \theta_c$ plotted versus the logarithm of the distance between two spheres $\ln r$. Then plot is fitted by a linear function with a slope of -2.42 ± 0.03 .

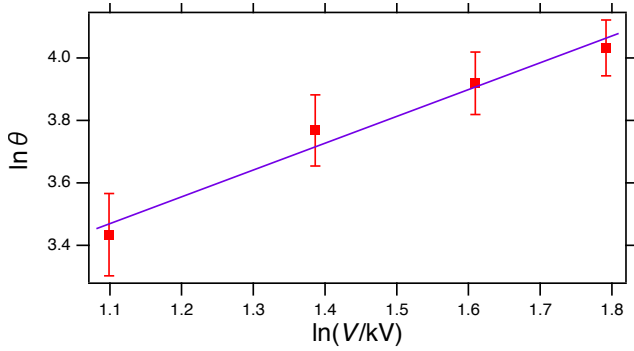


FIG. 6: The logarithm of the torsion angle $\ln \theta$ plotted versus the logarithm of the voltage $\ln V$. The plot is fitted by a linear function with a slope of 0.9 ± 0.1 .

which was 10 % different from the predicted slope.

As far as uncertainties were concerned, there were two uncertainties for angle measurements, one for position measurements, and one for voltage measurements. The position measurements were based on the limits of the centimeter, making the uncertainty simply ± 0.5 cm. The error of voltage measurement was due to the fluctuation of voltage, which was hard to quantify. I assumed the uncertainty in voltage measurement was small and the most significant uncertainty came from angle measurements. However, the uncertainty for angle measurement was hard to determine. The angle measurements were based on the limits of the torsion dial, making the minimum uncertainty $\pm 0.5^\circ$. Since the charge would gradually leak into the air, the uncertainty increased because of the leakage.

In order to quantify the leakage of the charge, we measured the time taken for pendulum to move back to its zero displacement every time the torsion force was decreased. The angle was plotted versus the time and Fig. 7 was the plot obtained. Since we expected the charge to decay exponentially with time based on Seaver's theory, an exponential function was used to fit the graph. As shown in Fig. 7, the plot was very well fit by the exponential function with a decay rate $\lambda = 0.018 \pm 0.001$ 1/s

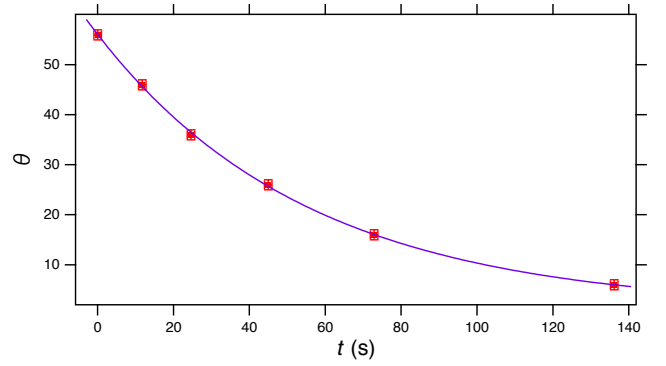


FIG. 7: The torsion angle θ plotted as a function of time t . The plot is fitted by an exponential function with a decay rate $\lambda = 0.018$ 1/s and a mean lifetime $\tau = 55.7$ s.

and the mean lifetime $\tau = 1/\lambda = 55.7 \pm 1.8$ s. The time taken for me to charge the suspended sphere, position the sliding sphere, and adjust the knob to balance the electrostatic force was about five to ten seconds. Hence, the leakage was estimated to be 10 % to 15 % of the charges. Based on this measurement, we estimated the uncertainty for θ to be $\pm 5^\circ$. Therefore, the uncertainty for $\ln \theta$ was calculated with the following equation:

$$\delta_{\ln \theta} = \frac{d \ln \theta}{d \theta} \delta_{\theta} \quad (10)$$

$$\delta_{\ln \theta} = \frac{1}{\theta} \delta_{\theta} \quad (11)$$

$$\delta_{\ln \theta} = \frac{5}{\theta} \quad (12)$$

Hence, based on this equation, the uncertainties for $\ln \theta$ were calculated using θ measured during experiment and were added to the plots as Fig. 5 and Fig. 6 showed.

V. CONCLUSION

The Coulomb's Law was successfully verified for our experiment. As shown in Fig. 5 and Fig. 6, the force between the two charges was directly proportional to their charges and inversely proportional to the distance squared. The slope of the linear fit we got for $\ln \theta_c$ plotted versus $\ln r$ was -2.42 ± 0.03 , which was 21 % different from the predicted slope, and the slope of the linear fit for $\ln \theta_c$ plotted versus $\ln V$ was 0.9 ± 0.1 , in which the predicted value was within our uncertainty. We measured the decay of charge and concluded that the charge decayed exponentially with a decay rate of 0.018 ± 0.001 1/s and mean lifetime of 55.7 ± 1.8 s. Hence, we estimated that about 85 % to 90 % of the charge was sustained when we measured the angle. Therefore, the leakage of the sphere was estimated to be 10 % to 15 %, which was not negligible. The leakage of the charge could explain the discrepancy between our experimental results and the expected results. The leakage could be affected by the

humidity in the room and the fabric we wore during the experiment. Thus, the experiment could be improved by performing it when the humidity was lowest during the year, or performing it in a draft-free room.

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VI. ACKNOWLEDGMENTS

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