Trapping Polystyrene Spheres Using "Optical Tweezers"

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Despite being massless, photons are able to interact with objects with mass because the photon/mass system must adhere to conservation laws. As a result, when a mass comes into contact with a laser, the object will cause the light to refract in particular directions, depending on the geometry and physical characteristics of the object. If the object is small enough and the laser powerful enough, the laser will sometimes push the object towards both its primary axis and focal point until the object becomes trapped within the light beam. Once trapped, the object can be manipulated with precision that far surpasses the capabilities of ordinary tools. This technique of trapping and moving small objects using radiation pressure is called "optical tweezing" and was first proposed by Arthur Ashkin in 1986. In this lab, I attempted to recreate this phenomena by trapping polystyrene spheres with diameters on the order of micrometers with a laser. The trapping was unsuccessful, but there were some noticeable fluctuations in the direction and magnitude of one sphere's momentum when it encountered the laser. Using data collected from a video of the sphere moving towards, through, and out of the laser, an average acceleration of $0.01 \pm 0.01 L/s^2$ was measured, where L is an arbitrary unit used to provide a scale to examine the motion of the particle. The presence of this acceleration indicated that there was indeed a force on the sphere attributed to the laser.

I. INTRODUCTION

Tools such as tweezers, pliers, and tongs are used in a wide range of activities and studies, whether it be for delicately manipulating bodily tissues during surgery or wrenching a nail out of a plank of wood in carpentry. Even in these dramatically different situations, the purpose of the tools are the same; to provide a firm grip on an object without sacrificing dexterity. Another commonality in the situations cited above are that the objects that are gripped by the tools are relatively large. An object on the scale of millimeters may seem small and difficult to grip even with tools, but they are still vastly larger than objects on the scale of micrometers. In order to manipulate objects of such small magnitude, it becomes necessary to work with exceptionally small tools and apparatuses that allow us to maneuver these tools with greater accuracy than that of which our hands are capable. One such method of manipulation is called "optical tweezing" .

Single-beam gradient force traps, also known as "optical tweezers" are microscopic re-creations of tools like tweezers, pliers, and tongs that employ light in place of solid material in order grip microscopic objects. This method of particle trapping was developed by Arthur Ashkin in 1986 [2]. The technique entails focusing a laser with ample intensity into a medium containing loose particles. When the light is incident with a particle, it exerts radiation pressure in the forms of reflection, refraction, or a combination of both. The radiation pressure guides the particle through the light beam into the area with the greatest intensity. For a laser, the region of greatest intensity is the middle of the beam. Therefore, conducting optical tweezing with a laser traps the particle in the middle of the laser. The light is not perfectly symmetrical at all times, however. As a result, the radiation pressure from the laser will cause minute pushes away from its center in varying directions. This causes small oscillations in the particle's position around the center of the beam.

Optical tweezing is a technique employed by various scientific fields in order to investigate microscopic objects. In medicine, it can be used to more precisely administer drugs to cancer cells in order to increase the efficacy of treatments [3]. In biology, it allows for the containment of microscopic specimens, such as bacteria like E. Coli, providing samples much more conducive to study [1]. In this experiment, a solution containing microscopic polystyrene spheres (PS spheres) was used to observe the trapping phenoma.

II. THEORY

Optical Tweezing is predicated on fundamental mechanical and optical laws. The primary foundation for the technique is the law of conservation of momentum. Put in simple terms, this law states that momentum cannot be created or destroyed; it can only be transferred between objects. The equation for the momentum of objects with mass is

$$\vec{p} = m\vec{v}, \tag{1}$$

where $\vec{p}$ is the momentum, m is the object's mass, and $\vec{v}$ is the objects velocity. Light, however, is quantized in massless particles called photons, so it cannot be modeled by equation 1. The solution to this issue lies in the general theory of relativity that was conceived by Albert Einstein in 1915 [5]. This theory describes mass as a form of energy called "resting energy", which is modeled by the equation

$$E = mc^2, \tag{2}$$

where E is the energy, m is the mass of the object, and c is the speed of light. By employing the Planck-Einstein relation

