

Gone With the Wind: An Investigation into the Flight Dynamics of Discs

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In our experiment we investigated, through theoretical and experimental means, the variables that affect disc flight. Disc flight is broken into two pieces that we analyze separately: aerodynamics and angular momentum. The aerodynamics provide the forces necessary to keep the disc aloft and include the lift, drag and aerodynamic torques. Experimentally, we proved that the disc's velocity and shape affect the aerodynamic forces. The angular momentum produces the crucial disc stability by diminishing the effects of precession. We investigated theoretically how the angular momentum alters the flight of the disc in the presence of external torques. Combining our theoretical and experimental investigations into the aerodynamics and angular momentum, we built the foundation necessary to analyze the flight dynamics of discs.

I. INTRODUCTION

Although the sight of Yale students throwing a pie tin around may seem strange, this was the origin of the Frisbee™, trademark Wham-O Manufacturing company, which we will also refer to as a disc. In the early 1900s Yale students would use their empty pie tins from the Frisbie Pie Company for entertainment. The pie tins had a similar weight and shape to the modern day Frisbee and therefore could maintain stable flight. The idea was picked up by Warren Francisconi and Walter Morrison who started making the discs from plastic. These two found little commercial success with their product, which became known as Pluto Platter, and eventually Morrison took the patented disc to the Wham-O Manufacturing company. In 1964 Wham-O improved the Frisbee's design, labeling the new disc a *professional* model disc, and sales began to soar [1]. The popularity of the Frisbee led to the invention of several sports with the two biggest being ultimate and disc golf. Ultimate is a team based sport in which two teams of seven players pass the disc back and forth from one end of a field to the other. Similar to football, one team is on offense and is trying to get the disc to the opposite endzone to score a point. The other team is on defense and is trying to intercept the disc or have the disc touch the ground so they gain possession of the disc. The other major disc based sport is disc golf. Unlike ultimate which involves a great deal of running, disc golf is all about precision throwing. Disc golf courses consist of a series of metal baskets that players must land their disc in using as few throws, or strokes, as possible. Similar to normal golf which uses different clubs, players use different discs depending on the distance to the hole and shape of the course. Companies now manufacture a large variety of discs such as driver, mid-range, and putter discs that have distinct properties that are ideal for certain situations.

As these sports continue to grow so does the need to analyze discs' flight dynamics. Ultimate is becoming a nationally recognized sport and has begun airing on ESPN [1]. If ultimate becomes as popular as football or

baseball there will be a huge demand for coaches and analysts. Studying how a Frisbee flies will allow for better instruction on how to throw discs accurately in different scenarios. Since the opposing team is constantly trying to intercept the disc, the thrower must be very careful in choosing the disc's flight path. If the disc flies too close to a defender or too far from the receiver the pass will not be completed. If the thrower understands how the disc will fly he can adjust the disc's speed and orientation to have it follow the optimal flight path. Analyzing disc flight dynamics can also help design new disc golf discs. We can use our knowledge of disc flight to alter the shape of the disc so it flies better in specific situations. Therefore this research has monetary value in manufacturing, especially considering that disc golf is a growing sport. In our experiment we focused on studying three disc golf discs because the information about the putter can be applied to the ultimate discs.

II. THEORY

In order to develop a theoretical interpretation for Frisbee flight we must consider what factors are involved in Frisbee flight. The aerodynamic shape and cavity on the underside of the Frisbee must produce lift in order for any flight to be achieved. However the lift also causes the Frisbee to turn over and lose flight. This is why a spin is imparted on the disc when it is thrown. The angular momentum of the spinning Frisbee stabilizes the disc and prevents it from turning over. In order to analyze the flight of a Frisbee, we analyze the aerodynamic and angular momentum effects separately.

A. Disc Terminology

Before diving into the theory behind disc flight, we will first define the disc orientation and terminology used to describe the disc and its motion. The disc is oriented with the plane of the disc lying in the xy plane as shown in

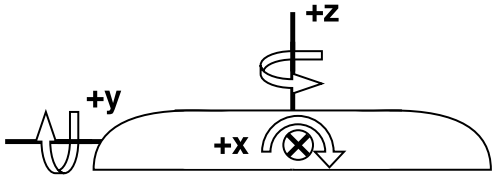


FIG. 1: The orientation of the disc in the x, y , and z axes. The positive x axis is into the page, the positive y axis is to the left, and the positive z axis is upward.

