How does tuned mass damper affect vibration with different viscosity and the mass ratio

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We determined how the tuned mass damper's viscosity and damper's mass affect the building's vibration. In the experiment, the main model structure is built of wood and five floors are connected by rubber bands to increase the vibration. The structure is attached to a moveable base to simulate the earthquake effect. The top floor of the building is a container with liquid and a pendulum which work as a tuned mass damper. In the experiment, we varied the mass of the pendulum and the liquid with different viscosity and record the building's motion with a camera.

I. INTRODUCTION

For tall buildings, the shock resistance of the building is one of the most important factors to consider for civil engineers. Earthquakes are the most well known natural shock causes which can be described by transverse waves and longitudinal wave. Longitudinal wave are waves in which the displacement of the medium is in the same direction. Transverse wave's oscillation direction is perpendicular to the wave's traveling direction. In the earthquake, transverse waves would shock the ground in the horizontal direction, and the longitudinal wave travels faster but due to its structure which can raise the ground up and down causes less damage then transverse waves. In another case, the wind can cause the same effects as longitudinal earthquake waves. Inside the metropolis area, there are many skyscrapers which cause a wind tunnel effect and make the wind extra fast relative to the plain area which can create the vortex behind the skyscrapers and make the building vibrate at across-wind side. The wind effect is similar to the longitudinal wave. For the people who spend a long time in tall buildings, even small amplitude vibration can cause motion sickness. The research shows how the constant vibration can cause health issues to people. Therefore, reducing vibration has been an important task for civil engineers for many years. [8]

There are many kinds of damper to reduce vibration of structures like passive, active, semi-active and hybrid system. Passive tuned mass damper means the motion of the damper is induced by the main structure's motion. Active tuned mass damper are coded within algorithm which detects the incoming motion of the building and produce the counter force against the motion. The semi-active tuned mass damper uses the passive tuned mass but able to adjust its damping constant and frequency. The hybrid system uses multiple kinds of tuned mass damper to reduce the vibration. The object we are studying in the experiment is the passive system: pendulum tuned mass damper. Pendulum tuned mass damper consists of a mass and a cable shown in Fig. 1. When the structure starts to vibrate, the pendulum will create a force counter against the motion of the structure which reduces the vibration.

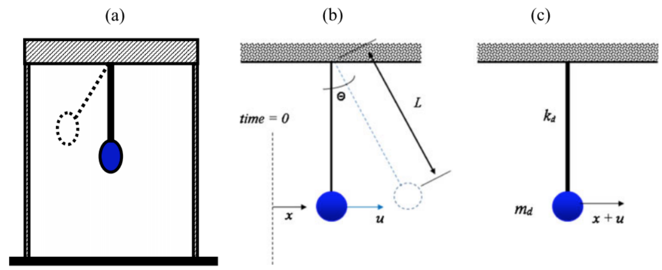

FIG. 1: (a)Pendulum tuned mass damper in an single-degree-of-freedom system, (b)Motion of a pendulum tuned mass damper, (3)Properties of a pendulum tuned mass damper u is the displacement of the pendulum, L is the length of the cable. Image is reproduced from Ref. [1]

The most famous case of using pendulum tuned mass damper is the Taipei 101 tower. The building was officially classified as the world tallest building by 2004.[2] Hurricanes occur often in Taipei which creates strong wind pressure and vortex which vibrates the tower. While building the tower, the top limit of the top floor acceleration is 5 cm/s^2. However, because of the hurricane, Taipei 101 tower's structure under the return period of the wind speed, the top floor acceleration can reach 6.7 cm/s^2. It is necessary to build a tuned mass damper on the tower. The tower's pendulum tuned mass damper is showed in Fig. 2. The whole structure consists of pendulum made up of many layers of blocks for building purpose. The Primary Hydraulic Viscous Damper is acting as a damper of the pendulum which transfers the kinetic energy into heat. The tuned mass damper decreased 40 percent of wind effect on the building.