$$E = hf, \quad (3)$$

which describes the energy (E) of a photon as being the product of the frequency (f) of the light containing the photon, multiplied by Planck's constant (h); it is evident that energy is a property of objects both with mass and without mass [6]. Based on this conclusion, it is reasonable that massless objects should also be able to have momentum and indeed this is the case, as shown by

$$\vec{p} = hf/\vec{c} = h/\vec{\lambda}, \quad (4)$$

where p is the momentum, h is Planck's constant, f is the frequency of the light, c is the velocity of the light, and λ is the wavelength of the light. If both the photons and the objects with mass have momentum, then according to the law of conservation of momentum, the net change in momentum must be zero when these objects interact. With this premise established, it is now important to focus on how photons and masses interact.

In optics, there are three ways in which light can interact with matter: absorption, refraction, and reflection. Absorption occurs when an object is capable of transforming all of the energy from a photon into some form of internal energy. Coal, for example, is a black body that absorbs light and converts its energy into thermal energy which, under certain conditions, will make the coal glow after enough energy has been absorbed. Blackbody radiation is beyond the scope of this experiment and because the polystyrene spheres were transparent, its presence in this experiment is negligible.

Refraction occurs when a photon passes through the surface of an object, but changes its path due to physical differences between the object and the medium the light was previously traveling through. These physical differences between different mediums are accounted for using a dimensionless constant called the index of refraction. An index of refraction is specific to a particular material and determines how much a photon is deflected from its original direction of propagation and how fast it can travel through a medium. Therefore, refraction causes changes to the photon's momentum, both in terms of direction and magnitude (see figure 1).

The index of refraction for a material is modeled by the equation

$$n = c/v, \quad (5)$$

where n is the index of refraction, c is the speed of light, and v is the speed of the light in the medium. Equation 5 is derived from Snell's law which describes the index of refraction of a material as the ratio of the sines of the angles of incidence and refraction. Snell's law can be written as

$$sin(\theta_1)/sin(\theta_2) = v_1/v_2 = \lambda_1/\lambda_2 = n_2/n_1, \quad (6)$$

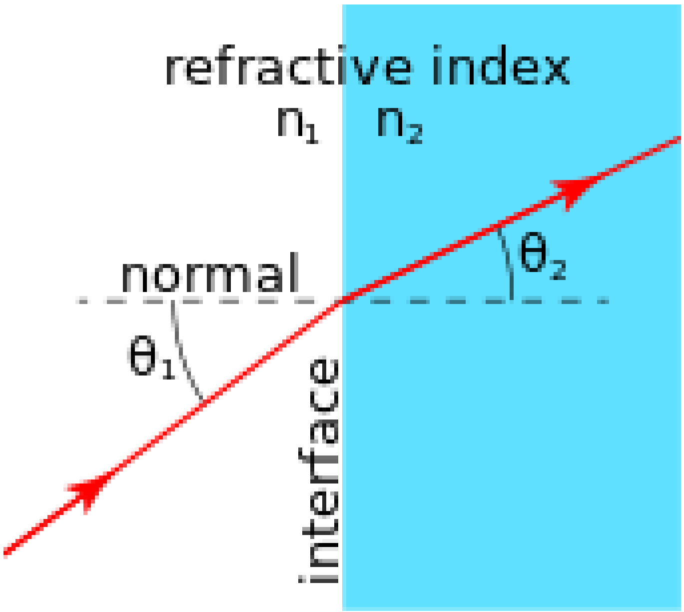

FIG. 1: A diagram depicting the refraction of a light beam as it crosses the interface between two different mediums. [8]

where θ_1 is the angle of incidence, θ_2 is the angle of refraction, v_1 is the speed of light in medium 1, v_2 is the speed of light in medium 2, λ_1 is the wavelength of light in medium 1, λ_2 is the wavelength of light in medium 2, n_1 is the index of refraction of medium 1, and n_2 is the index of refraction of medium 2 [9].

