Flips are Fun: A Study in the Conservation of Angular Momentum of a Non-Rigid Human Body

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Abstract

In this study, the human body's behavior during a back somersault was examined. Due to the laws of classical mechanics, once a body leaves the ground, all linear and angular momentum must be conserved until the application of an outside force or torque. Therefore, any change in angular velocity without application of an outside torque must be due to a change in the moment of inertia of the body. Using Tracker video analysis, a video of a back somersault was analyzed to determine the angular spin of the body and whether or not it was conserved. By simplifying all limbs to cylinders, the center of mass of each limb, as well as the mass of each limb, was used to determine the center of mass of the body as a whole for three key points in the somersault: the take off, the tuck or ball shape at peak height of the somersault and just before the landing. From calculating the body's overall center of mass and using the Tracker video analysis program, the angular velocity and inertia tensor of the human body was calculated. With the inertia tensor and angular velocity, the magnitude of the angular momentum of the body was found to be $L_1 = 27.89$, $L_2 = 57.409$, $L_3 = 24.85$, measured in kg M^2 where each number represents the three previous positions and all three angular momentum act about the axes of rotation. The results found show that angular momentum was not conserved and in fact doubles at the tuck phase of the rotation. This change in angular momentum was more than likely due to the inefficiency of the measuring method used during the video tracking of the several body parts of the body.

1 Introduction

The conservation of energy and momentum has been discussed since the age of Isaac Newton and his composition of the *Principia*. While Newton discussed conservation of energy and linear momentum in his experiments and writing, his only topic concerning angular momentum was his proof of the Law of Areas[1]. The trend of discussing important pieces of angular momentum without increased analysis did not stop with Newton but continued on. Scientists such as Leonhard Euler[2], Pierre-Simon Laplace, and Léon Foucault all discussed important parts to angular momentum, but it was not until William J. M. Rankine's 1858 *Manual of Applied Mechanics* that we received our modern definition of angular momentum[3].

From bicycles and gyroscopes to the orbit of planets, angular momentum and its conservation principles are very important in governing the movement of various objects and organisms. For any who may have seen an animal such as a squirrel or cat fall from a tree, than they have seen the abilities of the animal to adjust the position of their body mid-air in order to minimize damage or land on their feet. We can see humans doing something similar in many sports. From ice skating and dancing to diving and gymnastics, athletes use the change in body position in order to generate greater amounts of angular velocity. Due to the laws of the conservation of angular momentum, athletes are able to adjust their body position to increase their angular velocity.

Among the sports that use this physical law is a new sport known as Freerunning. Founded by David Belle and Sebastian Foucan, freerunning is a freestyle sport based in the movement of the body. The sport came from the French military technique known as parcour or parkour. This discipline was taught to french troops to ensure their ability to safely move in any environment quickly and efficiently. From this, movements from martial arts tricking, hip hop dance and gymnastics were adapted and the sport as it is known today was created [4].

Because freerunners perform movements either directly from or similar to gymnastics, we will be analyzing the change in body position of a parkour athlete in order to perform a back somersault. While similar studies have been conducted in the study of gymnastics movements, few have looked at the increase of angular velocity due to the change in body position and none have examined the techniques of freerunning athletes. In this study, we examine the changing inertia tensor of the human body and its efficiency in generating angular velocity. Using Tracker, a video analysis tool, a video of a backwards somersault was analyzed following the center of mass of the major limbs of the body(arms, legs, torso, etc.), measuring their position and angular velocity relative to a set axes at the bottom of the athlete's feet. By using the positions of the centers of mass for each limb, the body's overall center of mass and inertia tensor was calculated for the instant where the feet of the athlete left the ground, the time at which the greatest change in position occurs (in this case, the tuck position of the somersault) and the instant just before landing. Because all measurements were taken while airborne, the body can be treated as an isolated system and we can assume the momentum of the body is conserved.

2 Theory

All equations in this section were taken from [4]. In order to determine the body's overall center of mass and subsequently the inertia tensor, we must first determine the center of mass for all major limbs. Estimating each limb to be a cylinder of set length and radius, with the head being a sphere of set radius, the center of mass of each can be estimated to be in the center due to symmetry. Treating these limbs as point masses, we can use the center of mass equation,

