Conservation Laws in Symmetric Systems

Collin Hendershot
Physics Department, The College of Wooster, Wooster, Ohio 44691, USA
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Noether's theorem states that symmetric systems have a corresponding conservation law. The two experiments conducted in this lab looked at the conservation law associated with a rotational system. One experiment was conducted to find the moment of inertia of a spinning platter and the result for the apparatus used in this lab was $6.28 * 10^3 \pm 0.02$ kg m^2. The second experiment was conducted to check if the rotational system conserved angular momentum. The results from the experiment concluded that the rotational system did conserve 97% of the angular momentum which was the predicted result from Noether's theorem.

I. INTRODUCTION

In 1915, through the use of Lagrangian mechanics, Emilee Noether proved that symmetry in a system results in a conservation law for that system [1]. If the Lagrangian of a system is invariant due to a change in a variable there will be a value for the system that is conserved. This idea of the invariance of the Lagrangian for a system is the base for the conservation laws associated with translational, rotational, and many other types of transformations [2].

II. THEORY

The two experiments in this lab require an understanding about moments of inertia and angular momentum. The moment of inertia is related to the torque through the equation:

$$\vec{\tau} = I\vec{\alpha} \tag{1}$$

Where τ is the torque, I is the moment of inertia, and α is the angular acceleration. This relationship can be modified to:

$$I = \frac{\vec{r} \times \vec{f}}{\vec{\alpha}} \tag{2}$$

Where r is the radius and f is the force. Since the radius and the direction of the force are perpendicular, and the angular acceleration is in the same direction as the force the equation simplies further to (Figure 3):

$$I = \frac{r * f}{\alpha} \tag{3}$$

Since r and f are known values in this experiment the angular acceleration, α, is the value that must be measured to determine the moment of inertia. The second experiment checks for the conservation of angular momentum in the system. The eqution for angular momentum is:

$$\vec{L} = I\vec{\omega} \tag{4}$$

where L is the angular momentum and ω is the angular velocity. If angular momentum is conserved within a system, then a change in the moment of inertia will result in a change in the angular velocity such that the initail angular momentum is equal to the final angular momentum.

III. PROCEDURE

A. Apparatus

In order to calculate the moment of inertia for the spinning plate and to test for the conservation of angular momentum in a symmetric system a rotational apparatus, computer interface, and a pulley with a photgate were used. The rotation apparatus included a main platter with a step pulley(Figure 2), which was able to rotate, a pulley that was oriented so a mass can be hung over the side of a table, and a string. A photogate pulley in contact with the rotating platter was used to measure the angular velocity and angular acceleration (Figure 1).

B. Calculating the Moment of Inertia of the Main Platter

In order to calculate the moment of inertia for the main platter the angular acceleration of the main platter was recorded as the force from a hanging mass caused the main platter to rotate. A 50 g hanging mass was hung from a string that was tied to the step pulley and then run over the pulley that allowed the mass to hang over the side of the table. The main platter and the hanging pulley were kept level to ensure the maximum effieciency of the force from the hanging mass. The string was wound around the step pulley and then the main platter was released while the angular acceleration was recorded. Three runs were recorded for each of the three levels of the step pulley(Figure 2). The average angular momentum for each step was used to calculate the moment inertia of the main platter.

