# The Effect of Charge and Distance on the Electrostatic Force

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The purpose of this experiment was to find the relationships between the electrostatic force and charges found on two spheres and the electrostatic force and distance. This experiment was also used to calculate the Coulomb constant $k$. We found the relationship between force $F$ with charges $q_1$ and $q_2$ and distance $d$ to be $F \propto q_1 q_2/d^2$. We then calculated the proportionality constant, or Coulomb's constant, using two methods of calculating the charge on the sphere. One method was experimentally and the other was by calculating the theoretical capacitance of the spheres and using the applied voltage. When working experimentally, we found $k = (6.07 \pm 0.04) \times 10^9$ $\mathrm{Nm^2/C^2}$ which is within 32% of the accepted value and when working theoretically, we found $k = (7.27 \pm 0.05) \times 10^9$ $\mathrm{Nm^2/C^2}$ which is within 19% of the accepted value.

## I. INTRODUCTION

On dry days we experience static which can sometimes hold things together if the items are oppositely charged or push things apart if they are the same charge. We commonly see this with a plastic wrapper as we attempt to open an object. This force is the electrostatic force which Augustin de Coulomb was curious about. The electrostatic force can affect many different things in nature and in the lab.

In 1784, Coulomb investigated the electrostatic force on two bodies which acted as point charges. He did this using a torsion balance and found the force was proportional to the charges and inversely proportional to the distance squared between the point charges. These results give us what is now known as Coulomb's Law [1].

The purpose of the experiment was to verify Coulomb's law. We also wanted to calculate Coulomb's constant. This shows us how the electrostatic force is dependent on both the distance between the charges and the magnitude of the charges.

## II. THEORY

Coulomb experimentally found the electrostatic force $F$ using two point charges $q_1$ and $q_2$. He found

$$F \propto \frac{q_1 q_2}{d^2}, \tag{1}$$

where $d$ is the distance between the center of the charges. Coulomb was then able to calculate for the proportionality constant $k$, which is now known as Coulomb's constant to be equal to $8.99 \times 10^9$ $\mathrm{Nm^2/C^2}$. This gives the electrostatic force to be

$$F = k\frac{q_1 q_2}{d^2}. \tag{2}$$

### A. Force versus Distance

When investigating the force and distance relationship, we assume $q_1$ and $q_2$, to be constant. If we take the logarithm of both sides of Eq. 2 we find

$$\log(F) = \log\left(k\frac{q_1 q_2}{d^2}\right). \tag{3}$$

Using the rules of logarithms, the equation can be rearranged to

$$\log(F) = \log(kq_1q_2) - 2\log(d). \tag{4}$$

and the force is proportional to the angle $\theta$ needed to rezero the torsion balance which can be measured using the torsion balance, so

$$\log(\theta) \propto \log(kq_1q_2) - 2\log(d). \tag{5}$$

A correction factor $B$ also had to be applied to angles in the experiment because the spheres of radius $r$ are not point charges, but instead have charge around the outside of the sphere. As the spheres are brought closer, the charges redistribute themselves non uniformly, so the fact that the spheres are not point charges matters more at smaller distances. This correction factor was

$$B = 1 - 4\frac{r^3}{d^3}. \tag{6}$$

We multiplied our angles by $1/B$ to get our corrected angles $\theta_{co}$.

Now if the correction term is applied to Eq. 5, we find

$$\log\left(\frac{\theta}{B}\right) \propto \log(kq_1q_2) - 2\log(d). \tag{7}$$

If $\log(\theta/B)$ versus $\log(d)$ is plotted there should be a linear relationship because the $\log(kq_1q_2)$ term is constant and the slope should be -2.

### B. Force versus Charge

When looking at the force and charge relationship, we assume $d$ to be constant, so from Eq. 5 the $-2\log(d)$ term is constant. For the experiment $q_1 = q_2$, so using logarithm rules, Eq. 5 becomes

$$\log(\theta) \propto 2\log(q_1) + \log(k) - 2\log(d). \tag{8}$$

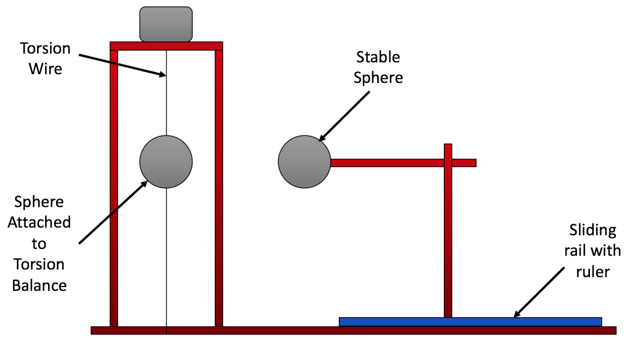


FIG. 1: Apparatus used to investigate the relationship between force and charge or force and distance.