$$X = \frac{1}{M}\sum_{\alpha} m_\alpha x_\alpha, \qquad (1)$$

to determine the center of mass of the human body overall. In this case, X is the x-axis position of the center of mass, M is the total mass of the body, m_α is the mass of one of the limbs and x_α is the single coordinate position of that point mass. This calculation was repeated for the Y and Z coordinates to find the three dimensional position of the center of mass. From Newtons laws of motion, when the athlete leaves the ground, the athlete's center of mass will travel similar to a projectile with no torque acting on the body. Because there is no external torque about the center of mass once leaving the ground, the angular momentum of the athlete is conserved. Knowing that angular momentum is conserved, we can look at how an athlete is able to change their angular velocity by changing their body position.
The angular momentum of any body can be found using the equation,

$$L_{total} = L_{orb} + L_{spin}, \qquad (2)$$

where L_{orb} is the angular momentum of the center of mass and L_{spin} is the angular momentum of the rotating body about the center of mass. In our case, because the center of mass will take a parabolic trajectory, L_{orb} is trivial and we will focus on L_{spin}. In order to find our L_{spin}, we can can use the equation,

$$L_{spin} = \sum_{\alpha} \left(\vec{r'}_{\alpha} \times m_{\alpha} \frac{d\vec{r_{\alpha}}}{dt} \right), \tag{3}$$

where $\vec{r'}_{\alpha}$ is the position of the center of mass for any particular limb relative to the body's overall center of mass, m_{α} is the mass of any particular limb and $\dot{\vec{r'_{\alpha}}}$ is the change in position of any particular limb relative to the body's overall center of mass. In order to move from the positions measured relative to some origin to our $\vec{r'}_{\alpha}$, we can use the relation

$$r'_{\alpha} = r_{\alpha} - R \tag{4}$$

where r_{α} is the distance from origin to the limb's center of mass and R is the position of the body's center of mass. With Eq. (3), we can use the position and linear momentum of the limbs to determine the angular spin of the body.

When observing a back somersault, it is seen that the angular velocity of the body increases during the tuck position of the somersault. This is due to the change in body position during the somersault. Using the equation

$$L = I\omega, \tag{5}$$

where I was the 3×3 inertia tensor of the human body and was the angular velocity of the human body, the angular momentum can be determined. Because the change in body position occurs while airborne, there is no torque on the body, meaning that the angular momentum is conserved. Because angular momentum is conserved, a change in the inertia tensor of the body from an initial flat body to a tight round ball must be occurring. Knowing this, we can calculate the inertia tensor during take off, at the position when the tuck is the tightest and just before landing. The inertia tensor acts on the 1×3 angular velocity matrix according to

$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}. \tag{6}$$

To calculate the inertia tensor, we can use the equations,

$$\begin{aligned} I_{xx} &= \Sigma_{\alpha} \left((y_{\alpha})^2 + (z_{\alpha})^2 \right) \\ I_{xy} &= \Sigma_{\alpha} \left(x_{\alpha} \cdot y_{\alpha} \right) \\ I_{xz} &= \Sigma_{\alpha} \left(x_{\alpha} \cdot z_{\alpha} \right), \end{aligned} \tag{7}$$

to find the first row of the inertia tensor. In order to find the second and third row of the tensor, the same process as above needs to be done for the y and z coordinates where we have the relations,

$$\begin{aligned} I_{xy} &= I_{yx} \\ I_{xz} &= I_{zx} \\ I_{yz} &= I_{zy} \end{aligned}, \tag{8}$$

and

$$\begin{aligned} I_{yy} &= \Sigma_{\alpha} \left((x_{\alpha})^2 + (z_{\alpha})^2 \right) \\ I_{zz} &= \Sigma_{\alpha} \left((x_{\alpha})^2 + (y_{\alpha})^2 \right). \end{aligned} \tag{9}$$

With Eq. (7). Eq. (8) and Eq. (9), we can calculate the inertia tensor for each specified position. Because the angular momentum is conserved, if we see a change in the inertia tensor, we should also see a change in the angular velocity to compensate as to maintain the constant angular momentum.

3 Procedure

3.1 Data Collection

For this experiment, a method to measure the mass, height and center of mass position was needed. Because angular momentum is reliant on both the mass and the position of the center of mass of an object, my body was used as a constant. Because I was used as the test subject, the mass, height and position of the center of mass of each body part could be measured. In order to determine the center of mass of each limb, they were broken into segments; forearm and hand, upper arm, thigh, calf and foot and torso, all estimated to be a cylinder. With the estimation of a cylinder with uniform mass distribution, the center of of mass of each must be in the center due to symmetry whereas the head is treated as a solid sphere of uniform mass.