II. THEORY

A tuned mass damper (TMD) is a device consisting of a mass, a spring, and a damper that is attached to a structure in order to reduce the dynamic response of the structure. "Tuned" means the frequency of the damper is adjusted into the structural frequency so that the structure is vibrating at that frequency, the damper could vibrate at the same frequency, but opposite direction and

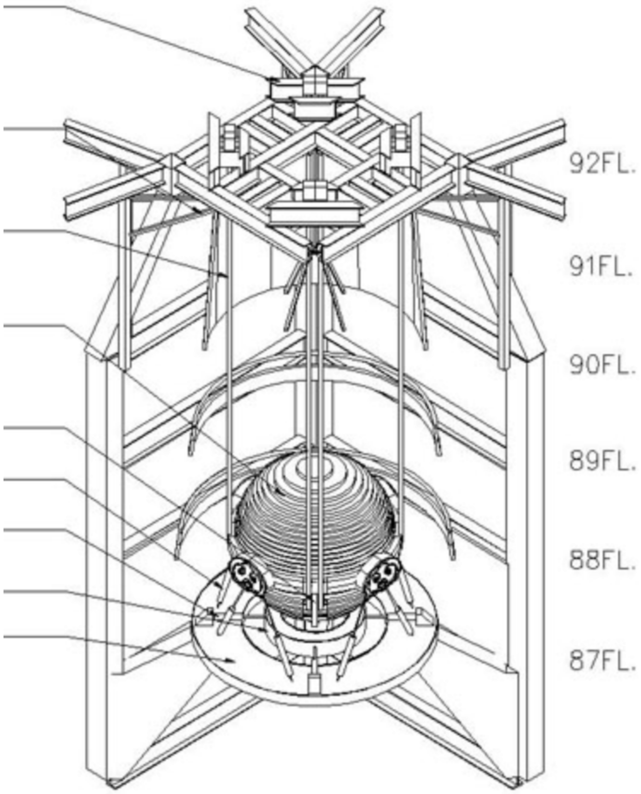

FIG. 2: Taipei 101 tower's pendulum tuned mass damper consist of the pendulum, supporting cables, support beam and Primary Hydraulic Viscous Damper. The figure is reproduced from Ref. [2]

resonate out of phase with the structural motion.[4] In my experiment, the damper I chose is pendulum damper which consists of a large mass block and a cable which attaches the mass block to the building. In reality, the pendulum damper is usually located on the top floor and the mass is usually about 3% of the total structural mass. It also requires a large space for installation thus creating construction constraints.

A. Damper

Simple harmonic oscillation is an ideal situation where no friction occur to slow the oscillation down. In reality, air, liquid and sliding friction on the surface can cause force against the motion.

In the experiment, the applied oscillation is a single degree of freedom which means the structure can only shake on one side. Therefore, we are considering an object in one dimension. For one degree oscillation, we can consider an object attached by a spring. The force applied by the spring is $-kx$ and the resistive force is $-b\dot{x}$. Based on Newton's second law, the motion can be expressed as:

$$m\ddot{x} + b\dot{x} + kx = 0 \tag{1}$$

In the equation, k is the spring constant, m is the mass of the object and b is the resistivity. It describes the single degree damped oscillation. In order make the resistivity clear and because the resistance is proportional to the mass of the object, damping constant is used here. Damping constant measures how strong the friction force is. It is defined to be:

$$2\beta = \frac{b}{m}. \tag{2}$$

Another important concept for the oscillation is the natural frequency which means the frequency of the oscillation where no friction exists. The natural frequency is defined as:

$$\omega_0 = \sqrt{\frac{k}{m}}. \tag{3}$$

By substitute the natural frequency and damping constant into Eq. 1, we can determine:

$$\ddot{x} + 2\beta\dot{x} + {\omega_0}^2 x = 0. \tag{4}$$

This equation is a second-order, linear, homogenous equation. Therefore, we can find a solution using this form:

$$x(t) = e^{rt}. \tag{5}$$

By substituting Eq. [4], we see that Eq. [5] satifies only if

$$r^2 + 2\beta r + \omega_0^2 = 0. \tag{6}$$

The solution to this equation is:

$$r = -\beta \pm \sqrt{\beta^2 - \omega_0^2}. \tag{7}$$

This determines the two solutions for r. We can solve fpr x, c to obtain :

$$x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}. \tag{8}$$

By substituting in the two solutions for r, the equation becomes:

$$x(t) = C_1 e^{(-\beta + \sqrt{\beta^2 - \omega_0^2})t} + C_2 e^{(-\beta - \sqrt{\beta^2 - \omega_0^2})t}. \tag{9}$$

