A Quantum Paradox: The Double Slit Experiment

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In this study, the double slit experiment was performed to observe the wave-like nature of photons. To test the wave-like nature of photons, both a red laser and dimmed green bulb were used as light sources. After passing the light sources through the double slit, both exhibited an interference pattern in their intensities that is characteristic of waves passing through a double slit. The laser exhibiting the interference pattern means that photons can interfere with each other like waves. The integrated count rate was used to determine the time in between each photon traveling from the bulb to the detector. This time was found to be five orders of magnitude greater than the time it takes for a photon to travel the distance from the bulb to the detector. This suggests a low probability that more than photon will ever be in the path at once. Therefore, the interference pattern observed is due to photons interfering with themselves.

I. INTRODUCTION

In 1804, Thomas Young published *Experiments and Calculations Relative to Physical Optics* in which he discussed his observations of light exhibiting constructive and destructive interference. He concluded that the nature of light must resemble the nature of sound since both have similar behavior [1] . The conclusion made determined that light behaves as a wave rather than a particle. However, in 1905 Albert Einstein published *Concerning an Heuristic Point of View Toward the Emission and Transformation of Light* in which he concluded that light is quantized through the photoelectric effect [2]. This suggests that light also behaves as a particle.

The intent of this study is to observe the wave-like characteristics of light. This was done by passing light through a double slit and analyzing the pattern in the intensities that result from the photons passing through the double slit.

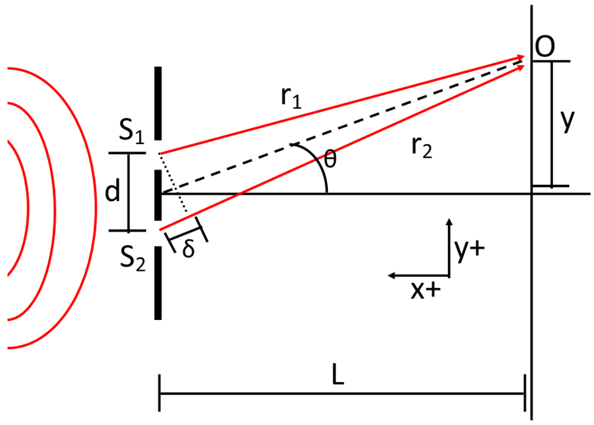

FIG. 1: Geometric diagram of light passing through a double slit (variables defined in derivation, adapted from MIT derivation [3])

II. THEORY

The behavior of light when passing through a double slit can be described through equations of the intensity of the light after passing through the double slit. To determine the intensity, the total Poynting flux, S, for the point of interest is needed and has the proportionality

$$S \propto E^2 = (\vec{E}_1 + \vec{E}_2)^2 = E_1^2 + E_2^2 + 2\vec{E}_1 \cdot \vec{E}_2, \qquad (1)$$

in which E is electric field and $\vec{E}_1$ and $\vec{E}_2$ are the electric fields from slits 1 and 2 respectively. To obtain the intensity at the point of interest, we take the time average of the Poynting flux, giving us the equation

$$I = \langle S \rangle \propto \langle E_1^2 \rangle + \langle E_2^2 \rangle + 2\langle \vec{E}_1 \cdot \vec{E}_2 \rangle. \qquad (2)$$

To take the time average of a function $f(t)$, the integral

$$\langle f(t) \rangle = \frac{1}{\Delta T} \int_t^{t+\Delta T} f(t')dt' \qquad (3)$$

is evaluated, in which ΔT is the time interval of interest. In Eq. 2, $2\langle \vec{E}_1 \cdot \vec{E}_2 \rangle$ is known as the interference term. This term makes maxima of a double slit intensity pattern have a greater magnitude than either of the single slit intensities at the same position. To determine a function for I containing easily measurable values, expressions for $\vec{E}_1$ and $\vec{E}_2$ are needed. For the purposes of this derivation, the origin is declared to be the point of interest labeled O in Fig. 1. This eliminates the spatial "kx" term from the traditional wave equation for an electric field and leaves the equations

$$\vec{E}_1 = E_\circ \sin(\omega t) \qquad (4)$$

and

$$\vec{E}_2 = E_\circ \sin(\omega t + \phi), \qquad (5)$$

in which $E_\circ$ is the amplitude of the electric field, ω is the frequency, t is time, and ϕ is the phase shift between $\vec{E}_1$ and $\vec{E}_2$ due to $\vec{E}_2$ having a longer path to traverse. With equations for $\vec{E}_1$ and $\vec{E}_2$, the total electric field E becomes