With a method of determining the center of mass position of each limb, several videos were taken of the

Figure 1: The three positions of a back somersault used for analysis, includes the markers of each limb's center of mass: upper arm (blue), forearm (white), head (yellow), torso (red), thigh (purple) and calf (green)

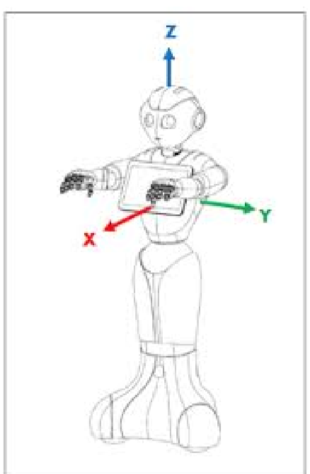

Figure 2: The illustration of the coordinate system used for this experiment, where this figure was reproduced from source [5]

back somersault action using a GoPro Hero 3 video recorder. The axis of rotation was determined to be the y-axis, with an assumption that the body is symmetrical about the vertical axis, the vertical was determined to be the z-axis and forward and backward movements were on the x-axis as illustrated in Fig. 2. With these axes, one of the videos with a view of the xz-plane was used to analyze the motion of the somersault as well as the position of the different limbs before, during and after the somersault. Because the goal was to analyze the spin angular momentum of the body as it rotates, the program Tracker Video Analysis and Modeling Tool was used to track the position of the center of mass of each limb. This was done by playing through the video frame by frame and marking the position of the center of mass of each limb in each frame as seen in Fig. 3. With the position of each limb's center of mass, Eq. (1) was used to find the x, y, and z components of the body's overall center of mass. Because the human body is not rigid,

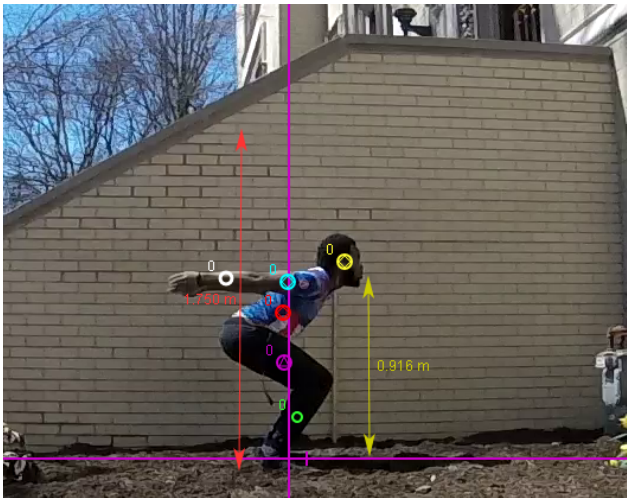

Figure 3: The beginning frame of the video used for analysis, includes the markers of each limb's center of mass: upper arm (blue), forearm (white), head (yellow), torso (red), thigh (purple) and calf (green)

this was calculated for the three points of interest, or three basic body positions of the human body during the somersault, separately. These points included the three positions of the body that were most frequent during the somersault, a flat body, a tucked or balled up body and a semi-bent body as seen in Fig. 1. For these three points, Eq. (7), Eq. (8), Eq. (9) were used to calculate the inertia tensor of the body.

With the inertia tensors, the Tracker program was used once again to determine the angular velocity of the torso and head of the body. Because the somersault is a 360 degree rotation of the body about the center of mass, the angular velocity of the head and torso system were multiplied with the inertia tensor of the body. To determine these angular velocities, the Tracker program used the change in position from the previous point relative to the origin. However, because the inertia tensor was calculated relative to the center of mass of the human body, the angular velocity must also be relative to this center of mass. In order to do this, the position of the center of mass for each previously specified body position was marked in the original axes of all other measurements before the orientation of the axes was changed to the center of mass of the body. With the axes adjusted and all previous points of the various centers of mass shifting relative to the change in axes, the Tracker program recalculated the various positions and angular velocities of the limbs. These values were used to act on the inertia tensors and find the angular momentum of the body.

4 Data and Analysis

4.1 Video Analysis

In order to analyze the angular momentum of the human body during a somersault, the mass and height of the body was needed. As specified in the procedure of this report, my own body was used for all experiments. Knowing that all video was of my own body, my mass and height was used for all following calculations. Being of a height of 175 cm and a mass of 65.3 kg, these measurements were further used to determine the mass and center of mass position of all limbs. Because the body is a nonrigid object, the separate limbs were assumed to be cylinders where their centers of mass position would always be in the center of each limb. The measurements of each limb were done from the nearest joint, where the length of the upper arm was measured from shoulder to elbow and forearm was measured from elbow to finger tip while the thigh was measured from hip to knee and calf was measured from knee to the ground. With these measurements, the center of mass of each limb was marked as seen in Fig. 3. Using data from [7], the average mass distribution of the 25 year old male was used to determine the mass of each of these limbs in Table II.