It is hard to understand the equation by just staring at this form, but if we make the damping constant into 0. The Eq. [9] becomes

$$x(t) = C_1 e^{i\omega_0 t} + C_2 e^{-i\omega_0 t}. \tag{10}$$

The Eq.[10] is just the solution for the simple harmonic wave. There are many different kinds of situations where the damping constant can be equal to the natural frequency or much larger than the natural frequency. In my experiment, the pendulum damper's weight is much smaller than the whole structural weight. Therefore, we are considering the weak damping, also called it under-damping.

B. Tuned Mass Damper

The reason people call it tuned mass damper is because it is tuned to the particular frequency which fits the structures natural frequency. Therefore it can counter the vibration when the natural frequency happens. It has been a long lasting task for engineering to prevent the building from vibrating into its natural frequency. There is a famous case called Tacoma Narrows Bridge, which was built in 1940 and collapsed in late 1940. Fig. 3 is the original picture of the Tacoma Narrows Bridge. In many engineering textbooks, Tacoma Narrows Bridge is the classical example of destruction caused by resonance. The high-speed wind caused aeroelastic flutter which is a dynamic instability of an elastic structure in a fluid flow, caused by "positive feedback between the body's deflection and the force exerted by the fluid flow."[6] When the flutter caused frequency matches the natural frequency of the bridge, it can produce resonance which has much higher amplitude than other frequency's oscillation. Fig. 4 shows how much the amplitude of the oscillation can increase when the resonance happens. Therefore, Tuned mass damper is a good solution to reduce the amplitude of the structure when the resonance effect occurs.

In Fig. 5, the figure gives the illustration of a single-degree-of-freedom tuned mass damper. In the figure, k and k_d are the stiffness of the main structure and damper which determines how much deformation a certain force can do to the structure, c and c_d are the damping coefficient of the main structure and damper. In the figure, m and m_d are the mass of the main structure and the damper. The displacements of the main structure and damper are u and u_d.

The frequency of the main structure and the dampers are ω and ω_d:

$$\omega^2 = \frac{k}{m}. \tag{11}$$

$$\omega_d^2 = \frac{k_d}{m_d}. \tag{12}$$

The damping coefficient for main structure and damper can be expressed as:

$$c = 2\xi\omega m. \tag{13}$$

$$c_d = 2\xi_d\omega_d m_d. \tag{14}$$

In the equations, ξ is the damping ratio of the structure. Damping ratio is the ratio between the damping coefficient and the critical damping. It is dimensionless which helps engineers to compare different structures' damping property.

Based on Newton's second law, the primary system's motion can be expressed:

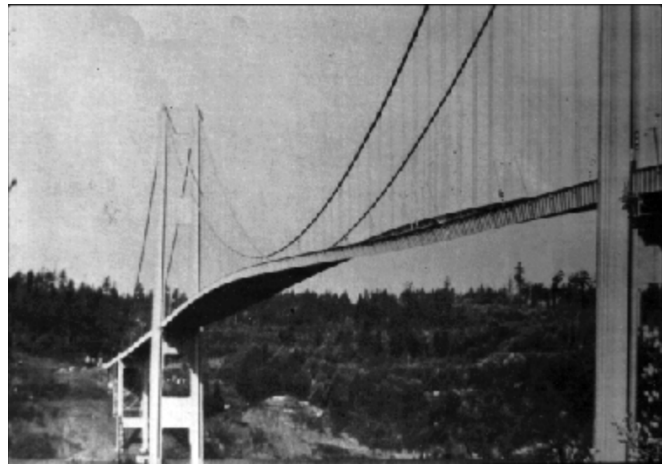

FIG. 3: The original Tacoma Narrows bridge. The figure is reproduced from Ref. [5]

$$(1+\bar{m})\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = \frac{p}{m} - \bar{m}\ddot{u}_d. \tag{15}$$

The tuned mass damper's motion is determined by:

$$\ddot{u}_d + 2\xi_d\omega_d\dot{u}_d + \omega_d^2 u_d = -\ddot{u}. \tag{16}$$

In the Eq. [15],$\bar{m}$ is the mass ratio between damper's mass and the main structure's mass $\bar{m} = \frac{m_d}{m}$. The purpose of adding the tuned mass damper on the structure is to limit the motion of the structure when it is subjected to particular excitation. The natural frequency of the building ω needs to be the same as ω_d. In order to reach the goal, Eq. [13] shows that specific mass m_d, stiffness k_d and damping ratio ξ_d.