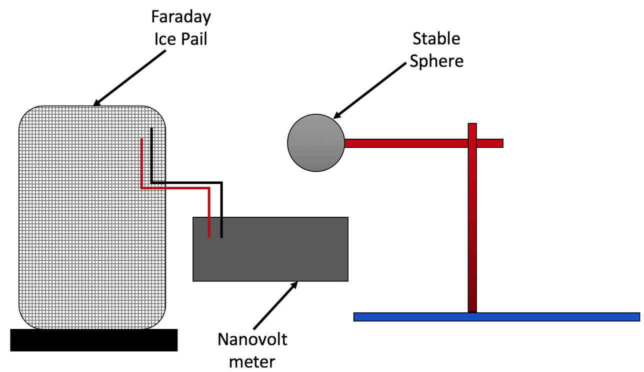


FIG. 2: Apparatus used to measure the charge on the sphere.

If $\log(\theta)$ versus $\log(q_1)$ is plotted there should be a linear relationship because $\log(k)$ will be constant since $k$ is always a constant and the slope should be 2. The correction coefficient is not necessary for the force versus charge relationship because the distance is held constant.

## III. PROCEDURE

### A. Apparatus

The apparatus was a Pasco apparatus as seen in Fig. 1 for investigating Coulomb's law and Coulomb's constant. Two spheres are used to act as point charges. One of these spheres was stable, so it could be moved a given distance from the center of the other sphere. The other sphere was attached to a torsion balance, so it can swing away from its equilibrium point as the electrostatic force acts on it. The stable sphere is attached to a sliding track to set it a given distance from the other sphere. A kilovolt power supply was used to apply a voltage to the spheres.

In order to observe the charge on the spheres, we used a nanovoltmeter to find the voltage across a Faraday ice pail $V_{ip}$ when we touched a charged sphere to the ice pail as can be seen in Fig. 2. A capacitance meter was used to measure the capacitance $C_{ip}$ of the ice pail system.

### B. Force and Distance Relationship

We wanted to investigate the relationship between the force and the distance. In order to do this, we applied the same voltage $V = 6.12$ kV to both of the spheres. The sphere were set at distances of 20, 14, 10, 9, 8, 7, 6, and 5 cm apart. For each distance, five tests were done and then averaged.

The torsion balance was zeroed prior to each test. When the spheres were brought to the given distance, the torsion balance was then rezeroed by turning the wire an angle $\theta$. The spheres were grounded and recharged between each test. The angle was recorded and plotted versus the distance squared to see the relationship. A plot was also made of $\log(\theta)$ versus $\log(d)$ to observe the relationship. The corrected angles $\theta_{co}$ were also plotted in the same way.

### C. Force and Charge Relationship

We also wanted to investigate the relationship between the force and charges on the spheres. The distance was kept constant and the voltage applied to the spheres was varied from 3-6 kV. We completed five tests for each voltage.

The torsion balanced was zeroed before each test and then once the voltage was applied to the sphere, the balance was rezeroed and the angle was recorded. The spheres were grounded and recharged between each test. A plot was made of the logarithm angle versus the logarithm of the voltage to see the relationship between the force and the charge.

### D. Finding Coulomb's Constant

Now that the dependence the force has on the distance and the magnitude of the charge can be seen, we wanted to calculate Coulomb's constant. In order to do this, we had to calculate the torsion constant $K_{tor}$. Gravity was used as the force on the torsion balance by applying 0, 20, 40, and 50 mg masses $m$ to the sphere attached to the balance and rezeroing the balance for each mass, so $K_{tor}$ could be found. The mass was converted to weight by $F_g = mg$, where $g$ is the gravitational acceleration on earth. The torsion constant is then $K_{tor} = F_g/\theta_{tor}$, where $\theta_{tor}$ is the angle when zeroing the balance. The tosion constant was used to calculate the force in newtons instead of degrees by $F = K_{tor}\theta_{co}$.

The charge on the spheres was also needed in order to calculate Coulomb's constant. We attached the Faraday ice pail to a nanovoltmeter and a capacitance meter in parallel, so the capacitance on the system could be found.