Fig. 1. The axes are set up so that the positive x direction is into the page, positive y is to the left and positive z is up. We will always assume that the disc is traveling in the positive x direction meaning the velocity vector of the disc is pointing in the positive x axis. Since the Frisbee can freely rotate about each axis, there are three angular velocities p, r , and s which correspond to the x, y , and z axis respectively. With the axes defined, the vocabulary used to describe the disc consists of several terms. The leading edge is the front rim of the disc pointing in the positive x direction while the trailing edge is the rim pointing in the negative x direction. The angle of attack α is the angle between the plane of the disc and the velocity vector of the disc. The center of mass is where the force of gravity acts and is always in the center of the disc. The center of pressure is where the aerodynamic forces act and does not have to be in the center of the disc. The center of pressure can change throughout the disc's flight and is dependent on the angle of attack [2]. We also have terminology for the specific rotations of the disc. If the disc rotates about the y axis, causing the leading edge to move up or down, the motion is called pitch and the angle produced by this rotation is the pitch angle. If the disc tilts around the x axis the motion is called roll and has a corresponding roll angle.

B. Aerodynamics

One aerodynamic force is drag which acts directly opposite the velocity vector of the disc. There will always be a drag force F_D acting on the disc and can be modeled using the equation

$$F_D = \frac{1}{2} \rho v^2 A C_D. \quad (1)$$

In this equation ρ is the density of the air, v is the velocity of the disc, A is the surface area of the disc, and C_D is the drag coefficient. In typical aerodynamics the drag coefficient is determined by three factors: Reynolds number, spin parameter, and angle of attack. The Reynolds number is the ratio of inertial forces to viscous forces and the spin parameter is the ratio of rotational velocity to translational velocity. Through previous experimentation by Potts and Crowther it was determined that the Reynolds number and spin parameter have negligible effects on the

drag coefficient of the disc [2]. Removing these two variables the drag coefficient is a quadratic function of the angle of attack.

Another aerodynamic force is the lift which acts perpendicular to the velocity vector of the disc. The lift is the force that allows the disc to stay aloft for extended periods of time and is the backbone behind flight. Physically the lift is caused by the cavity on the bottom of the Frisbee and the pressure difference caused by the Bernoulli principle. When air hits the leading edge of the disc the air splits and travels above and below the disc. The cavity on the bottom side of Frisbee catches air traveling below the disc and forces it downward at the trailing edge of the disc. This downward stream of air generates a lift on the Frisbee. As the angle of attack increases, more of the air stream is deflected downward increasing the lift. The fluid above the disc follows along the curved surface and meets with the air that travelled below the disc. Therefore the air flowing above the disc must travel at a faster velocity to reconnect with the air below the disc. According to the Bernoulli principle the faster moving air will cause a low pressure region and the slow moving air will cause a high pressure region. Since the pressure above the disc is significantly less than the pressure below the disc, the air will push up resulting in lift on the Frisbee. Similar to drag it can be modeled using the equation

$$F_L = \frac{1}{2} \rho v^2 A C_L \quad (2)$$

where F_L is the lift force and C_L is the lift coefficient. The lift coefficient is a linear function of the angle of attack which means the lift force is also a linear function of the angle of attack.

The other aerodynamic effects are the moments, also known as torques, acting on the disc. During its flight a disc can experience torques about any of its three major axes which can greatly alter the disc's course. These moments are due to the complicated pressure distribution on the disc and the lift and drag forces. The pressure distribution can be uneven over the surface of the disc generating a torque. The reason lift and drag produce a torque is because they act on the center of pressure rather than the center of mass. When the center of pressure coincides with the center of mass the lift and drag forces will not produce a moment on the disc. However the center of pressure changes during the flight and typically does not coincide with the center of mass. Since we have three axes of rotation, we have three moments acting on the disc which are the pitch P , roll R , and spin down S moment. The pitch moment is associated with a torque around the y axis and a positive moment means the leading edge tilts upward. The roll moment is associated with a torque around the x axis and a positive moment means the Frisbee rolls to the right. The spin down moment is a torque along the z axis and counteracts the spin of the disc. These moments are calculated