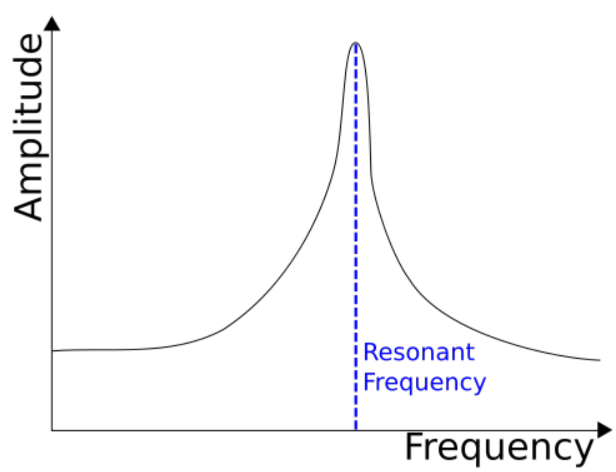

FIG. 4: The figure shows when the driven frequency matches the natural frequency, the resonance can cause high amplitude oscillation. The figure is reproduced from Ref. [6]

C. Illustration of the model

The model I built is a four-floor building with a pendulum damper on the top floor. Each floor is connected by the rubber band, therefore the floors are movable. The bottom floor is connected to the base which is tied to a rotor. During the experiment, the rotor produces simple harmonic waves. When the building is being shaken, each floor would wobble side to side. The resonance occurs when the frequency of wobble floors matches the frequency of the driven base. As the top floor wobble to the right side, the base is moving toward the left side. The force produced by the top floor hitting the bottom is much greater which can explain why the resonance can be so destructive. The illustration of the model is shown in Fig. 6

III. PROCEDURE

A. Building The Main Structure

My main structure is made out of the wood board. The wood board is 3 ± 0.5 mm thick. The first step, I cut three strips of wood with 4 inch width and 24 inch length. The board was not flat, it was a bit curved. Therefore, I pinned many wooden blocks between each wall to hold the board and also leave space for nails which is going to be used to hang rubber bands. In the end, I cut the structure into 4 floors and put spacers between each floor at the horizontal direction, so when I apply the vibration, it does not vibrate sideways.

During the experiment, the main structure is connected to a platform that oscillates by a rotor. The rotor is powered by an adjustable power source which can adjust the frequency of the oscillation. For the length of the rod used to connect the platform can be adjusted to change the amplitude of the wave. Fig. 7 shows the schematic of the whole structure.

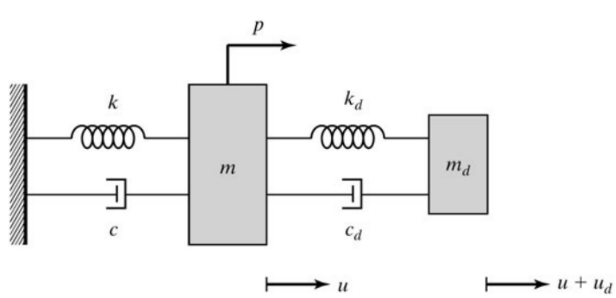

FIG. 5: The figure gives an illustration of a single-degree-of-freedom tuned mass damper. The figure is reproduced from Ref. [4]

B. Data Collection

The goal of the experiment is to find out how the pendulum damper would work to reduce the amplitude of the main structure's vibration with different weights of damper within different viscosity liquids. I placed a LED light on the top of the building to increase the displacement and record the motion of the LED light by a camera. I used a program "Tracker" to analyze the motion of the LED light. During the experiment, I used water and detergent as the liquid and used 100 grams and 50 grams weight blocks acting as a pendulum. The weight block is tied by strings and the other end of the string goes through the cap of the container. During each trial, I recorded the displacement of the LED light the horizontal direction which is approximate a sinusoidal wave. To compare the amplitude of the waves with different

