Falling into Chaos

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Falling objects follow a path determined by the outside forces acting upon them. Some trajectories are predictable and are consistent if the initial conditions remain unchanged, but in some cases these trajectories can evolve into chaos. This experiment changes the material and initial angle that the object is dropped at and records the trajectory of its descent. It was found that lighter materials are more susceptible to outside forces, mainly air resistance, because they have less of a gravitational force to counteract collisions with air molecules. The initial angle causes trajectories to have more variation and increased separation over time.

I. INTRODUCTION

Gravity is the force that makes objects fall. In a vacuum, objects will fall straight down unless an outside force acts upon them. In the vacuum case, the trajectory follows a specific pattern where, no matter the initial conditions, the object should fall straight down from the point at which they are released. When not in a vacuum, the outside force affecting an object depends on the material that the object is passing through. The molecules that make up the material interact with the object and exert a force, changing its trajectory. For this experiment, the major force affecting the objects is air resistance, which may seem like a minor addition, but this addition has the ability to send a simple system into chaos depending on the initial conditions. The small molecules in the air exert a changing force on the falling object determined by the surface area. The variability in trajectories leads to the definition of chaos, where the slightest change in initial conditions can have a dramatic effect on the total outcome of a situation.

The purpose of this experiment is to test different initial conditions and see how they affect the trajectories. The conditions that will be focused on are the initial angle that the object is dropped at and the material that will be dropped. The object will be held in place at desired angle using a vacuum pump and the trajectory will be recorded as the object is released.

II. THEORY

As objects fall in a material the outside forces alter the direction of the object that is falling. The more it rotates during the descent, the more the outside forces affecting it change. The surface area that is pointed in the direction of motion changes as the object rotates. The larger the surface area directed down causes more collisions with air molecules that in turn slow the object as it falls. If the force across the surface is unbalanced, the object will be begin to rotate. The rotation would change the surface area and directly affect the force, slowing the object and causing a change in trajectory. This should cause different materials dropping at different angles to have varying trajectories. The different materials each have their own thickness, weight, and density; all of which will affect the motion during the fall. Heavier materials will generate a strong gravitational force and in turn be less affected by the outside force of the air molecule collisions. The lighter materials will have the reverse effect and be greatly affected by the air and have more variation between each trajectory.

A. Reynolds Number

A main component that is measured for a falling object is its Reynolds number. The Reynolds numbers is a ratio between inertial forces and viscous forces. The Reynolds number is commonly denoted as R and is defined by

$$R = \frac{\rho\ u\ L}{\mu} = \frac{u\ L}{\nu},$$

where ρ is the density of the fluid the object is passing through, u is the velocity of the fluid with respect to the object, L is related to the size/shape of the object, μ is the dynamic viscosity of the fluid, and ν is the kinematic viscosity of the fluid. The Reynolds number is used to give a numerical representation of the drag forces acting on an object. A low Reynolds number corresponds to laminar flow, meaning there is smooth, constant fluid motion. A high Reynolds number corresponds to turbulent flow, which is categorized by chaotic flow instabilities. The Reynolds number helps explain the observed motion of the objects in this experiment since as the object rotates due to outside forces, its surface area changes causing its velocity to change. This directly affects the Reynolds number and can lead to a change in its value that could cause a simple smooth flow to become chaotic.

III. PROCEDURE

For this experiment, sheets of different materials were dropped at different initial angles and the trajectories were recorded using a camera. An important aspect of this experiment was properly dropping the sheets. Certain drop methods, such as using two fingers to hold the