using the equations

$$P = (C_{P0} + C_{P\alpha}\alpha + C_{Pp}p)\frac{1}{2}\rho v^2 Ad \quad (3)$$

$$R = (C_{Rs}s + C_{Rr}r)\frac{1}{2}\rho v^2 Ad \quad (4)$$

$$S = (C_{Ss}s)\frac{1}{2}\rho v^2 Ad \quad (5)$$

where d is the diameter of the disc and the rest of the undefined coefficients are experimentally calculated. These moments, along with lift and drag, make up the aerodynamic forces acting on the disc.

Strangely the spin of the disc has *no* effect on the lift or drag and has been shown to have minimal effects on the pitch and roll moments [1]. One aerodynamic force that is heavily influenced by spin is the Robins-Magnus force. Also known as the Magnus effect, the Robins-Magnus effect occurs when an object's spin creates a force perpendicular to the object's velocity. The Magnus effect has been studied extensively on tennis balls and baseballs however this force has been experimentally shown to be negligible on a Frisbee for low spin parameters [2]. Despite the spin not influencing the aerodynamic forces on a Frisbee, it is incredibly important to a Frisbee's flight as we will discuss in the next section.

Understanding the lift, drag and three moments are essential to explaining the motion of the disc. Moreover the angle of attack, velocity and disc area are present in all of the aerodynamic force equations meaning these variables are significant. For our experiment we will primarily look at the effects of the angle of attack and velocity because the surface area is the same among the discs we are testing. We will also look for how differences in the bottom cavity and leading edge affect the aerodynamics and prove that angular velocity has no effect on the lift and drag.

C. Angular Momentum

Angular momentum is the key to a Frisbee's stable flight. The angular momentum \vec{L} is the rotational equivalent of momentum and is equal to the moment of inertia I times the angular velocity $\vec{\omega}$ and points in the direction of the axis of rotation. For example, if the disc is rotating counterclockwise around the z -axis the angular momentum points in the positive z direction. Angular momentum can be maximized by altering the shape of the Frisbee to increase the moment of inertia and maximizing the angular velocity. The reason angular momentum is so important for stable flight is because it resists changes in the disc's orientation. According to the law of conservation of angular momentum, if there are no external torques acting on the disc the angular momentum will remain constant. The \vec{L} will always point along the z axis of rotation and the disc will be unable to tilt in any direction. Obviously during a Frisbee's flight there are going to be external torques acting on the disc which

will alter the direction and magnitude of the angular momentum.

As stated before there are several aerodynamic torques acting on the disc that will cause a rate of change in angular momentum: $d\vec{L}/dt = \vec{M}$ where \vec{M} is the sum of all moments acting on the disc. Naturally the magnitude of the angular momentum will decrease over time as the angular velocity slows across the duration of flight. However, the direction of the angular momentum has much more variability in the way it changes. This change in direction is known as precession $\vec{\Psi}$ and can greatly disrupt Frisbee flight. With a large rate of precession $d\vec{\Psi}/dt$ the disc will wobble along the x and y axis and in most cases turn over and fall to the ground. Luckily the rate of precession can be counteracted by increasing the angular velocity. The rate of change in angular momentum is equal to the rate of precession times the angular momentum [2]. Considering the magnitudes of these variables and using the dot for the time derivative, we have the equation

$$\dot{\Psi}L = \dot{\Psi}I\omega = M \quad (6)$$

From this relationship we know for a given moment an increase in angular velocity will cause a decrease in the rate of precession. Reducing the rate of precession vastly improves the stability of the disc and allows the disc to maintain its initial orientation.

