Characterizing Germanium with the Hall Effect

Samuel Nash
Department of Physics, The College of Wooster, Wooster, Ohio 44691, USA
(Dated: May 10, 2018)

This experiment investigated properties of germanium using the Hall effect. A sample was placed into a Hall effect module, different currents run through it and using the resulting Hall voltage the sample was characterized. The first experiment described n-type germanium in terms of carrier concentration. The n-type germanium was found to have a carrier concentration n of 8.402×10^{20} m^{-3}. The second experiment determined the band gap of germanium by heating an undoped germanium plate and measuring voltage as the plate cooled. The band gap was found to be 0.727 ± 0.001 eV versus the accepted value of 0.66 eV, suggesting a percent error of 10.6 %.

I. INTRODUCTION

The Hall effect was discovered in 1879 by Edwin Herbert Hall. The Hall effect is produced when an electrical current is passed through a sample placed in a magnetic field. The Hall effect can be used to determine several qualities of the sample. The first is how electrical current carried through the material by electrons, or by positive holes, the absence of electrons. The second is the density of the carriers, or the amount of mobile charge carriers per unit volume of material. Third, is what the density of the carriers, the amount of mobile charge carriers per unit volume of material, is. Finally, the equipment we are using can be used to determine the band gap between the conduction and valence band of the material [1].

The characteristics the Hall effect defines have several significant uses including the study of semi-conductors. The band gap allows for materials to be sorted into insulators, relatively large band gaps, and semi-conductors, which have relatively small band gaps. The charge carrier capacity is useful for transistor construction. The identification of charge carriers allows for the sorting of n-type and p-type semiconductors, in which current is either carried by electrons or alternatively by holes (the lack of electrons).

II. THEORY

A. Energy Gap, and types

How can the lack of an electron carry a current? In order to answer this, how electrical current is carried must be understood. In a normal conductor, the conduction electrons may move freely without energy barriers. In a semiconductor however there is an energy barrier, or band gap. This band gap is a separation of energy levels from the valence band, where the unexcited electrons are, to the conducting band, where excited electrons can move freely. Electrons cannot move in the valence band because they cannot occupy the same space as another electron. As opposed to the negatively charged electron, the positively charged hole is the lack of an electron in the valance band. Electrons move into the hole; however because there are so many electrons in the valence band, it appears as a moving hole that can carry current. To make these holes in the semiconductors an impurity is added that removes an electron.

There are three types of semiconductors. Undoped semiconductors are materials such as silicon that are made into pure semiconductors. These materials should have no imperfections, and should follow band gap theory exactly. If the base semiconductor material is silicon which has four valence electrons, a P-type semiconductor may be produced by creating a hole with deliberate doping with small amounts of boron or aluminum. N-type semiconductors are made the same way as P-types, but instead of adding boron or aluminum, phosphorous is used to add an electron which will move into the conducting band.

B. Hall Effect

The cause of the Hall effect is the interaction of a magnetic field with the electron current moving within the sample material. The magnetic field is perpendicular to the current, and creates a force on the charge carriers, that is perpendicular to the current, and magnetic field. This moves the charge carriers in the material and creates a potential that can be read as a voltage. The force is given by,

$$\vec{F} = e(\vec{v} \times \vec{B}) \tag{1}$$

where $\vec{F}$ is the Lorentz force vector, e is the charge of an electron, $\vec{v}$ is the velocity of the charge carriers, and $\vec{B}$ is the vector of the magnetic field [2]. A schematic of the geometry is shown in Fig. 1.

In the case of the Hall effect $\vec{v}$ is the drift velocity, and is related to the current I by,

$$v = \frac{I}{neA}, \tag{2}$$

where n is the density of mobile charges, and A is the cross-sectional area.

Assuming B and I are perpendicular Eq. (1) can be rewritten as,

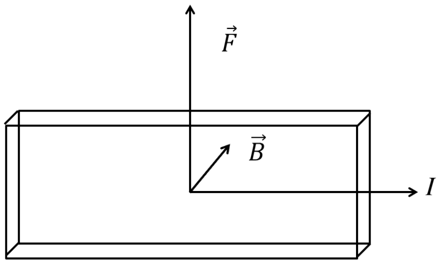

FIG. 1: A figure showing the Lorentz Force, and how the the current, magnetic field and force applied to the electron interact.

$$F = \frac{IB}{nA} \tag{3}$$

where F is the over all force exerted on the electrons by the magnetic field. What Eq. (3) says is that in order to increase the force on an electron, increase the current or the magnetic field.