$$E = \vec{E_1} + \vec{E_2} = E_\circ(\sin(\omega t) + \sin(\omega t + \phi)) \\ = 2E_\circ \cos\left(\frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right) \tag{6}$$

after several steps of algebra and a trigonometric identity. Substituting the equation for E into the equation for intensity in Eq. 2 and the equation for the Poynting flux in Eq. 1, the intensity becomes

$$I \propto \langle E^2 \rangle = 4E_\circ^2 \cos^2\left(\frac{\phi}{2}\right) \left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right) \right\rangle \\ = 2E_\circ^2 \cos^2\left(\frac{\phi}{2}\right). \tag{7}$$

To obtain constructive interference, a phase difference of $\phi = 2\pi$ would correspond to a path difference of $\delta = \lambda$, where δ is the difference in distance the two electric fields have to travel, shown in Fig. 1, and λ is the wavelength of the light. This correspondence gives us the relation

$$\frac{\delta}{\lambda} = \frac{\phi}{2\pi}. \tag{8}$$

The aprroximation for δ,

$$\delta = r_2 - r_1 \approx d\sin(\theta), \tag{9}$$

can be made if the distance between the slits and detector, L in Fig. 1, is significantly greater than the distance between the two slits, d. Using this approximation, ϕ becomes

$$\phi = \frac{2\pi d}{\lambda}\sin(\theta). \tag{10}$$

This can be substituted into Eq. 7 and the coefficients can be combined into one constant $I_\circ$, making the equation for the interference pattern for light passing through a double slit

$$I = I_\circ \cos^2\left(\frac{\pi d \sin(\theta)}{\lambda}\right). \tag{11}$$

Through a similar derivation, the intensity patter for light passing through a single slit is found to be

$$I = I_\circ \left(\frac{\sin(\alpha)}{\alpha}\right)^2, \tag{12}$$

in which

$$\alpha = \frac{\pi a}{\lambda}\sin(\theta) \tag{13}$$

and a is the width of the slit [3]. To obtain the final equation for the double slit intensity pattern, equations 11 and 12 are multiplied together for the equation

$$I = I_\circ \left(\frac{\sin(\alpha)}{\alpha}\right)^2 \cos^2\left(\frac{\pi d \sin(\theta)}{\lambda}\right). \tag{14}$$

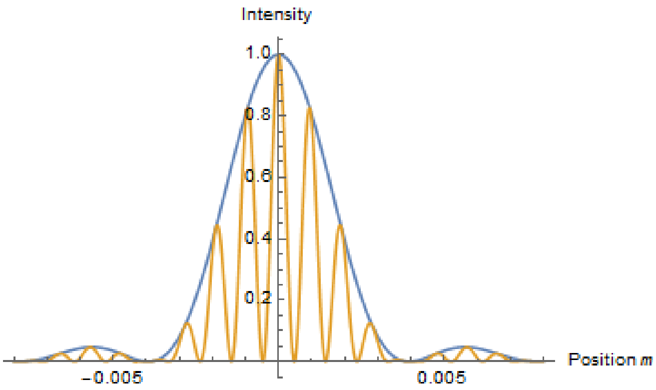

FIG. 2: Predicted intensity patterns for light passing through a single slit (blue) and a double slit (yellow) for an $I_\circ$ value of 1, x-axis being position and y-axis being intensity

Eq. 14 is plotted along with Eq. 12 in Fig. 2 as a comparison of the double slit pattern to the single slit pattern for identical parameters in both equations. This plot shows that the double slit intensity has an interference pattern due to its equation's cosine term, but is constrained by the single slit pattern term its equation contains.

III. EXPERIMENTAL

A. Apparatus

The interfaces shown in Fig. 3 was used for measuring the single slit and double slit intensities. The photodiode on the photon counting module was connected to the multimeter. The signal from the photodiode to the multimeter was read as a voltage on the multimeter. To measure the intensity pattern of the single slit, the photomultiplier was connected to the oscilloscope and pulse counter and sent signals in the form of a voltage. The pulse counter read voltages above the set voltage for discrimination as a pulse. Control of the slits which the light passed through was done using the components of the apparatus light path shown in Fig. 4. The path was used to control what position of the intensity pattern of the light reached the detector and whether the light passed through a double slit or single slit.