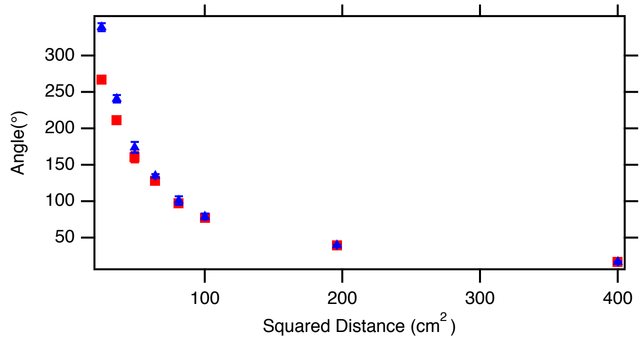


FIG. 3: The angle measured versus the distance squared with the corrected angle is in blue triangles and the original angle is in red squares. This shows the relationship between force and distance is $F \propto 1/d^2$.

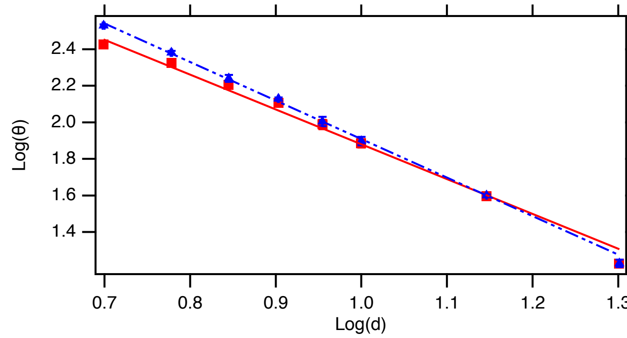


FIG. 4: The logarithm of the angle versus the logarithm of the distance with the corrected angle in blue triangles and a dashed line and the original angle in red squares. The slope of these lines is equal to the exponent $n$ in the equation $F \propto 1/d^n$. For the corrected angle $n = -2.107 \pm 0.018$ and for the original angle $n = -1.905 \pm 0.017$. which are both with 5% of the accepted value of 2.

After finding the capacitance of the system, one of the spheres was charged and touched against the ice pail to observe the voltage across the ice pail. The charge $q$ was then found by $q = C_{ip}V_{ip}$.

Another method of calculating the charge was by calculating the capacitance $C$ of the sphere

$$C = 4\pi\varepsilon_0 r, \tag{9}$$

where $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m and is a constant based on the material of the sphere. We then calculate the charge $q = CV$, where $V$ is the voltage applied to the sphere. Force versus $q^2/r^2$ was plotted for both of our values of $q$ to find Coulomb's constant.

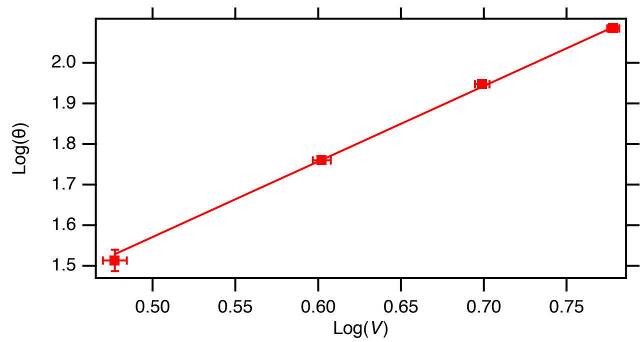


FIG. 5: The logarithm of the force measured as an angle versus the logarithm of the voltage applied to the spheres. The slope $n$ is the exponent for $q^n$ and is $1.86 \pm 0.08$ which is within 7% of the accepted value of 2. This shows the relationship between force and charge to be $F \propto q^2$ with $q = q_1 = q_2$.

## IV. RESULTS AND ANALYSIS

### A. Force and Distance Relationship

We observed the relationship between distance and force. The force as measured in the angle $\theta$ versus the distance squared was plotted in Fig. 3. This shows us that our data is inversely proportional and most likely has a $F \propto 1/d^2$ relationship. We plotted both the corrected in blue triangles and the original angles in red squares.

We want to confirm the force versus distance relationship is a inverse squared relationship, so the logarithm of the force versus the logarithm of the distance was plotted in Fig. 4. The slope of the graph is now the exponent of the distance. For the corrected angle, the slope is $-2.107 \pm 0.018$ which is within 5% of the accepted value of 2 and the line is the blue dashed line. The slope for the original angle is $-1.905 \pm 0.017$ which is also within 5% of the accepted value.

### B. Force and Charge Relationship

We observed the relationship between the charge and the force. The force as measured by the angle from the torsion balance $\theta$ versus the charge which was applied to the sphere was plotted in Fig. 5. The slope of the fit is $n$ for $q^n$ and $n = 1.86 \pm 0.08$. This is within 7% of the accepted value of 2 and shows that the force is directly proportional to the charge of each of the spheres.