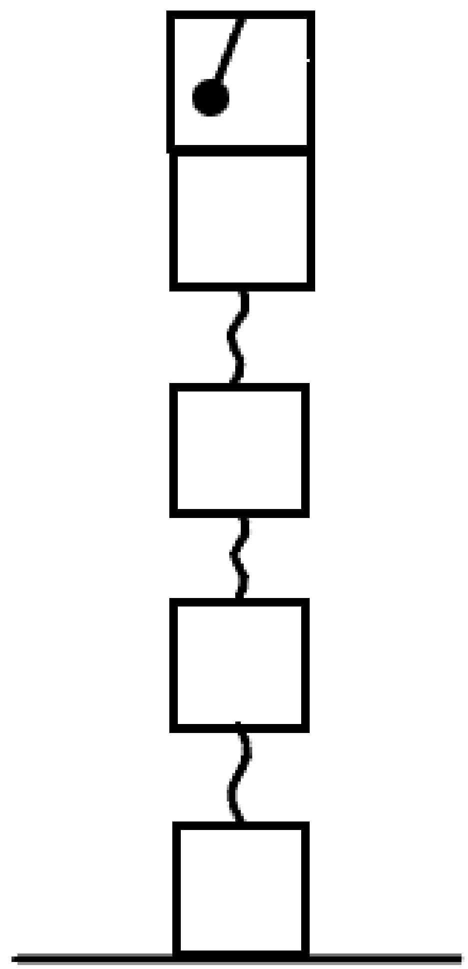

FIG. 6: The figure gives illustration of an single-degree-of-freedom tuned mass damper

TABLE I: Trials with different damper's masses and liquids

	Mass of the damper(g)	Liquid
Trial 1	50	water
Trial 2	50	detergent
Trial 3	100	detergent

liquids and pendulums, I used the standard deviation of the waves.

IV. ANALYSIS

In the experiment, the data I collected is the displacement of the LED light. I have done three different trials. The specific combination of liquid and damper is showed in Table. [1].

At the beginning of the experiment, I was expecting the 50 grams of pendulum within water can cause a good damping effect during the oscillation. Fig. 8 shows the displacement of the LED light under different damping

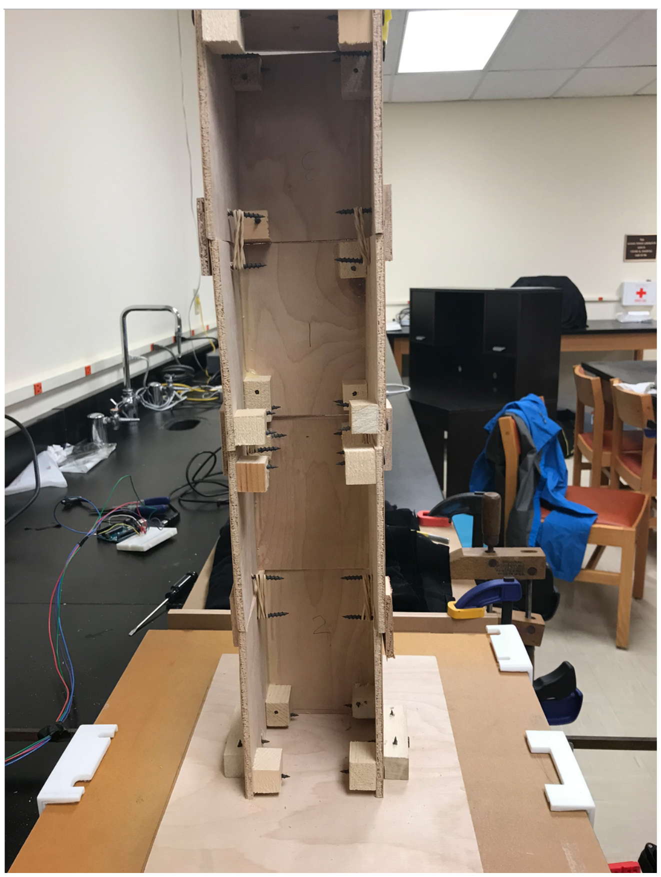

FIG. 7: The picture of the apparatus taken from the left side of the main structure. The figure illustrates how the building is built by three pieces of wood and connected by the wooden blocks with nails. Each floor is connected by rubber bands to increase vibration.