B. Data Acquisition

The shutter to expose the detector to the photomultiplier tube was closed until later noted, allowing light to reach the photodiode. Also, the high voltage dial was set to zero until later noted. The path cover was removed and the laser turned on. A white notecard was placed in the path between the detector and the double slit. The notecard was removed and the

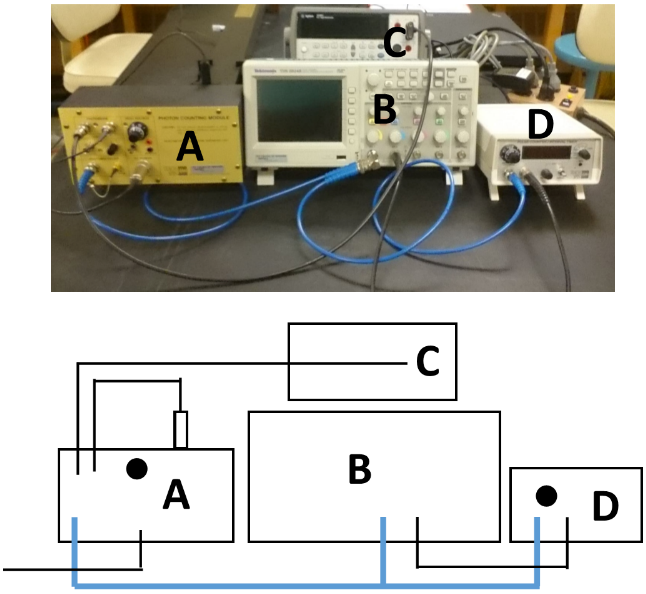

FIG. 3: Apparatus Interfaces A: Photon Counting Module, B: Oscilloscope, C: Multimeter, D: Pulse Counter

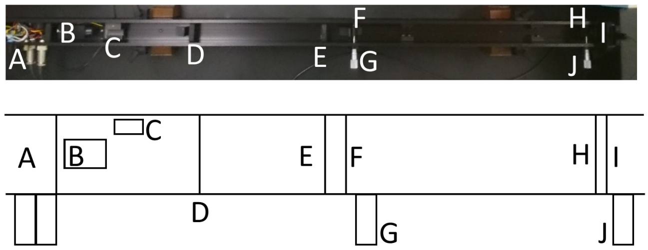

FIG. 4: Apparatus Light Path A: laser and bulb controls and power sources attached, B: bulb with green filter, C: laser, D: single slit for spatially selecting light to proceed to the double slit, E: double slit, F: slit blocker, G: slit blocker micrometer, H: detector slit, I: detector, J: detector slit micrometer

path was closed and locked. The multimeter was turned on and set to read up to 10 V DC. The theory predicts that the intensity of a double slit interference pattern at its central maximum intensity will be greater than double either of the individual single slit intensities at the same position of the pattern. This is predicted by the interference term in Eq. 2. The central maximum of the double slit intensity pattern for the laser was found and the single slit intensities were measured for the same position of the detector slit.

Intensity readings were taken on each side of the central maximum of the double slit and single slit intensities for the laser at intervals of 0.05 mm and 0.1 mm respectively. Those measurements were taken to observe the pattern that occurs in the intensity of both slit configurations. The shutter to the photomultiplier was opened to use the dimmed bulb with a green filter as a light source. The oscilloscope and pulse counter were used to detect individual photons as pulses. Counts were taken in a 10 second time frame. Having both slits unblocked, the central maximum pulse count was found and intensity reading were taken on each side at intervals of 0.1 mm to observe the intensity pattern.

IV. RESULTS AND ANALYSIS

The results of the test to show the intensity of the light change from the central max of the double slit intensity to the single slit intensities for the same detector position are shown in Table I. Even though the detector slit remained in the same position for each intensity reading, the single slit intensities are less than half the intensity of the double slit. The intensities of the single slits do not simply add together, as seen in Eq. 1 and Eq. 2. This shows the interference term causing the intensity to more than double when going from a single slit to the double slit for the same detector position.

TABLE I: Double slit central max test

Slit	Intensity (V)
Double	3.917±0.001
A	0.8975±0.0001
B	1.0852±0.0001

For each of the three intensity patterns recorded, plots of the data as well as plots of the theoretical pattern on top of the data are shown. The data plot of the single slit intensity pattern in Fig. 5 has the characteristics expected of a wave passing through a single slit from the theory. To test the theory farther, a predicted plot of the pattern was created in Fig. 6 using the specifications given for the apparatus [4]. The predicted plot was overlaid the data obtained with the central maximum normalized. The theory appears to predict the data well with the data curve being just slightly broader than the predicted curve.