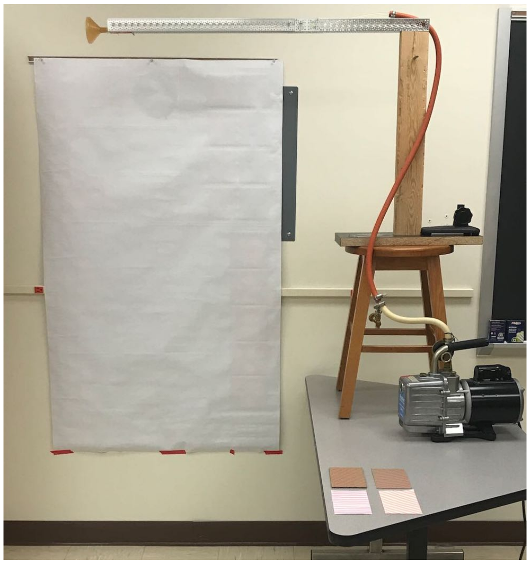

FIG. 1: Vacuum pump apparatus for dropping sheets. A vacuum pump is connected to a tube that had a funnel attached to the end. The funnel is shown here at 90°, but is capable of being adjusted to all angles between 0° and 90°. Different 9x15 cm sheets used shown on the table. From left to right and top to bottom, the sheets are cardboard, cereal box, notecard, and cardstock.

card and then releasing impacts the drop angle. Dropping them with no influence on initial angle or velocity was accomplished using a vacuum pump system. The vacuum pump was connected to a funnel and also had a release valve that allowed for the vacuum power to be reduced for lighter materials. This was necessary because some of the lighter materials would be slightly sucked into the funnel instead of being held firm against it. This was attached to a stand that allowed for the sheets to be dropped at an appropriate height for varying trajectories to be observed. The end of the funnel was loose so that the angle could be adjusted throughout the experiment. A white background was put up and the sheets were colored so that they would stand out during recordings. The apparatus is shown in Fig. 1 along with the sheets used. The drops were recorded by a camera and imported into Tracker. Each video was manually tracked by clicking on the object in each frame. The tracking outputted x and y coordinates for the object that could be exported to Igor for further analysis. A total of 10 trials were done per material for each angle.

The materials used were a piece of cardboard, a cereal box, a notecard, and cardstock. These were cut into a 9x15 cm^2 sheet so that they would show up in the recordings and also be small enough to see varying trajectories at an approximate 2 m drop. These materials were selected for the varying rigidity and weights. The weight takes into account the gravitational force changing based on mass, so the heavier materials, the cardboard and the cereal box, were less affected by air resistance. On the other hand, the notecard and the cardstock were less rigid and were able to slightly bend during descent, which can cause chaotic trajectories. Due to the set-up of the apparatus, materials lighter than cardstock were not suitable to be used since the vacuum pump would too strong and would suck them in and not allow for a proper drop. Materials heavier than the cardboard were not used because they were too heavy for the vacuum pump, so the drops could not be controlled.

The initial angles chosen were 0°, 30°, 60°, and 90°. The actual angles measured end up being 0°, 33°, 62°, and 90° because of how the apparatus was setup. The funnel was loose so that it could be adjusted, but this made it hard to get a precise angle, so the closest approximate angle to the desired was used.

IV. RESULTS

Initial trials started at a horizontal drop. Due to the balance of the air resistance acting on the object, if the object falls flat, it will remain approximately flat throughout its descent. This was tested and proven before moving on to different angles. The horizontal trials were all approximately the same and show little variation. This helped confirm the main idea behind the project and allowed for continuation to trials at different angles.

A. 90° Trials

Trials were done at 90° and are shown in Fig. 2. The progression of trajectories is shown from left to right and the separation is also shown. The separation was calculated by selecting a single trajectory and measuring the distance between it and all other trajectories at the same time. These separations were averaged to get a general idea of the variation between all of the trajectories. The separation does not follow the same general trend as the trajectories, but this could be attributed to the random choosing of a trajectory to base the separation on. If the chosen trajectory was an outlier, then the separation will be greater than if a trajectory in the general trend was chosen. Regardless of the chosen trajectory, the separation is shown to diverge as time increases. The trajectories show an evolution of increasing chaos. They begin following simple paths and a general trend, but as the materials get lighter the paths become more interesting and lead to more variation between each.