With a specified mass and position of center of mass for each limb, Eq. (1) was used for the x, y, and z coordinates to find the center of mass of the human body. Because the human body is a nonrigid body, the center of mass moves with the orientation of the body's limbs. Knowing this, a center of mass was found for all specified points of interest, assuming symmetry about the y axis, and was recorded in Table III.

Table 1: The specific mass of each major limb of my body.

Body Part	Mass(kg)
Upper arm	2.575
forearm and Hand	2.26
Thigh	5.796
Calf and Foot	3.919
Head	6.914
Torso	28.35

Table 2: The center of mass position of my whole body at the three chosen key points.

Body Position	CM Position(m)(x,y,z)
Take off	(0.0101,0,1.320)
Tuck	(-0.044,0,1.543)
Landing	(0.137,0,1.06)

These calculations for the center of mass(CM) showed that, relative to the specified origin, the position of the CM moves farther up the body. In analyzing the change in position of the center of mass as the body achieved a tucked position, or curled position, the body changes from the simplified shape of a cylinder to the shape of a sphere. Because the position of the CM of a cylinder and sphere are different, this shift is an expected result.

After finding the center of mass positions for each body position, because the goal was to analyze the spin of the body, the center of mass of the body was used as a new origin for the calculations of the inertia tensor and angular velocity. Using Eq. (4) and the calculated centers of mass were used to find the positions of the limbs r'_α relative to the center of mass position R, the results of which were recorded in Table IV.

The positions calculated in Table IV were used in Eq. (7), Eq. (8), and Eq. (9) to calculate the inertia tensors needed to find the angular momentum. The resulting inertia tensors were

$$\begin{aligned} I_1 &= \begin{pmatrix} 10.56 & 0 & 2.64 \\ 0 & 11.09 & 0 \\ 2.64 & 0 & 1.56 \end{pmatrix} \\ I_2 &= \begin{pmatrix} 2.8 & 0 & -0.94 \\ 0 & 3.89 & 0 \\ -0.94 & 0 & 3.24 \end{pmatrix} \\ I_3 &= \begin{pmatrix} 3.83 & 0 & -1.92 \\ 0 & 6.26 & 0 \\ -1.92 & 0 & 2.94 \end{pmatrix}, \end{aligned} \tag{10}$$

where all measurements are of units in kg$\cdot$ m^2. With these inertia tensors, I_1 is the take off position, I_2 is the tuck position, and I_3 is the landing position. In examining these results, it is clear that the inertia tensor of the take off was the greatest where as the inertia tensor of the tuck was the lowest. In comparing these results with the simplified forms, a solid cylinder about its central diameter and a sphere respectively, the moment of inertia of a sphere is indeed smaller than that of a cylinder.

Not only did this shift match the expectations of their simplified forms, but with the assumption that angular momentum is conserved also holds in this case, if the inertia tensor decreases, the angular velocity must increase to compensate. However, in order to find the angular momentum of the body at each body position, the angular velocity of the body must be found. In order to find the angular velocity of the body, the system was simplified to the head and torso.

When doing a somersault, the rotation is judged by the ability of the head and torso system to do a complete rotation. Because of this consideration, to determine the angular velocity of the body, a straight line was drawn from the torso's center of mass to the head's center of mass, going through the body's overall center of mass, which in this case is the origin. Using trigonometry, the angle at which this line lay with respect to the set axes was calculated. With this calculation, the same was done for a frame before and after the frame of the chosen position. Once this was done, the difference in angles was found between the chosen frame and the following frame, as well as the chosen frame and the previous frame. These differences in angle were then divided by the time passed

Table 3: The calculated y-direction inertia tensor, angular velocity of the head torso system and the angular momentum of the whole body.

Body Position	$I_{yy}(m \cdot r^2)$	ω(Rad/s)	$L_y(kg \cdot m^2/s)$
Take off	11.09	2.5	27.9
Tuck	3.9	14.8	57.4
Landing	6.26	-4.0	-24.85

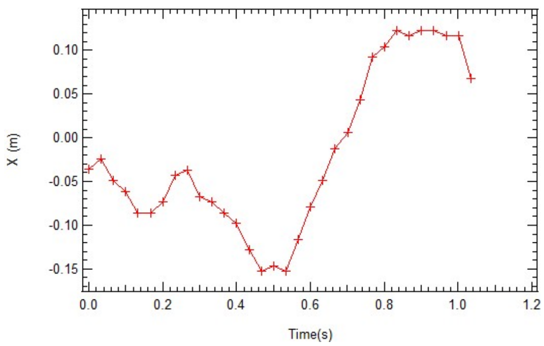

Figure 4: The position of the torso's center of mass in the x-direction as a function of time.