Although the rate of precession can be minimized, precession is still one of the most important factors to consider when analyzing flight dynamics. While too much precession is detrimental to flight, a small amount of precession is necessary to prevent the disc from pitching and flying straight into the ground. When a disc is thrown with standard speed, angle of attack and angular velocity, the lift initially generates a negative pitch moment on the disc. If the disc were not spinning, the leading edge of the disc would tilt downward and the disc would fly into the ground. However the spin prevents the disc from pitching downward and instead causes the disc to roll [1]. This roll is due to the precession of the angular momentum by the pitching moment. The angular momentum rotates in the direction of the pitching moment and this rotation produces a perpendicular roll moment. For example, if the disc is rotating counterclockwise it has angular momentum pointing in the positive z direction. If a negative pitching moment, which points in the negative y direction, were to act on the disc, the angular momentum would rotate about the x axis towards the negative y direction. In accordance to the right hand rule, there would be a roll moment pointing in the positive x direction. Therefore the disc would have a positive roll moment and tilt to the right. We must also note that the disc can prevent roll moments and produce a pitch moment. If a roll moment were to act on the disc the angular momentum would rotate to cause a pitching moment. The introduction of angular momentum causes the disc to rotate on the axis perpendicular to the moment

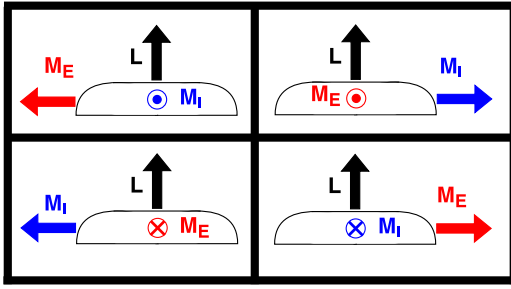


FIG. 2: The four motions for a counterclockwise rotating disc where L is the angular momentum, M_E is the external moment, and M_I is the induced moment.

acting on the disc. Fig. 2 shows the four possible motions for a counterclockwise rotating disc depending on which way the moment acts. While these four motions are accurate they are a simplification. It is rare that the moment would act perfectly along one axis especially with the center of pressure's irregular movement. In this case the induced moment will act along an arbitrary vector and cause the disc to pitch and roll simultaneously. Fortunately for throwers, much of this complicated motion can be inhibited by increasing the angular velocity.

The angular momentum generates the stability necessary for Frisbee flight. Discs with higher angular velocity are less susceptible to precession and the moments acting on them. The precession that does occur rotates the disc along the axis perpendicular to the angular momentum and the acting moment. With the center of pressure moving around the disc, the precession is difficult to determine because a moment can produce both pitch and roll. For our experiment we will attempt to observe this phenomenon and record how the angular velocity dampens the precession.

III. PROCEDURE

A. Apparatus

In order to observe the aerodynamic and angular momentum effects on the Frisbee, we had to construct an apparatus that would spin the disc and allow the disc to tilt. Our apparatus was comprised of several components from Servocity that are listed and shown in Fig. 3. The 15 inch aluminum channeling, from which the rest of the apparatus hung, was suspended between two ring stands. The channeling was either held tightly between the ring stands or held loosely on a bar placed across the ring stands. The tube clamps were then fastened to the bottom of the aluminum channeling with a 1/2 inch shaft placed through the clamps. This shafting acted as the axis of rotation for the disc. Two pillow block ball bearings were slid onto the center of the shafting and connected to more aluminum channeling. This suspended

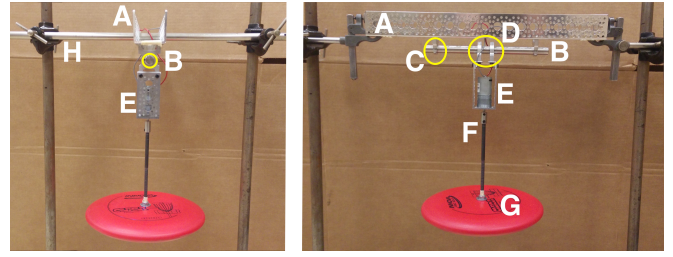


FIG. 3: The experimental apparatus which consists of A) 15 inch aluminum channeling, B) 1/2 inch shafting, C) tube clamps, D) pillow block ball bearings, E) 970 rpm economy motor held in aluminum bracketing, F) 1/4 inch shaft with screw threading, G) disc, and H) metal bar that the aluminum channeling rotates around. The right picture shows the apparatus held rigidly and the left picture shows the apparatus free to rotate.

aluminum channeling served as a mount for the 970 rpm economy motor that could rotate back and forth using the ball bearings. A 1/4 inch shaft with screw threading was then attached to the motor and the disc was held onto the shaft using nuts and washers. The motor was hooked up to a DC output power supply that controlled the spin of the disc. The voltage output of the power supply was read by a multimeter giving us a measurement for the angular velocity. A multi-speed and multi-angle AirKing model 99602ELA1 fan generated a wind stream that acted on the disc and a camera was placed perpendicular to the apparatus to record the disc's motion. The entire setup for our experiment is shown in Fig. 4. This setup allowed us to vary the height, distance from wind source, wind speed, angle of attack, and angular velocity to test the theoretical behavior of the disc.