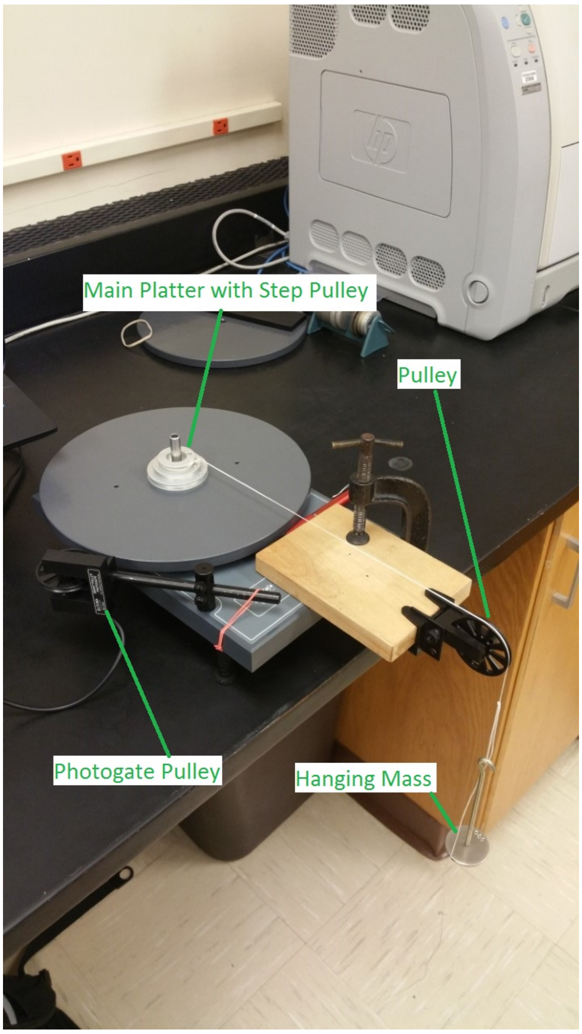

FIG. 1: The general apparatus used for the two experiments.

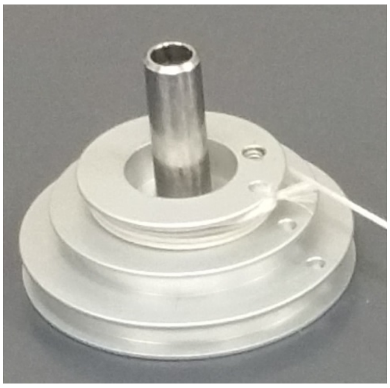

FIG. 2: A close view of the step pulley.

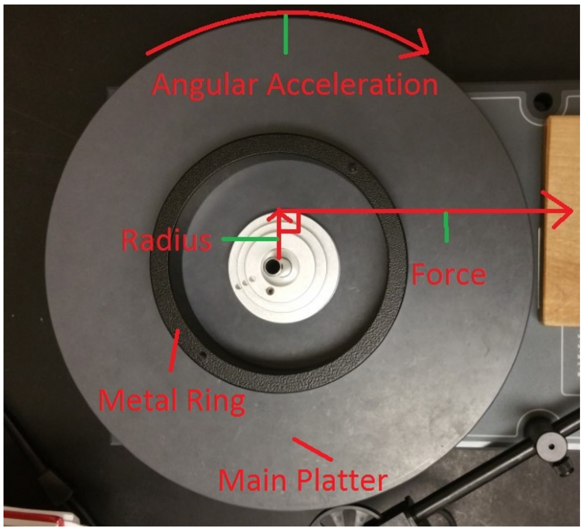

FIG. 3: The rotational apparatus with the ring, which was used to check for the conservation of angular momentum.

C. Testing for the Conservation of Angular Momentum

To test for the conservation of angular momentum in this rotational system the string that held the hanging mass in the previous apparatus was removed so that the apparatus included the main platter with the photogate pulley (Figure 3). The main platter was spun and the photogate pulley was set to record the angular velocity of the main platter. After the main platter was rotating a metal ring was dropped onto the platter so that the centers of the ring and the platter aligned. The angular velocity measurements from the photogate pulley were used to find the angular velocity of the main platter just before the ring was dropped and just after the ring was dropped. These angular velocity measurements were used to calculate the angular momentum of the system before and after the ring was added.

IV. ANALYSIS

A. Calculating the Moment of Inertia

To calculate the moment of inertia the average angular velocities from the runs for each step of the step

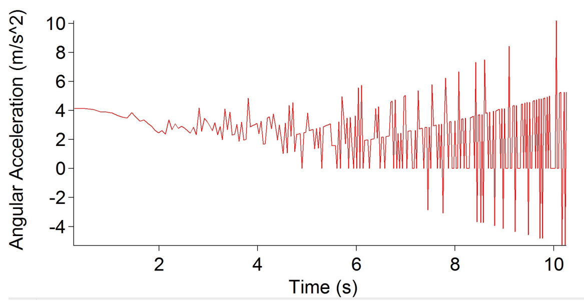

FIG. 4: This graph shows the angular acceleration recordings for the rotational apparatus.

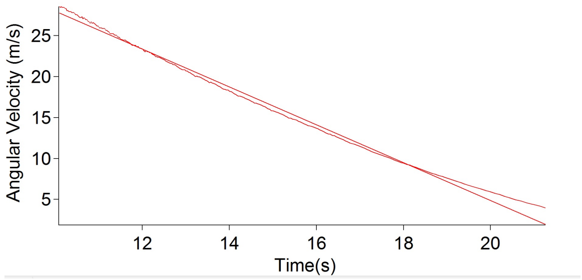

FIG. 5: This graph shows the velocity measurements that where used to calculate the friction of the apparatus.