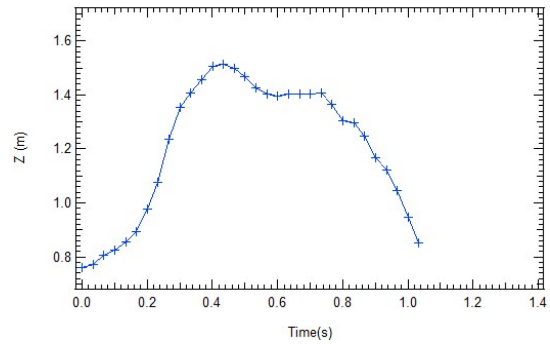

Figure 5: The position of the torso's center of mass in the vertical or z-direction as a function of time.

between frames. The resulting angular velocities were then averaged, giving the results found in Table III.

Because the body rotated about the y-axis exclusively, all of the angular velocity was in the y-direction. Because all of the angular velocity was in the y-direction, when the inertia tensor acts on the angular velocity vector, the only result that will be non zero will be in the y-axis. The y-axis row being the middle row, it was clear that the product between the center value and the angular velocity value would give the angular momentum of the body.

With several specified angular velocities and inertia tensors, Eq. (5) was used to find the angular momentum for the three different body positions, where each is referred to as the take off, tuck, and landing positions respectively. The results from Table III suggest that angular momentum was not conserved. If angular momentum were to have been conserved, these values would have been either closer to one another or exactly the same. In examining the magnitudes of the take off and landing angular momentum, they are within 10% of each other, showing the possibility of conservation. In observing Fig. 4 and Fig. 5, the negative sign of the landing angular momentum is explained by the seeming decrease in the change in both the x and z positions of the body from $t = 0.8$ to $t = 1.0$. However, in examining the angular momentum during the tuck position, the value found was more than double that of the take take off angular momentum and 2.3 times greater than the landing angular momentum.

Because we expect angular momentum to be conserved, the results found were used to investigate the reliability of the tools used. The increase in angular momentum was more than likely due to a multitude of errors in calculating the angular velocities and the position of the center of mass of the body. In calculating the angular velocities, after repeated analysis of the change in the angle of the head body system, the tuck position continuously resulted in an exceedingly high angular velocity while the landing position resulted in a negative velocity, indicating that the body began rotating in the opposite direction. This error in analysis is more than likely the cause of the error in angular momentum but along with this error is the calculation of the center of mass position. Because the human body is a nonrigid object, the configuration of the limbs actively changes the body's overall center of mass. Because the center of mass for each position was considered to be constant, the change in the angle of the head torso system could possibly be incorrect as well. In acknowledging these failures in analysis, a second method of analysis was used. Whereas the previous analysis measured all angular velocities from the center of mass, this method analyzed the rotation of the body relative to the original origin at the athlete's feet. In doing this, more reliable data was hoped to be collected. However, the resulting angular velocities were not only very similar to the initial quantities found, but the angular velocity of the landing position was once again calculated to be negative. With these results, it was concluded that the use of Tracker video analysis would not be appropriate or functional for this type of analysis.

5 Conclusion

The stunts and actions performed by both gymnasts and freerunners are incredibly impressive and complex. In studying the back somersault, performed by both such athletes, the conservation in the body's spin angular momentum was examined. By estimating each limb of the human body to be several cylinders connected at the knees, hips, shoulders and elbows, the center of mass of each limb was determined with Tracker video analysis and was used to find the total body's center of mass at three key points: the take off, tuck or curl of the body at peak height and the landing. With this, all center of mass positions were recalculated relative to the body's center of mass at these three points. From these positions, the inertia tensors were calculated and the Tracker program was used to determine the angular velocity relative to each body position. The resulting angular momentum were $L_1 = 27.89$, $L_2 = 57.409$, $L_3 = 24.85$, all in the y-direction and have units of kg$\cdot m^2$. These values suggest that the angular momentum of the body is not conserved during a somersault. This increase in angular momentum was more than likely due to the functionality of the Tracker video analysis. In using this Tracker program and two separate methods of determining the head torso system's angular velocity, both such methods resulted in unrealistically high angular velocities at the tuck position of the somersault and negative angular velocities during the landing position of the somersault, both of which lead to the non-conservation of momentum.

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