Because an electrical force arises from the displacement of electrons by the magnetic field, force can be rewritten in terms of the potential,

$$F = \frac{Ve}{w} \tag{4}$$

where w is one component of A, that is perpendicular to both I and B, V is the voltage [2].

Finally Eq. (3 and 4), can be written as,

$$V_H = \frac{IB}{ned} \tag{5}$$

where V_H equals the Hall voltage that is produced by the experiment, and d is the thickness of the plate that is parallel to $\vec{B}$ [4]. What this says is that in addition to more current or a stronger magnetic field increasing the Hall voltage, a thicker sample, or a sample with a higher carrier capacity will have a smaller Hall voltage.

The Hall coefficient R_H can be be found through,

$$R_H = \frac{V_H d}{BI} \tag{6}$$

and can be used to calculate the carrier capacity n, [4]

$$n = \frac{1}{R_H e}. \tag{7}$$

In order to calculate the band gap, the equation

$$\sigma = \frac{lI}{AV_H} \tag{8}$$

was used, where σ is conductivity, l is the length of the metal plate, and A is the cross-sectional area the current flows through.

Another way to calculate σ is,

$$\sigma = \sigma_0 \exp \frac{E_g}{2kT} \tag{9}$$

where σ_0 is the intrinsic conductivity, E_g is the energy of the band gap, T is the temperature and k is Boltzmann's constant [5].

Taking the natural log of both sides of Eq. (9) produces of more usable form of the equation,

$$ln(\sigma) = ln(\sigma_0) + \frac{E_g}{2kT}. \tag{10}$$

What this equation says is that as the sample gets hotter the conductivity will go down.

III. PROCEDURE

A. N-Type Germanium

For the n-type germanium experiment a board which mounted a sample plate was placed into the Hall effect module. The board was connected to a current source built into the Hall effect module. The current flowed across the sample. A voltage meter was connected perpendicular to the current to measure the Hall voltage. A set of two magnetic coils was used to generate the constant magnetic field that induces the hall voltage. The apparatus is shown in Fig. 2.

To collect data the current through the coils was maintained at 0.7 A throughout the experiment. The current across the germanium plate was set to -30 mA, and the voltage was zeroed. The board was lowered between the coils and the resulting voltage was recorded. In addition the magnetic field was measured with a gauss meter. The process was repeated incrementing up to 30 mA in steps of 5 mA.

After the Hall voltage has been measured the board current was set to 5 mA and the sample was heated to about 160° C. The heater was turned off, and the voltage and temperature were measured and recorded from a video recording [4].

B. Band Gap

The measurement of the band gap was similar to the process used for the n-type germanium. The key differences were that the plate was not placed in a magnetic field during the band gap experiment and that the voltmeter was set up to read the voltage in the same direction as the current. The plate was heated, and then a video was taken of both the temperature and the voltage [5].

FIG. 2: The experimental apparatus for the experiment. From the left is the gauss meter, the volt meter, the Hall effect module, with a board inserted, the magnetic coils, and the power source for the coils. Credit to PHYWE [3]

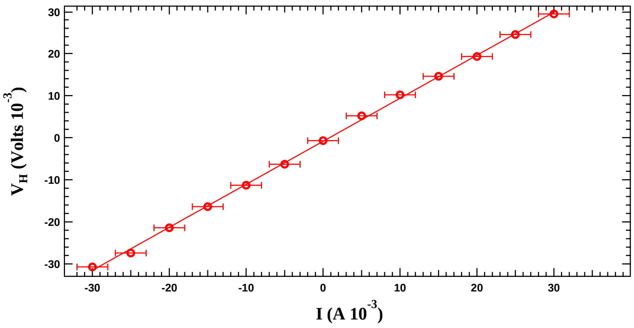

FIG. 3: A graph of the Hall voltage versus the current run through the sample. The y-intercept is $(8 \pm 1) \times 10^{-4}$ V, and the slope is 1.022 ± 0.009 A^{-1}. This graph is used to find the proportionality factor α.

IV. DATA AND ANALYSIS

A. N-Type

The measured Hall voltage is plotted as a fuction of current in Fig. 3. The first piece of data to be found was the proportionality factor α which relates current and voltage. The proportionality factor α is the slope of the graph in Fig. 3, the graph is the Hall voltage versus the current. The slope of the line is a good indicator of what α is equal to in this case because it is well fit, α is 1.022 ± 0.009 A^{-1}.

The second data point to be found was the Hall coefficient R_H. From R_H the carrier concentration of the sample could be found. To get n from R_H Eq. (7) and Fig. 4 were used. The graph shows the Hall voltage multiplied against the thickness of the plate versus the magnetic field multiplied by the current, the slope is R_H, and in this case data is well fitted by the slope.