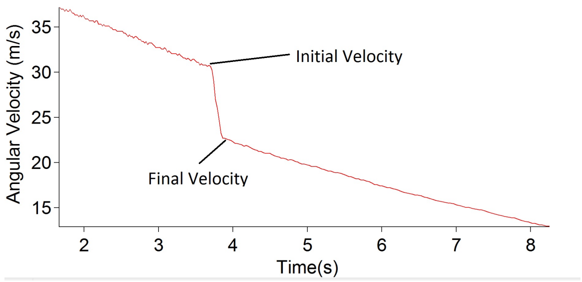

FIG. 6: This is the plot used to find the velocity before and after the ring was added to the apparatus.

pulley were substituted into Equation 3(Appendix A). As pictured in Figure 4 the photogate pulley started to reach its limitation of recording around two seconds into the run because after that time the photogate pulley recorded multiple negative angular acceleration values. This would have meant that the platter changed its rotational direction throughout the run and this behavior was not visually observed. The angular velocities used for the calculations were the resulting means from the first two seconds of recording. These three calculated values of angular acceleration provided an average moment of inertia of $2.02 * 10^3 \pm 0.02$ kg m^2. This experimentally calculated value was not equal to the provided moment of inertia of the main platter of $7.50 * 10^3 \pm 0.02$ kg m^2.

The difference between the experimental value and the given value for the moment of inertia was believed to have originated from the frictional forces of the apparatus. The effect of friction was studied using the plot shown in Figure 5 and there was an acceleration of -2.31 $\frac{\text{rad}}{\textbf{sec}^2}$. When this acceleration was included in the calculation the calculated value for the moment of inertia was $6.28 * 10^3 \pm 0.02$ kg m^2, which was closer to the given value, a difference of $1.22 * 10^3$ kg m^2.

B. Checking for the Conservation of Angular Momentum

To check for the conservation of angular momentum in the system the angular velocity just before the ring was added to the system and just after it was added were substituted into Equation 4(Appendix B). Figure 6 pictures the data from one run that was used for the angular momentum calculations. The angular momenta of the system before and after the ring was dropped were compared. In order to achieve the most accurate results the given moment of inertia for the main platter and ring were used in the calculations. The angular momenta before and after the addition of the ring were almost equal, an average difference of 0.008 $\frac{kgm^2}{s}$, thus the system conserved 97% of angular momentum. The small difference between the momenta was possibly a result of the frictional forces of the system and the ring being dropped slightly out of line with the center of the platter. To limit the effects of an off center ring drop many runs of the experiment were recorded.

V. CONCLUSION

An experiment to calculate the moment of inertia of a spinning platter and an experiment to test for the conservation of angular momentum were conducted for this lab. The experiment to find the moment of inertia of the main platter resulted in a value of $6.28 * 10^3 \pm 0.02$ kg m^2. The experiment that was conducted to check for the conservation of angular momentum in the system concluded that the rotational system conserved 97% angular momentum. This result matches the prediction for a symmetric rotational system resulting from Noether's theorem. Both experiments results were affected by the frictional forces associated with the apparatus, limitations of the recording devices, and human error.

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[1] Emily Noether, *Invariante Variationsprobleme*(Math-Pys Klasse, 1918)
[2] John R. Taylor, *Classical Mechanics*(University Science Books, 2005)

VI. ACKNOWLEDGEMENTS

I thank Dr. Leary for assitance with the experiments and analyzing the results.

VII. APPENDIX

A. Calculating the Moment of Inertia

$r = 0.015m,\ f = 0.49N, \alpha = 0.44\frac{\text{rad}}{\textbf{sec}^2}$
These values are substituted into Equation 3:

$$I = \frac{(0.015)(0.49)}{0.44} = 2.11 * 10^3 \pm 0.02\text{kg m}^2 \qquad (5)$$

B. Calculating the Angular Momentum

$I = 7.50 * 10^3 kgm^2,\ \omega = 30.80\frac{\text{rad}}{\textbf{sec}}$
These values are substituted into Equation 4:

$$L = (7.50 * 10^3)(30.80) = 0.231\frac{\text{kg m}^2}{\text{s}} \qquad (6)$$