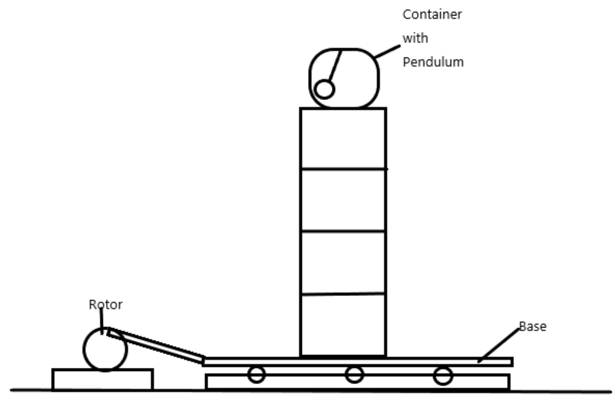

FIG. 8: The rod connects the platform and the rotor. The three wheels are within the tracks that are carved into the base wood plate. The platform is placed on the wheels to reduce the friction. The main structure is screwed on the platform.

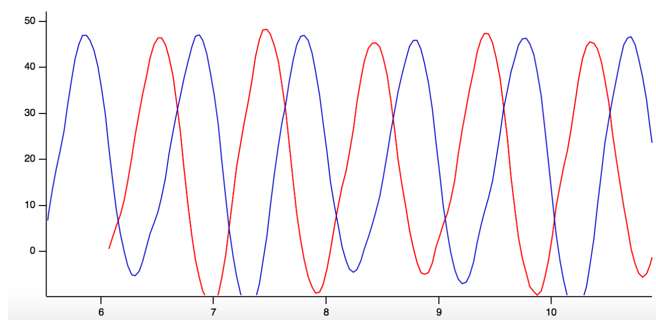

FIG. 9: The blue curve represents the displacement of the LED light with the 50 grams pendulum free, the red curve represents the displacement of the LED light with the 50 grams pendulum fixed

situations with the same output by the rotor at 7.5 A of current.

From the graph, there is nearly no difference in the amplitude of the waves. Both waves' standard deviation and the difference between the minimum point and maximum point of the curve are about the same which shows the damper does not work. During the trials, I was able to observe the motion of the pendulum, the pendulum swings along the building which means it is not producing a counter force against a building's motion.

During the second trial, the liquid I chose is detergent and 50 grams pendulum. The damping effect can be observed on the graph and when I turned the power source from 7.5 A to 8 A which increases the rotor's output and increased the frequency of the shaken wave, the structure with the pendulum free is able to withstand the oscillation, the structure with the pendulum fixed collapses. Fig. 9 shows the displacement of the LED light for this trial. Table. [2] shows the standard deviation of curves

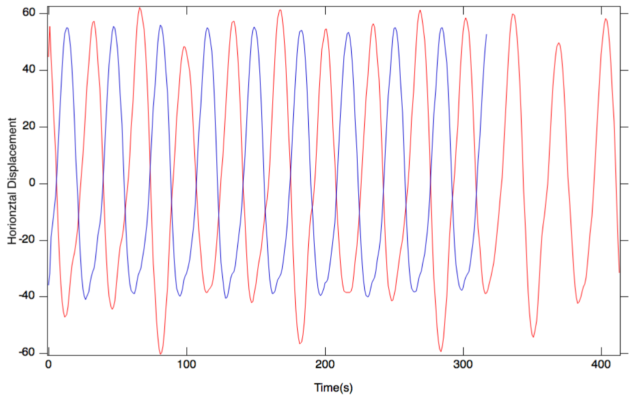

FIG. 10: One trial of data with the blue curve represents the displacement of the LED light with the 50 grams pendulum free, the red curve represents the displacement of the LED light with the 50 grams pendulum fixed. The amplitude(displacement) of the vibration on the top of the building is reduced by the damper.