The data plot of the double slit intensity pattern in Fig. 7 has the interference and constrained characteristics predicted in the theory. This behavior shows light behaving as a wave. To farther test the theory, a manipulate plot of the theoretical equation for the double slit intensity in Fig. 8 was made in Mathematica and was plotted over the data. The manipulatable value used was the slit width, the rest of the values were those provided. The value was manipulated until the predicted plot best fit the data. The data fit best with the value for the slit width set to 9.0×10^{-5} m, which has a percent error of 6 % to the provided value of 8.5×10^{-5} m.

The intensity pattern from the bulb through the double slit also exhibited the interference pattern expected for a wave seen in Fig. 9. Since the light from the bulb was filtered to green light, the scale is more compressed than that of the red laser due to green light having a higher wavelength. While the intensity pattern in Fig. 9

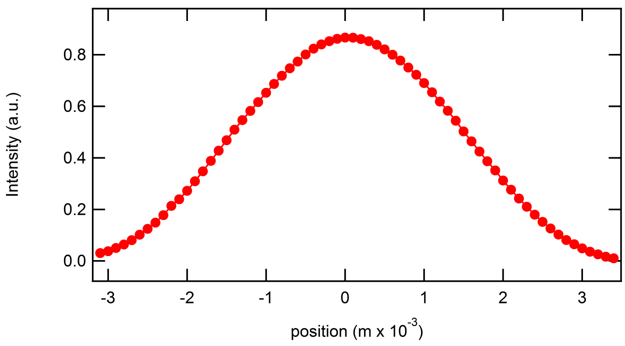

FIG. 5: Single slit intensity pattern from a red laser, experimental intensity pattern

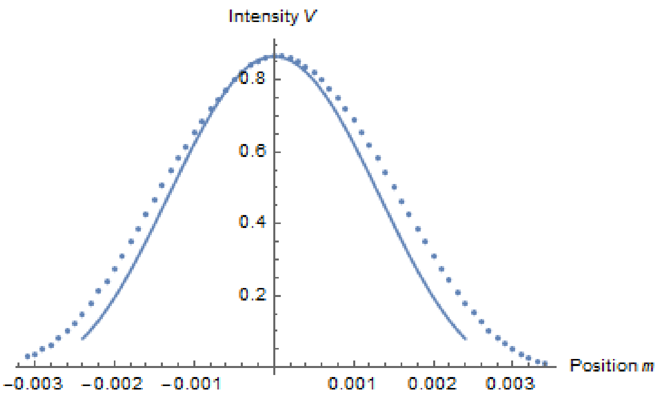

FIG. 6: Single slit intensity pattern from a red laser, predicted intensity pattern on top of data points, x-axis: position (m), y-axis: intensity (V)

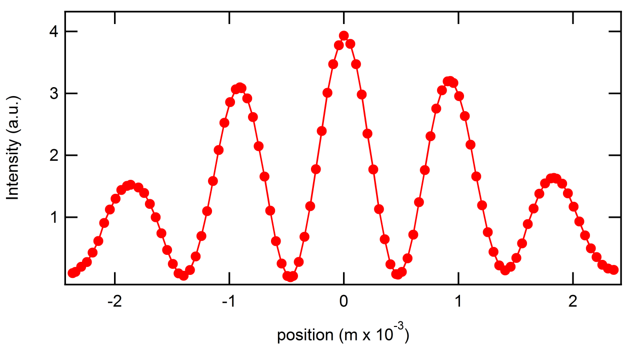

FIG. 7: Double slit intensity pattern from a red laser, experimental intensity pattern

exhibits general shape as the laser plots, the curves are less smooth. The less smooth curvature is likely due to a decrease in accuracy from the low discrimination value used during the photon counting. While the accuracy was lowered, intensities of the plot were increased which makes the intensity pattern more visible even if it is less smooth. After the value obtained for the slit width from Fig. 8 was used to create the predicted intensity pattern shown in Fig. 10, the predicted pattern matched closely

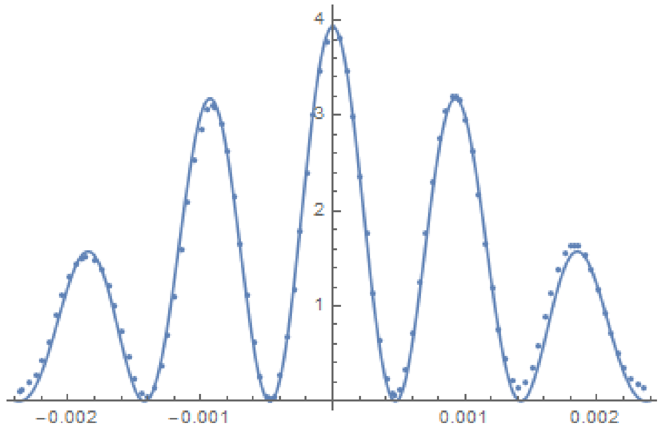

FIG. 8: Double slit intensity pattern from a red laser, fit intensity pattern on top of data points

with the intensities obtained.