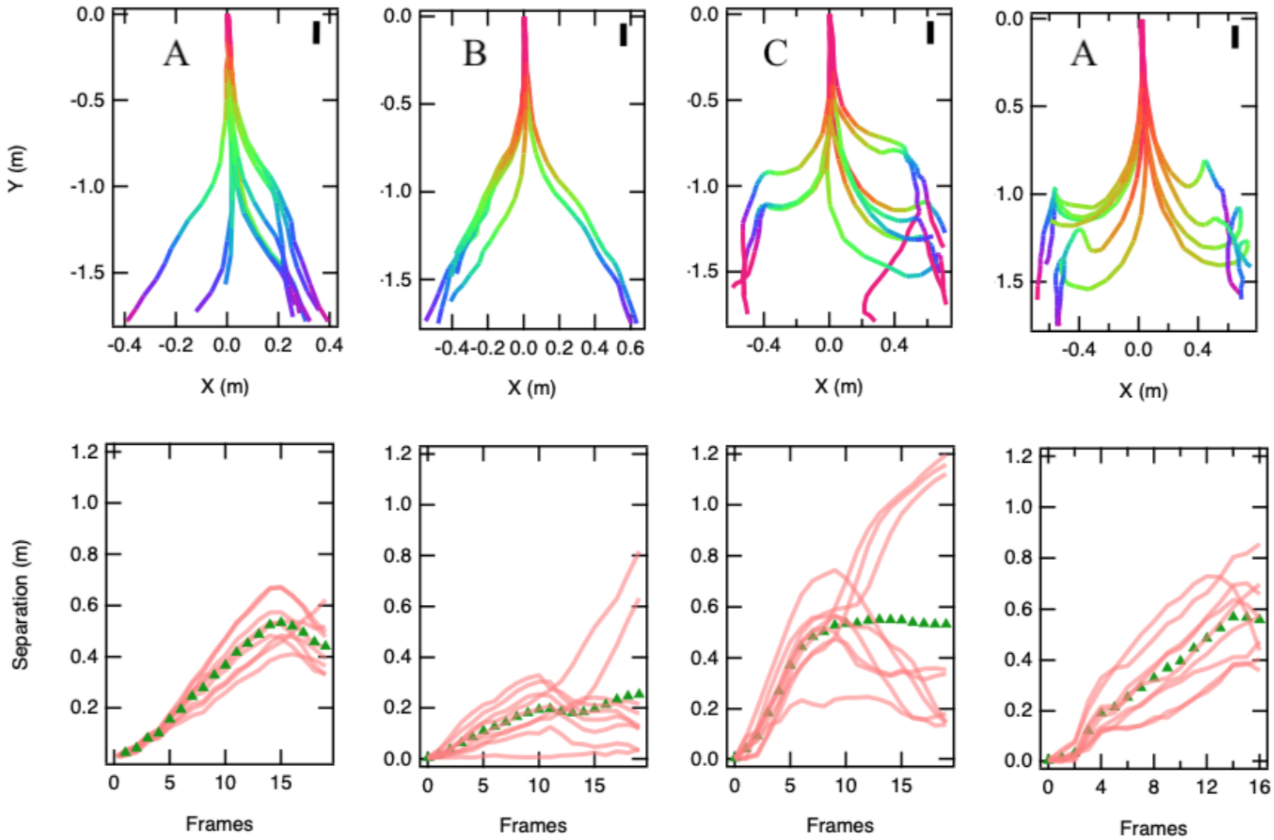

FIG. 2: Plots of trajectories of 10 trials each falling at a 90° and the separation between trajectories as time progresses for A) Cardboard B) Cereal Box C) Notecard and D) Cardstock.. Color scale shows the time evolution of each trial. The green triangles show the average separation between trajectories. Each separation plot corresponds to the trajectory above it. Line in the top right represented the angle that the object is dropped at.