Reflection is a special case of refraction in which a photon collides with an object such that the angle of incidence is equal to the opposite of the angle of refraction. This changes the direction of the photon's momentum, but the magnitude remains the same because the photon stays within the same medium throughout its motion (see figure 2).

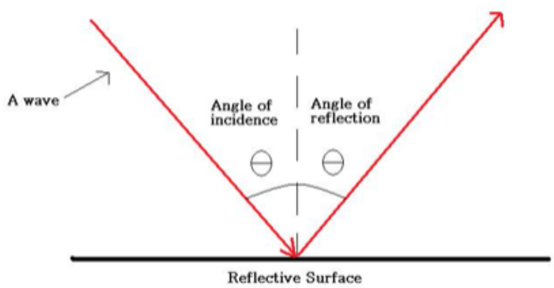

FIG. 2: A diagram depicting the reflection of a light beam as it reaches the interface between two different mediums. [7]

The law of conservation of momentum coupled with the law of refraction provides the conditions required for optical tweezing. In figure 3, a polystyrene sphere is shown in the process of migrating toward the center of the laser.

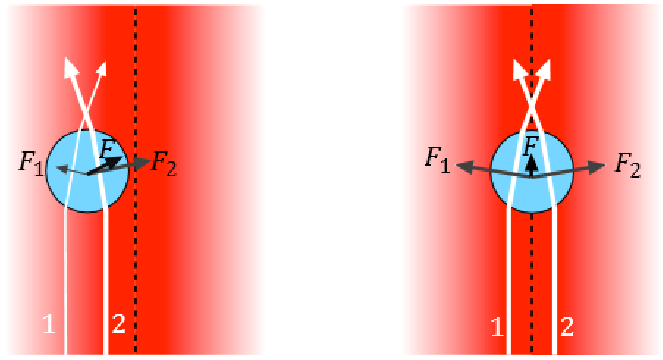

FIG. 3: A depiction of the radiation pressure pushing a polystyrene sphere toward the center of a laser. [1]

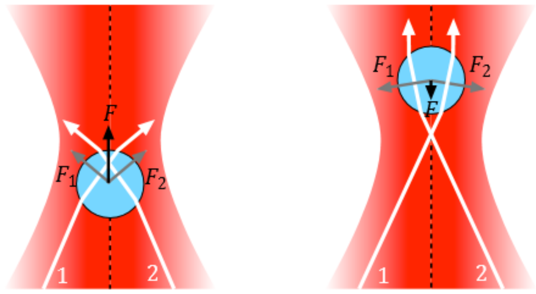

FIG. 4: A depiction of the radiation pressure pushing a polystyrene sphere toward the focal point of a laser. [1]

As the figure indicates, some of the light from the laser penetrates the sphere and because the PS sphere has a different index of refraction than the air that surrounds it, the laser light refracts at the air/sphere interface. The geometry of the sphere causes the less intense light toward the outside of the laser to refract toward the center of the laser, whereas the more intense light closer to the center of the laser refracts toward the outside of the laser. The momentum of the light/sphere system must be conserved so if the light was originally moving in the $+y$ direction, then when it is deflected to $\pm x$ direction, the sphere must counteract the x component of the photon's momentum by moving in the $\mp x$ direction. The intensity of the light that is refracted away from the laser is greater than the intensity of the light that is refracted toward the laser, so according to the law of conservation of momentum, the more intense light will cancel out the x component of the momentum of the less intense light. This means that the sphere only has to counteract the difference of the x components of momentum between the more intense and less intense light. The sphere will continue to move toward the center of the laser until there are equal volumes of the sphere on each side of the laser's primary axis. At this point, the x components of momentum will be completely balanced by the laser light alone, so the sphere will no longer have to move in the $\pm x$ directions.