The camera took video perpendicular to the axis of rotation. The computer program Capstone analyzed the video, using its built in angle tool, to determine the angle the disc rotated away from its original position. If the axis of rotation was perpendicular to the disc's relative velocity, the angle measured was a pitch angle. When the

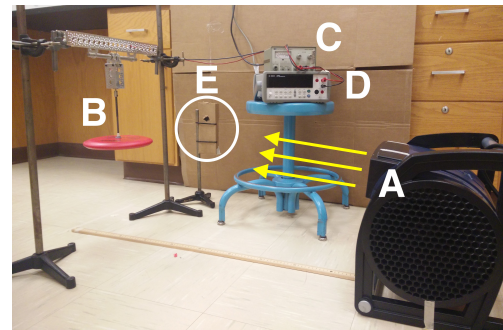


FIG. 4: The full experimental setup where A) is the multi-speed fan, B) is the experimental apparatus, C) is the power supply, D) is the multimeter, and E) is cardboard casing which held the recording device. The yellow arrows represent the direction of wind flow from the fan.

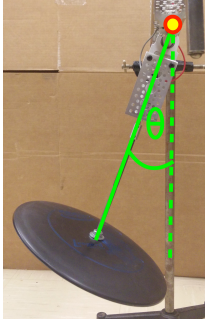


FIG. 5: The pitch angle θ generated by wind acting on the disc.

axis of rotation was parallel to the disc's velocity then we measured the roll angle. Fig. 5 shows an example of a pitch angle. During data collection, the pitch angle was measured for the aerodynamics experiments and the roll angle was used in the angular momentum experiment.

B. Discs

Each of our experiments were performed on three different Innova disc golf discs: a Leopard driver, a Shark mid-range, and an Aviar putter. Each disc is designed for a specific purpose in disc golf. The driver is made for maximum distance, the putter is made for maximum control and mid-range is a combination of both distance and control. These specific purposes are created by altering the edge and cavity of the disc. Drivers tend to have a sharper point on the edge while putters have a rounded edge. The cavity is the smallest for the driver and largest for the putter. For both these properties the mid-range disc is in between the driver and putter. Fig. 6 gives a visual representation of the edge and cavity for the three different discs.

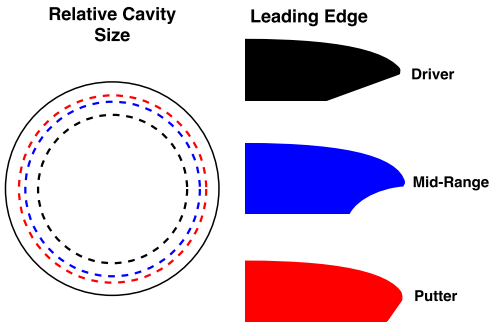


FIG. 6: A visual representation of the edge of each disc and a diagram of the cavity size for each disc. The area inside the dotted circle represents the cavity of the disc meaning the driver, black, has smallest cavity and putter, red, has largest cavity.

C. Aerodynamics

The first part of our experiment examined the aerodynamic forces acting on the disc. The aluminum channeling was held rigidly in between the two rings stands limiting rotations along the x axis. The disc could only rotate along the y axis perpendicular to the wind. The wind represented the relative disc velocity so our measurements were pitch angles. We collected multiple data runs for varied wind speed, angular velocity, and angle of attack for the different discs. The wind speed was changed by adjusting the speed setting of the fan and moving the disc closer to the fan. The angle of attack and angular velocity were both held constant and an anemometer was used to measure the wind speed acting on the disc. We then held the angle of attack and wind speed constant and increased the angular velocity by increasing the voltage to the motor. To change the angle of attack, the fan was tilted vertically from -5° to 20° and the distance between fan and disc was held constant. In order to keep the distance constant the disc's height above the fan was equal to $k\sin\alpha$ and the distance from the fan's base to ring stands was $k\cos\alpha$ where k is the distance between the disc and fan. Keeping the distance constant meant we could keep the wind speed the same and the spin was held constant as well. We repeated this procedure for the three discs.