B. 62° Trials

Trials were also done at 62° as shown in Fig. 3. Again the evolution of the trajectories is shown from left to right. The separations follow a trend except for the first one where the path chosen to find the separation was the single outlier, causing the resulting separation to be far greater than expected. Another aspect that leads to the separation varying from the expected greater separation is that during some trials, the sheet left the screen, so it does not complete its path and will continue to remain in its position while the others move farther away. The appearance of chaos is apparent in both the notecard and the cardstock, where the trajectories began to have more variation at the same initial condition.

C. 33° Trials

The last angle tested was at 33° and is shown in Fig. 4. For this trial, only the notecard and cardstock trials are shown because the cardboard and the cereal box remained consistent throughout the trials, as they were too heavy to have noticeable variation being dropped at this height. The notecard and cardstock were light enough to have variation shown in each trials. The separation of both of these were similar and the wide range shows the variation in trajectory.

D. General Trends by Material

Overall, at each angle the variation was more apparent at lighter materials. They were more susceptible to air resistance and have more variation trial to trial than the heavier materials. The general trends show that the separation between trajectories is divergent in each case showing that this system is susceptible to chaos. This is shown through the trials of the lighter materials, where the trajectories are inconsistent. This variation is caused by the sheets beginning to tumble and flutter. The rotation causes the air to hit a certain side more than another and causes the sheet to take a different path depending on the initial tumble which can be attributed to the weight

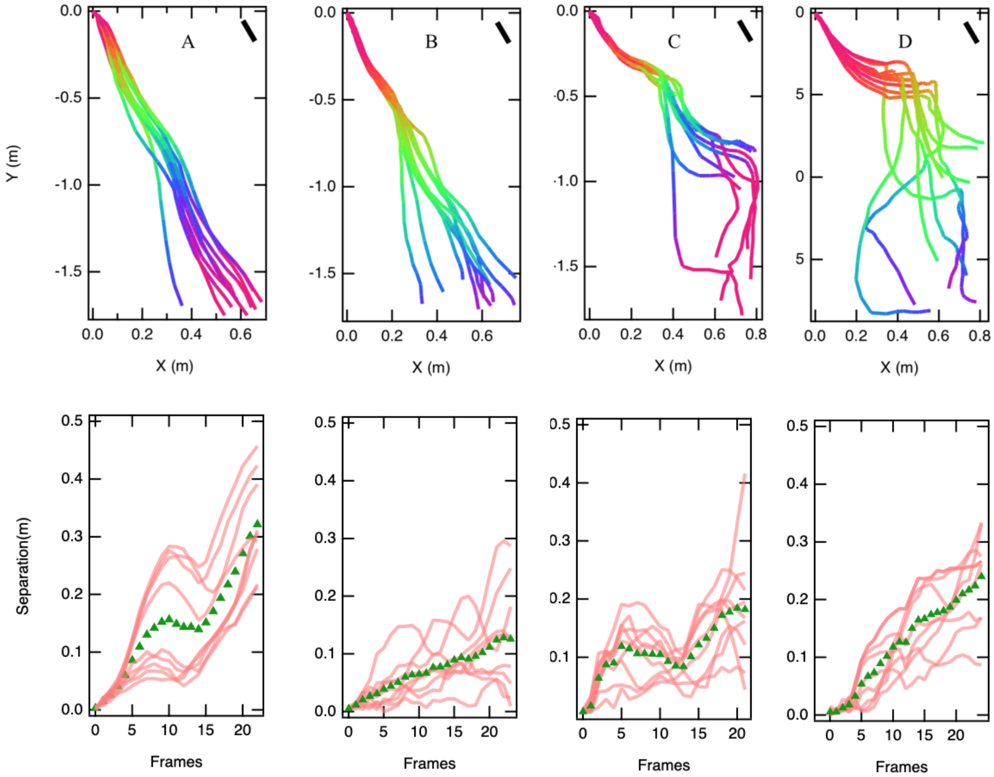

FIG. 3: Plots of trajectories of 10 trials each falling at a 62° and the separation between trajectories as time progresses for A) Cardboard B) Cereal Box C) Notecard and D) Cardstock.Color scale shows the time evolution of each trial. The green triangles show the average separation between trajectories.

of the object. The heavier sheets take more force to tumble, while the lighter sheets are highly susceptible.