According to figure 4, the sphere will move in the ± y direction depending on the sphere's index of refraction. If the index of refraction of the sphere is greater than the index of refraction of the air, then the speed of light will be less in the sphere meaning that the magnitude of the light's momentum will decrease. This will cause the sphere to move in the $+y$ direction in order to conserve momentum. Conversely, if the index of refraction of the sphere is less than the index of refraction of the air, then the speed of light will be greater in the sphere which indicates that the magnitude of the light's momentum will increase. The sphere will move in the $-y$ direction in order to counteract this change and conserve momentum. Regardless of direction, the sphere will move along the primary axis of the laser until it reaches the focal point of the laser. The focal point is the location along the laser's path where the intensity is greatest because the radiant energy of the laser is being focused at a very small point. As the sphere approaches the focal point, the light will gradually refract more and more until it essentially comes to rest at the focal point.

As the sphere moves towards the primary axis of the laser and the focal point of the laser, it's velocity and momentum will approach zero. Once it reaches these points, the sphere will oscillate very slightly about the two points because the lasers are not perfectly concentrated in an infinitesimally small area. The motion of the sphere will mimic a mass on an oscillating spring and a value that is analogically equivalent to a spring constant can be calculated for the system. In the same way that the spring constant is not a characteristic of the mass, the constant in this case is not a characteristic of the sphere. The constant is actually a result of the laser having a Gaussian profile. If we consider the potential energy of a spring given by the equation

$$U = kx^2 \tag{7}$$

the spring/mass system will have the greatest potential energy at the points furthest from its equilibrium position and the greatest kinetic energy at the equilibrium position (when U=0).

Likewise, the sphere will have the greatest potential energy when it is near the fringe of the laser because the entirety of the laser light passing through it will be refracted away from the laser. As stated earlier, when the light is refracted away from the laser, the sphere is pushed toward the center of the laser in order to conserve momentum. As the sphere approaches the center of the laser, the light starts to refract in patterns that cause the restoring force from the laser on the sphere to decrease. This indicates that the potential energy (according to equation 7) is decreasing and the kinetic energy is increasing in order to conserve energy. The intensity of the laser is a Gaussian, meaning that it can essentially be modeled by a parabolic curve when the displacement of the sphere from the center of the laser is relatively small. This parabolic implies that the displacement from the center of the laser (x or y depending on whether the sphere is moving with respect to the primary axis or the focal point) abides by a power of 2. Therefore, the model of the potential energy given in equation 7 is accurate in describing the energy of the sphere/laser system.

III. PROCEDURE

The apparatus consisted of the following components connected to an optics breadboard (the order in which the components appear loosely follows the order through which the light encounters them): Laser, Mirror, Mirror, Iris, Converging Lens, Linear Polarizer, Dichroic Mirror, Converging Lens, Camera, Iris, Microscope Objective (100x), Sample Holder/Adjuster, Diverging Lens/Converging Lens (facing laser and backlight respectively), 20 Watt Backlight, Backlight Power Supply, Laser Power Supply.

Using the multitude of mirrors, lenses, and other optical components, the laser was guided into the microscope objective. To ensure the laser was shining into the objective at maximum intensity, the irises were closed one at a time and used to align the mirrors. The objective focused the laser and directed it onto the sample. This provided a light beam with greater intensity which made it much more capable of trapping a sphere. Additionally, the objective allowed the camera to take focused videos of the trapping attempts on the sample using the "Photobooth" application. The position of the slide was manipulated by toggling the three adjustment screws on the sample holder. The back-light was kept on throughout the duration of each trial in order to provide the camera with a clear, bright picture. The camera could also be adjusted slightly through twisting it from side to side to make the picture clearer and eliminate dark spots.

Once the laser was in focus on the sample, I was able to further maneuver the sample and search for PS spheres to trap. Throughout the process, many spheres were found to be moving through the solution. By taking the laser slightly out of focus, I was able to increase the diameter of the beam. In order to trap the spheres the beam was placed in their path so that when they entered the fringes of the laser, they would experience a force pulling them toward the center of the laser and ideally trapping them within the beam.

IV. DATA AND RESULTS

In practice, the trapping proved difficult and no actual trapping occurred throughout the duration of the lab. There were several instances when the laser did appear to at least slightly influence the speed and direction of some of the PS spheres. When any of these phenomena were observed, the videos were inserted into "Logger Pro" and the motion of the spheres were measured by tracking them across the screen frame by frame and periodically placing points on their path as they moved towards, through, and out of the laser.

