Conserving angular momentum of two spinning disks before and after collision

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This experiments goal was to show that angular momentum is conserved in a system. When a weighted plate is dropped on a spinning disk the momentum of the original plate and the two after will be the same. Because the inertia changes from the increased weight and the speed decreases, momentum will then change proportionally. From this experiment the moment of inertia was found to be $(9.26 \pm 0.01) \times 10^{-3}$ kg m s^{-1}, with a 23% error from the literature value of 7.5×10^{-3} kg m s^{-1} for the disk. Using the inertia we can find the momentum of the disk from the speed of the spinning disk. The momentum before and after a collision were recored and compared to each other, a 6.24% difference was found between the momentums of the apparatus. This is shows that momentum is conserved and if not for friction and other factors there would be no loss.

INTRODUCTION

In 1918, Amalie Emmy Noether published a paper describing the relationships and symmetry of laws of conversations. Noether′s Theorem is a useful mathematical expression that finally showed how conservation of energy, linear momentum, angular momentum, and electric charge relate to each other[1]. This lab will focus on the conservation of momentum and proving it with a spinning disk and counter weights. With the conservation of momentum, many things can be proven and it is important to many experiments. Conservation of angular momentum is a useful property used in many different things such as figure skating, tops, and spinning stars. It is integral to many different research projects with spinning systems.

THEORY

The conservation of angular momentum states that momentum can be exchanged between objects, but the total momentum of the system remains the same[3]. When a torque is applied on matter in a closed system, an equal and opposite torque is applied on some other matter in the system. Using the rotational form of Newtons second law, the torque τ and the angular momentum L can be related by the time derivative of the momentum. When the motion is about a fixed axis

$$\frac{d\vec{L}}{dt} = \vec{\tau} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt}, \tag{1}$$

and with these expressions the moment of inertia and can be found. To find the moment of inertia, the torque and the angular speed are needed, where the torque is

$$\vec{\tau} = \vec{r}_s \times \vec{F}. \tag{2}$$

Using the Eqns. (1) and (2), an experiment can be done where the speed of an accelerating rotating disk is measured. The torque is being exerted by the falling mass over the pulley and onto the spindle, of radius r_s where the string is wrapped around. Yet the total torque will be over the whole disk, of radius r_d, so the total expression for momentum of inertia is

$$I = \frac{(\vec{r}_s \times \vec{F})\vec{r}_d}{\vec{\alpha}}, \tag{3}$$

where F is the force applied on the disk from the falling mass and α is the acceleration of the disk. Once the moment of inertia is found it can be used to prove that total momentum is conserved. The angular momentum is defined as

$$\vec{L} = \int \vec{r} \times \vec{v} dm = I\vec{\omega}. \tag{4}$$

The equation for the momentum, after a collision, will be the same as Eqn (4), but the moment of inertia will be the addition of both disks on the apparatus. Once the momentum is found before and after they are compared. If it was a perfect system there would be no energy is lost in the collision.

PROCEDURE

To test the conservation of angular momentum, the moment of inertia first needs to be found. A disk was set up with a spindle that a string could be wound on. A weight of 250 g was hung on the string and pulled over a pulley to decrease friction and have a constant force spinning the disk. Figure 1 shows the apparatus and Pasco interface used to record the data. Figure 2 shows a diagram of the apparatus and labeling the parts. To find the amount of friction on the disk and pulley system, the apparatus was allowed to spin with more and more weight until the velocity never increased or decreased while spinning. When the weight was released and allowed to fall it would spin the disk with a constant downward force. A smart pulley with a photogate was held tightly to the

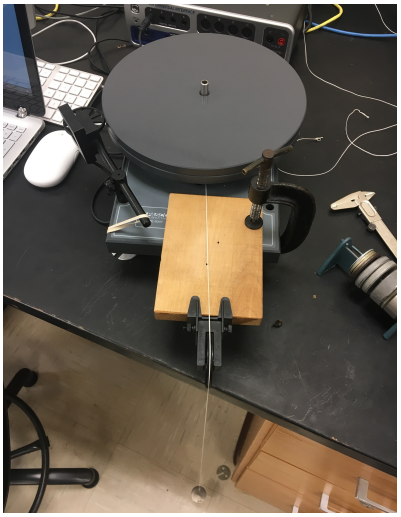

FIG. 1: Conservation of momentum spinning disk apparatus

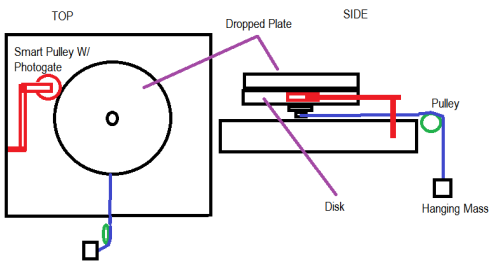

FIG. 2: Diagram of the apparatus

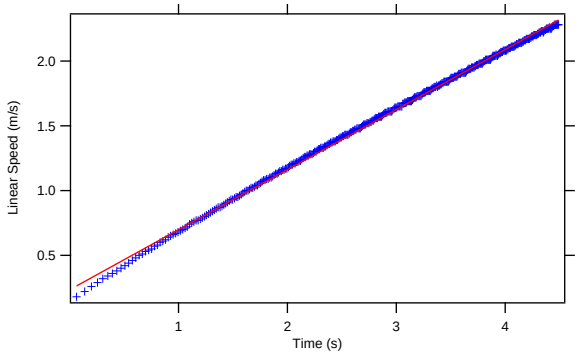

FIG. 3: Linear speed versus time plot with a linear fit line. The slope of the this graph will yield the acceleration of the system which is then used to find the moment of inertia.