To show that the light is dim enough such that there is only one photon at a time in the path, the period of the central max photons was calculated by integrating over the pulse pattern to obtain the total number of pulses. These values are shown in Table II. The time for a photon to travel the 1 m of the path is five orders of magnitude smaller than the time in between the pulses. This suggests that the probability of more than one photon being in the path of travel at once is small. Since there is only one photon in the path at a time to pass through the double slit and the interference pattern us still observed, the conclusion can be drawn that not only do photons' probability distributions interfere with each other, but they can interfere with themselves as well.

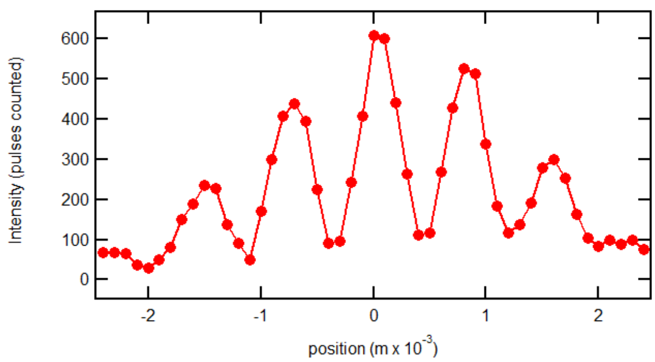

FIG. 9: Double slit intensity pattern from a dim, green light bulb, experimental intensity pattern

TABLE II: Time frame of pulses in the path of travel

Integrated Pulses	Pulse Period (s)	1 m Travel Time (s)
10 572	9.46×10^{-4}	3.34×10^{-9}

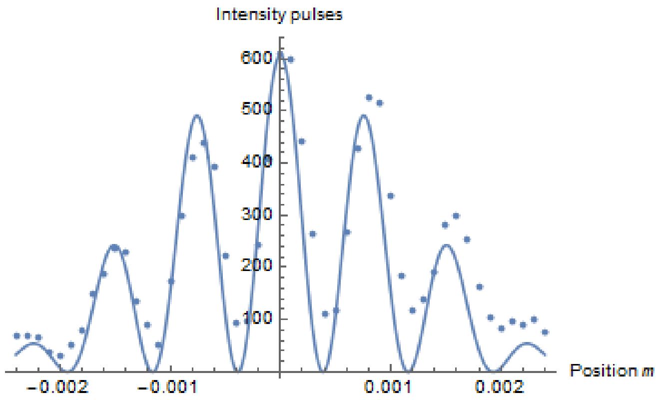

FIG. 10: Double slit intensity pattern from a dim, green light bulb, predicted intensity pattern on top of data points

V. CONCLUSION

The double slit experiment was performed using a laser and a dimmed light source to test the wave-like nature of photons. The dimmed light source was found to be dim enough such that the period between pulses was five orders of magnitude greater than the time it takes for a photon to travel from the bulb to the detector. The laser exhibited the predicted intensity interference pattern expected if light behaves as a wave. The dimmed light source with single photons passing through the slit exhibited the same behavior, suggesting that not only do photons interfere with each other's probability distributions, they also interfere with their own after passing through a double slit.

VI. FUTURE WORK

Future work could include examining the behavior of photons as they pass through the two slits by placing detectors on the slits to know which slit the photon passes through. I know that its behavior changes once it is observed, but it is still a compelling idea to examine.

VII. ACKNOWLEDGMENTS

I sincerely thank Dr. Cody Leary for being patient with me as I was persistent to set up the new pulse counting software for the first time as well as my fear of harming the apparatus. I thank Professor Grugel-Watson for providing a department computer and assisting with the installation of the new counting software.

[1] T. Young, *Phil. Trans. R. Soc. Lond.*, **94**, 1-16 (1804).
[2] A. Einstein, *Annalen der Physik*, **17** (6), 132-148 (1905).
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