The units used to describe their motion were based upon the diameter of the bright spot as shown in figure 5 in the black circle. For ease of calculation, I approximated the diameter of the of the section of laser contained in the black circle to be equal to the sphere contained in the red circle. This is not a completely accurate result because the sphere is out of focus and both the sphere and laser were slightly more oblong than circular, but the little bit of error present in these approximations was taken into account when determining the error in the placement of the dots on the sphere's path. I estimated the error in the vertical positions and horizontal positions of the sphere throughout its movement to be approximately $\pm 0.5L$ where L is the approximate diameter of the bright circle at the center of the laser.

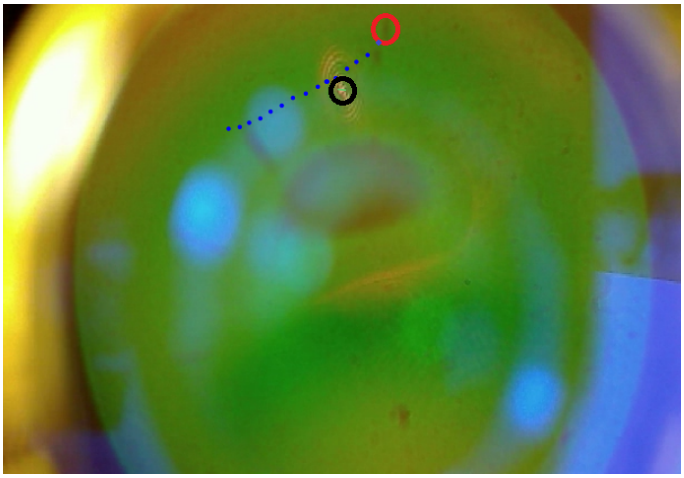

FIG. 5: This figure shows the points along the PS sphere's path that were used to calculate it's velocity (blue dots), the PS sphere that was tracked (red circle), and the bright spot in the middle of the laser (black circle).

Logger Pro provided a table containing the time, x position, y position, x velocity, and y velocity at each point that was marked along the path. In order to calculate uncertainty in the x and y positions, graph 6 was created in Igor Pro and uncertainty bars were added with the aforementioned $\pm 0.5L$ values. Then, in order to get the average x velocity and y velocity, the slopes of the y position versus t curve and the x position versus t curve were measured. Using this method, the average x velocity was calculated to be $0.70 \pm 0.03L/s$ and and the average y velocity was calculated to be $0.38 \pm 0.02L/s$.

To calculate uncertainty for the x and y velocities, the uncertainty value 0.5 was imposed the x position and y position waves. Then, the Differentiate command in Igor Pro was used to differentiate both waves with respect to time. This provided an x velocity wave with uncertainty and a y velocity wave with uncertainty. By subtracting the x velocity and y velocity waves given by Logger Pro from the velocity waves with uncertainty, the x velocity

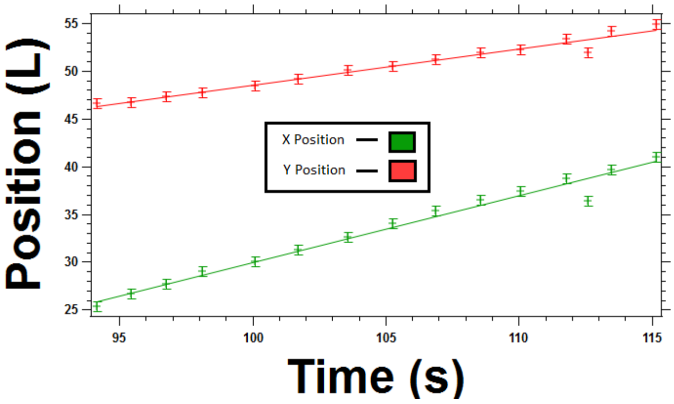

FIG. 6: A graph of the horizontal and vertical positions of the PS sphere with respect to time.