Since R_H is $(7.43 \pm 2) \times 10^{-3}$ Vm/TA the equation for n is Eq.

$$8.41\times10^{20}\ \text{m}^{-3} = \frac{1}{7.43\times10^{-3}\ \text{VmTA}\times1.602\times10^{-19}\ \text{C}} \qquad (11)$$

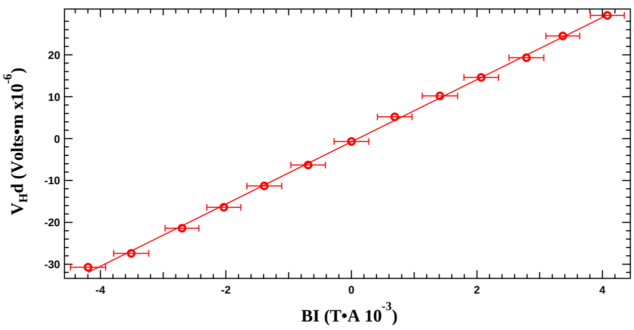

FIG. 4: A graph voltage times distance, versus current times magnetic field. The y-intercept is $(-8 \pm 2) \times 10^{-7}$ Volts $\times$ meters, and the slope is $(7.43\pm2)\times10^{-3}$ with units of Vm/TA.

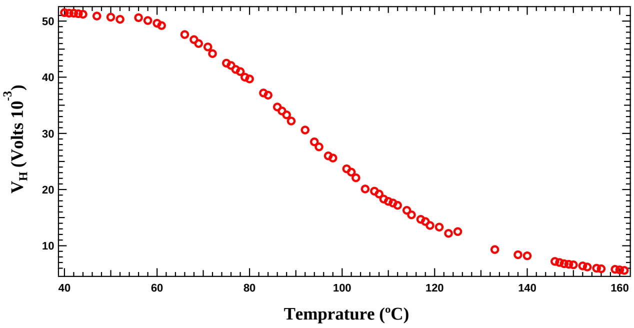

FIG. 5: This is the graph of voltage vs temperature in the n-type germanium sample.

This gives the carrier concentration as 8.41×10^{20} m^{-3}, which is within an order of magnitude of what the manual reports on for a different, but likely similar sample.

Voltage decreases nonlinearly with increasing temperature Fig. 5. As the material warms up more electrons are freed to move around, which consequently lowers the potential difference across the board.

B. Band Gap

The band gap of undoped germanium was found to be 0.70 eV. The raw data was voltage versus temperature. Voltage was converted into conductivity via Eq. (8) and the absolute temperature was inverted and multiplied by 1000. Next the natural log of σ was taken. These were graphed against each other to give Fig. 6.

The slope of the figure is given by Eq. (10). By multiplying the slope by -2k the bandgap, E_g is found to be 0.727 ± 0.001 eV.

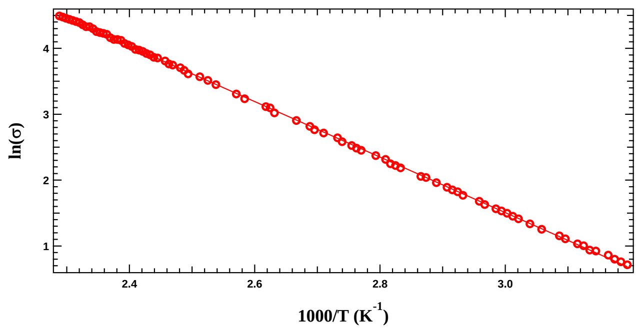

FIG. 6: A graph of the natural log of σ against $1000/T$. The y-intercept is 14.15 ± 0.02 and the slope is -4.214 ± 0.006 K^{-1}
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V. CONCLUSION

A. N-Type

I was primarily interested in finding n-type carrier capacity for my sample: 8.402×10^{20} electrons per meter. There is no board to compare this against as the carrier capacity is related to the samples production, and depends on the material that the germanium was produced with. A potential improvement to the experimental method employ equipment to force consistency stability into the experiment, for example on the Hall module the current run through the sample fluctuated.

B. Band Gap

The band gap was 0.727 ± 0.001 eV compared to an accepted value of 0.66 eV [6]. I found the percent error to be 10.6 %. Sources of error could include inaccuracies in the measurements. For example the temperature gauge of the Hall module was not accurate. During the cooling cycle the temperature would go up. In addition, the gauge only displayed temperature to the ones place, as it did not have decimal measurements.

VI. ACKNOWLEDGMENTS

I would like to acknowledge our Junior IS instructor Dr. Susan Lehman. I would also like to thank our TA's Zane Thornburg, and Gabriel Gou.

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