### C. Finding Coulomb's Constant

The torsion constant is found by calculating the angle needed to zero the balance as mass is added to the system. We then found the weight of the mass and plotted weight versus the torsion angle in Fig. 6. The torsion constant was found to be $K_{tor} = (1.37 \pm 0.02) \times 10^{-6}$ N/deg.

We found the charge through two methods, experimentally and theoretically. Experimentally, the capacitance of our ice pail system was found to be $C_{ip} = 5 \pm 1$ nF. A charged sphere was touched against the ice pail and the voltage across the ice pail was found to be $V_{ip} = 2.8 \pm 0.5$ V. Using the equations $q = C_{ip}V_{ip}$ we found the

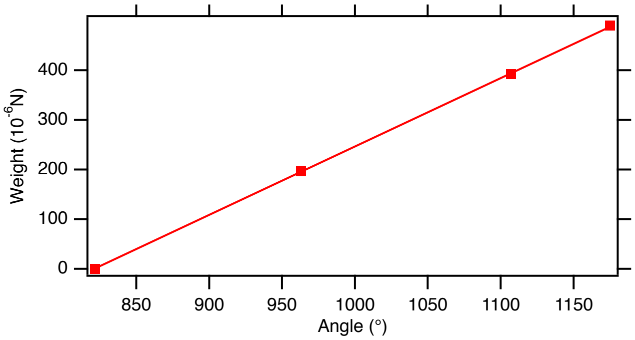


FIG. 6: The weight versus the torsion angle. The slope gives us the torsion constant $K_{tor} = (137 \pm 208) \times 10^{-6}$ N/deg.

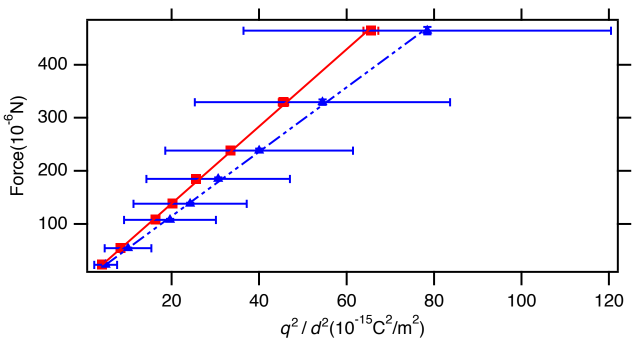


FIG. 7: The force measured in newtons versus the charge squared over the distance squared. The red squares are the calculated charge using the capacitance of the sphere and the blue triangles are the charge calculated using the ice pail. The red and blue lines indicate the linear fits to the calculated and measured charge points. Coulomb's constant for the theoretical capacitance was found to be $k = (7.27 \pm 0.05) \times 10^9$ Nm$^2$/C$^2$ and Coulomb's constant for the ice pail was found to be $k = (6.07 \pm 0.04) \times 10^9$ Nm$^2$/C$^2$.

charge to be $q = 1.4 \times 10^{-8}$ C.

From the experimental results, the force versus the charge squared over the distance squared was plotted in the blue triangle and dashed line in Fig. 7. This gives us the Coulomb's constant for the slope as $k = (6.07 \pm 0.04) \times 10^9$ Nm$^2$/C$^2$.

Theoretically, we found the capacitance of the spheres by Eq. 9. The capacitance of the sphere was then $C = 2.09 \times 10^{-12}$ F. Using the theoretical capacitance and the applied voltage of $V = 6.12$ kV, we find the charge to be $q = 1.28 \times 10^{-8}$ C.

From the result for charge, the force versus the charge squared over the distance squared was plotted in the red squares and line in Fig. 7. This gives us the Coulomb constant to be $k = (7.27 \pm 0.05) \times 10^9$ Nm$^2$/C$^2$.

## V. CONCLUSION

We verified the relationships Coulomb found in Eq. 1. We also then calculated for the Coulomb constant. We found the Coulomb constant to equal $k = (6.07 \pm 0.04) \times 10^9$ Nm$^2$/C$^2$ when we measure the charge. This value is within 32% of the accepted value. If we use the charge we calculated theoretically, we find Coulomb's constant to be $k = (7.27 \pm 0.05) \times 10^9$ Nm$^2$/C$^2$. This value is within 19% of the accepted value.

We see a large amount of uncertainty in our charge calculated from the ice pail in the blue triangles and dashed line in Fig. 7. This uncertainty is due to the measurement of the charge. The nanovoltmeter was only precise to 0.5 V, so this caused a large amount of uncertainty.

[1] A. de Coulomb, "First memorandum on electricity and magnetism," *History of the Royal Academy of Sciences*, pp. 569–577, 1785.