E. General Trend by Angle

As shown, the material has an effect on the overall trajectory regardless of the angle. To show the effect of the different angles, a single material is shown at different angles in Fig. 5. This plot shows how the trajectories become more and more spread out as the angle increases from horizontal to vertical. This makes sense, since the original case is the flat case where the object should fall straight down without much variation. Here at vertical the sheets have the ability to go almost every direction when released.

V. CONCLUSION

The effects of the change in material and the initial angle greatly influence the outcome of the trajectory. The lighter materials are much more susceptible to chaotic trajectories while the gravitational force generated by the heavier materials allows for much more predictable trajectories. The differences can be seen as the angle increases as well. The combination of the changing angle and material show the importance of both in determining the overall trajectory. The divergence of the separation is apparent as the increases from horizontal to vertical. The separation also increases with the lighter materials due to the greater variation in their trajectories. Overall results were limited by the current experimental setup, but more results could be produced with improvements. The main improvement would to film from farther back and have a greater initial drop height. When the angle was increased, there were many trials where the sheet would leave the field of view. This leads to a problem because some of the most interesting trajectories are the ones where it has enough forces acting on it to cause to leave the field of view. Another improvement would be to find light materials that has a small amount of thickness so that they are more viewable on video because the notecard and the cardstock became hard to track when falling parallel to the camera. The addition of a second camera would be an interesting development to the project because then a 3D trajectory could be tracked would add additional variation to each trajectory. Slight improvement could be made to the apparatus to get exact angles instead of close approximations and would allow for more angles to be tested.

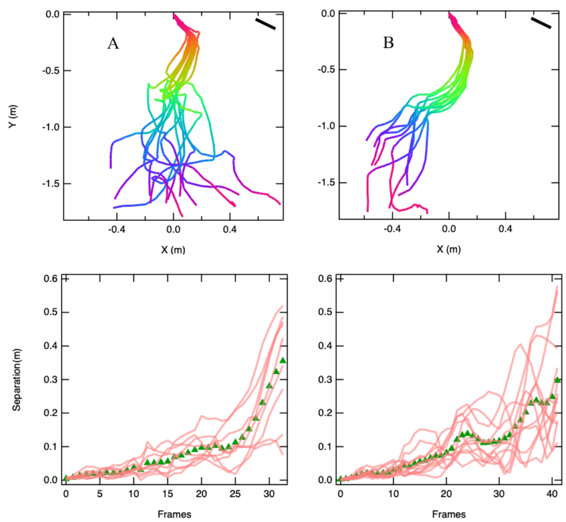

FIG. 4: Shows trajectories of 10 trials each falling at a 33° and the separation between trajectories as time progresses for A) Notecard and B) Cardstock. Color scale shows the time evolution of each trial. The green triangles show the average separation between trajectories.

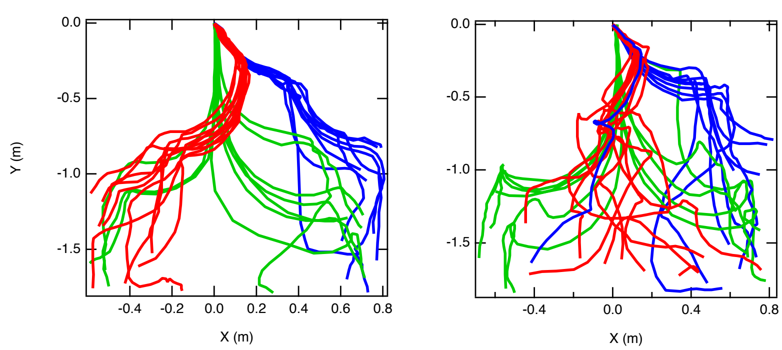

FIG. 5: Comparisons of the trajectories for the notecard(left) and cardstock(right) at the angles 33°(red), 62°(blue), and 90°(green).

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