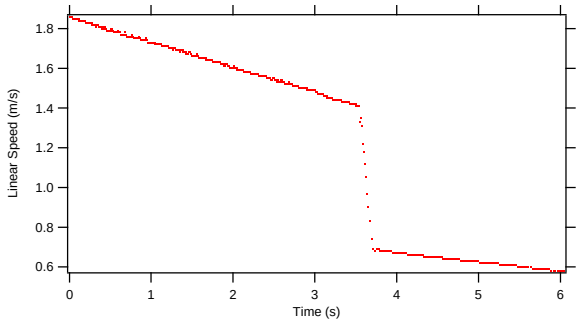

FIG. 4: Linear speed versus time plot. The dramatic drop in velocity corresponds to the collision of the disks and how momentum will be conserved before and after the collision.

disk so it rotated at the same rate, this was used to measure the velocity and time of the disk. Using the speed and time of the disk the acceleration can be found from the slope of velocity versus time graph. Next, the disk was spun and allowed to spin freely, then another disk was dropped on top of it to test the momentum before and after. This quickly changes the moment of inertia and speed of the system, but they should change proportionally to each other. This was used to show that momentum is the same before and after the collision.

RESULTS AND ANALYSIS

The first part of the experiment was looking at the moment of inertia of the spinning disk. To start, the friction offset was needed to be determined. It was found that 20 g of weight was needed to keep the disk rotating at a constant velocity, so for every test the amount of weight used in experiment had 20 g of it being lost to friction. An example of a plot with the velocity and time of the accelerating disk with a hanging mass is seen in Figure 3. The total moment of inertia was found to be $(9.26 \pm 0.01) \times 10^{-3}$ kg *ms^{-1} with a 23% error from the literature value of 7.5×10^{-3} kg *ms^{-1}. With this information, the momentum can be found when a collision happens. The moment of inertia of the plate that will be dropped on the disk was found to be $(2.06 \pm 0.01) \times 10^{2}$ kg *ms^{-1}.

To prove that angular momentum is conserved a disk was allowed to spin without any force acting on it. The momentums just before and after the collisions were recoded using the data from just before and after the crash. The last data point that is still in the linear fit of the line before the initial drop off was used for the inertia before, and the first data point after the steep slope that starts the linear fit was used for the inertia after. The values of the momentum are found in Table 1. The velocity versus time graph of the collision is shown in Fig. 4

TABLE I: The momentum values of the system before and after the weighted plate is dropped on.

Run	L before (kg m s^{-1})	L after (kg m s^{-1})	Percent different
1	0.0111	0.0115	3.5 %
2	0.0123	0.0128	1.6 %
3	0.0129	0.0138	13%
4	0.00902	0.00948	5.0%
5	0.0131	0.0142	8.1%

From the results, one can see that the momentum is conserved and the momentum on average changed 6.24% from the crash. The momentum after was shown to be slightly higher on average. I do not know why this happened, the mass of the plate could have been lower than reported so the inertia increased. If the system had no outside variables and the disk fell perfectly so it did not bounce or slide when it fell, then the momentum would have no change.

Error

The error in this lab occurs because of the apparatus and friction acting on it. Accounting for the friction during the moment of inertia, data could only be done in increments of certain sizes and the exact amount needed might not have been possible to achieve. This may have been the reason why the moment of inertia could have been off by 23%. The velocity of the disk and time were measured accurately with the photogate with low error. The largest error, and the most likely reason the momentum varied so much between runs, was how the plate fell on the disk and losing energy and not instantly sticking. The apparatus was also not exactly level, so I attempted to level it as much as possible and got it close, but not perfect.

CONCLUSION

From the results of finding the moment of inertia, we can see that this is experiment is not perfect and the results will deviate from the theory. Looking at the moment of inertia we found that the main disk had a value of $(9.26 \pm 0.01) \times 10^{-3}\ kg * ms^{-}1$ with a 23% error, and $(2.06 \pm 0.001) \times 10^{-3}\ kg * ms^{-}1$ for the plate that was dropped on the disk. The deviation from the literature value is mostly likely from friction and not having a perfectly level apparatus. When looking at the conservation of angular momentum, we can see that in reality its difficult to show without loss of energy. On average momentum changed by 6.24% before and after the collision of the disks. Despite the fact that this is a change, it is close enough that the error in the lab can explain the deviation. Thus we can see that momentum in a system is conserved.

ACKNOWLEDGMENTS

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[1] "Emmy Noether: German Mathematican", Encyclopeaedia Britannica, accessed March 7, 2018 https://www.britannica.com/biography/Emmy-Noether

[2] Physics 401 Junior Independent Study manual *The College of Wooster Physics Department*, 90, (2018).

[3] "Angular Momentum" Wikipedia, accessed March 7, 2018, https://en.wikipedia.org/wiki/ Angularmomentum-Conservation of angular momentum