uncertainty wave and y velocity uncertainty wave created. Instead of making a graph with the velocity components with uncertainty, a wave was created that contained the magnitude of the velocity with uncertainty. The magnitude without uncertainty was calculated using the x velocity and y velocity waves that Logger Pro had output based on the points that had been selected along the sphere's path. Substituting these waves into the equation

$$V_{mag} = \sqrt{V_x^2 + V_y^2}, \tag{8}$$

where V_{mag} is the magnitude of the velocity, V_x is the x velocity, and V_y is the y velocity yielded a wave of the magnitude of the velocity without uncertainty. In order to add the uncertainty to this wave, equation 8 was duplicated, but instead of plugging in the x velocity and y velocity waves, the x velocity uncertainty and y velocity uncertainty waves were substituted in. This outputted a wave containing only the uncertainty in the magnitudes. The magnitude wave versus time was plotted and error bars were added to the points using the magnitude uncertainty wave. The resulting was graph 7. The last five data points represent the points in the video when the sphere was entering, moving through, and exiting the laser. Large differences in the speeds for these points when compared to the points prior to the sphere entering the laser were present. This indicates that there was a force due to radiation pressure acting on the sphere. Though the force may not have been strong enough to trap the sphere, it was ample to cause some changes in its motion. Taking the slope of the magnitude of velocity as a function of time graph provided the average acceleration. This acceleration was $0.01 \pm 0.01 L/s^2$. The nonzero value of this acceleration is representative of a small, yet present force from the laser acting on the sphere.

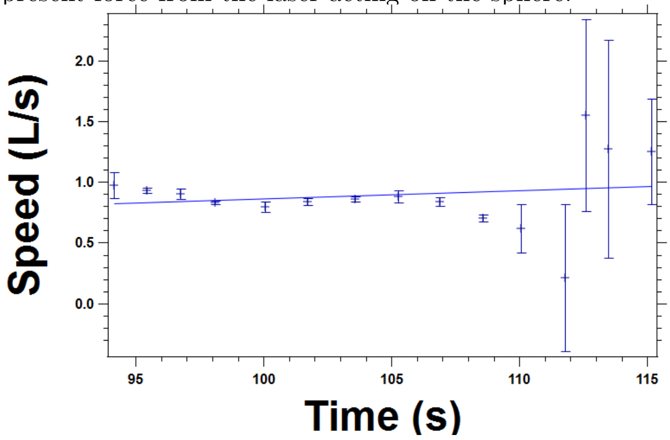

FIG. 7: A graph of the speed of the PS sphere with respect to time.

V. CONCLUSION

Though a particle was not successfully trapped using the optical tweezers technique, the results demonstrate that a laser can indeed be used to affect the motion of a particle of mass, despite the light itself being massless. The nonzero acceleration value of 0.01 ± 0.01 from graph 7 is indicative of a force acting on the sphere, albeit a rather small one. This implies that under different conditions, perhaps with a stronger laser, a better aligned apparatus, or smaller spheres, trapping of the particles using only radiation pressure is feasible. In order to obtain better results in the future, it may be useful to use thinner cover slips so as to further limit refraction due to oil/glass interface, create solutions with a greater number of spheres to allow for an easier collection of data and greater sample size, and a more stable optical breadboard set. One of the more common issues encountered during this lab was the looseness of the components in the apparatus. On several occasions, a slight nudge to the table or a component would cause it to become misaligned.

There are also various potential research projects that could stem from the optical tweezing technique. Further advances could be made in incorporating it into modern medicine in the form of gene therapy in order to prevent genetic diseases, developing some form of accelerator that could trap an object and gradually increase the intensity off the laser in order to propel the object forward, and perhaps even hologram technology that suspends and illuminates particles in the air via radiation pressure.

[1] Jr. IS Lab Manual *Ashkin: Optical Tweezers* (College of Wooster Physics Department, 2017).

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[3] *Tweezing and Manipulating Micro- and Nanoparticles by Optical Nonlinear Endoscopy* (Gu, Min, Hongchun Bao,

Xiaosong Gan, Nicholas Stokes, and Jingzhi Wu, 2014).

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