TABLE II: Standard Deviation For Pendulum Free And Pendulum Fixed Trials

	Sdev Pendulum free	Sdev Pendulum fixed
Trial1	33.7	36.4
Trial2	35.1	36.2
Trial3	34.6	35.9
Average	34.4	36.2

with three trials for damper and without a damper.

The standard deviation of the pendulum free trial shows 4.9% of reduction in the displacement. The standard deviation is not convincing about the damper's effect and one part of the reason is the total displacement for half period is about 90 units, but the reduced vibration for the building itself might be more prominent relative to the whole wave's length. Another important factor is the difference between a minimum point and maximum point for the curves. Table. 3 and Table. 4 shows the maximum and minimum point of the curves. The data points are presented in Table. 3 and Table. 4 are the horizontal direction coordinates on the tracker program.

From the minimum point and maximum point tables, the data shows 13.0% reduction in vibration.

During the third trial, I used 100 grams pendulum with detergent. Fig. 10 shows the curves for two different

TABLE III: Min And Max Point For Pendulum Free Trials

Pendulum Free	Max	Min	Difference
Trial 1	55.8	-41.1	96.9
Trial 2	58.2	-50.0	108.2
Trial 3	58.9	-52.6	111.5
Average	57.6	47.9	105.5

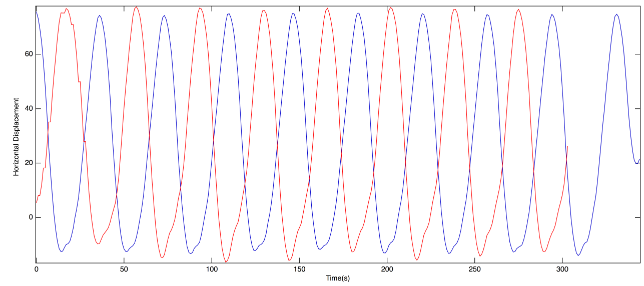

FIG. 11: The blue curve represents the displacement of the LED light with the 50 grams pendulum free, the red curve represents the displacement of the LED light with the 50 grams pendulum fixed

TABLE IV: Min And Max Point For Pendulum Fixed Trials

Pendulum Fixed	Max	Min	Difference
Trial 1	62.2	-60.4	122.6
Trial 2	59.2	-56.0	115.2
Trial 3	61.8	-64.4	126.2
Average	61.0	60.3	121.3

conditions with pendulum free and pendulum fixed. By comparing the sdev of both curves and minimum and maximum points, the 100 grams pendulum does not help the structure's vibration.

In the theory section, the Eq. [15] shows the main structure's motion is related to the damper's mass, the stiffness of the damper and damping ratio of the damper. However, in the experiment, the structure's stiffness and damping ratio are hard to find. Between each floor, they are tied by rubber bands which change their elasticity over time. My data shows how wrong damping ratio and wrong mass ratio could cause the damper to work less effectively.

V. CONCLUSION

In the experiment, I test three different dampers which are water with 50 grams pendulum, detergent with 50 grams pendulum and detergent with 100 grams pendulum. The detergent with 50 grams pendulum works best with 13% vibration reduction. The failure of the other two combinations could be due to the wrong mass ratio and not enough damping ratio. In conclusion, the right damping ratio, mass ratio, and stiffness are necessary to reduce structural damping. The natural frequency of the building is another important factor in order to damp out the resonance where the complex structure has complex resonance condition with multiple peaks.

In order to make the experiment more effectively, it is important to choose the right material. In my experiment, I choose the wood as my main structure. However, the wood itself is not flat and the idea of using rubber bands make the experiment inaccurate due to rubber bands losing its elasticity over time and their elasticity are not equal. Using plastic tubes acting as main structural with constant stiffness can help make the experiment more accurate and able to use the theory we discussed. Another improvement is using motion sensor can help improve experiment's accuracy. The camera's frame capturing speed is limited which makes the analysis not as accurate as a motion sensor. The last improvement is using 3D printing to make a better-sealed container, so the motion of the liquid would not affect the experiment. Because the moving liquid can be another damper.

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