The reason we see this pitch angle is because of the aerodynamic forces acting on the fixed disc. Normally these forces would cause the disc to tilt along the y axis of the disc. However the disc is fixed to the shaft preventing this rotation. Instead the aerodynamic forces cause the rotation around our apparatus's axis of rotation. The apparatus then settles into a stable point where the torques produced by the lift and drag are balanced out by the torque produced by the force of gravity. Fig. 7 is the free body diagram for our apparatus. Using the compli-

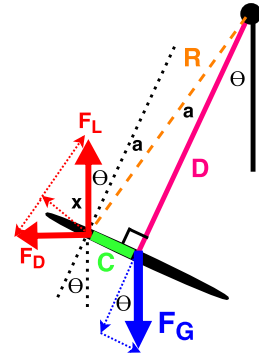


FIG. 7: The free body diagram for our apparatus where θ is the pitch angle, F_G is the force of gravity, C is the distance between center of pressure and center of mass, D is the distance between the axis of rotation and center of mass, R is the distance between the axis of rotation and center of pressure, a is the angle between R and D , and x is the angle between the lift force and perpendicular component to R .

cated geometry of the system, we can calculate that the sum of the two aerodynamic forces will be proportional to the pitch angle. Furthermore, since both aerodynamic forces are proportional to velocity squared, the pitch angle should also be proportional to velocity squared. The derivation for this relationship can be seen in Section VI. An important note is that this derivation is only valid when the angle of attack is zero. Changing the angle of attack changes the geometry of the system drastically.

D. Angular Momentum

The second part of our experiment tested the effects of angular momentum on the disc. Our apparatus was held loosely by a bar that was threaded through holes in the channeling. The channeling was able to rotate around the bar. This rotation was the relative disc velocity meaning the apparatus's axis of rotation was parallel to the relative velocity. Therefore we could measure the roll angle generated by an external moment. We did this by placing weights at one end of the aluminum channeling which caused the apparatus to tip around the bar. When the weight was removed the channeling rotated back to equilibrium and in doing so generated a moment on the disc. We then observed how the angular momentum produced a roll angle. For each disc, the roll angle was measured for different angular velocities and weights, corresponding to different moments.

The reason we see a roll angle is due to the precession acting on the fixed disc. When the channeling swings, the precession on the disc wants to rotate the disc along the x axis but the nuts and washers prohibit this motion. Thus the precession produces the roll angle on the apparatus's axis of rotation. Fig. 8 shows the motion of the apparatus and the moments acting on the system. An important

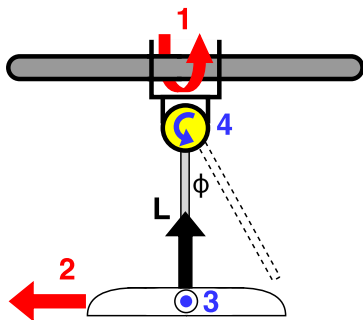


FIG. 8: The motion and moments experienced by the apparatus. L is the angular momentum of the spinning disc. 1) is the rotation of the aluminum channeling. 2) is the pitching moment generated by the channeling's rotation. 3) is the resultant roll moment on the disc. 4) is the rotation about the axis highlighted in yellow due to the roll moment. The dotted region represents where the motor shaft will rotate to and ϕ is the roll angle.

note is this experimental method does not settle into a stable point. Each time the aluminum channeling swings it produces a moment on the disc. So we measure the first roll angle experienced by the initial swing of the aluminum channeling and nothing after that.

IV. RESULTS AND ANALYSIS

A. Aerodynamics

1. Wind Speed

Before discussing the results we will detail a few problems were encountered in the data collection. The fan did not produce a perfect wind stream so the pitch angle fluctuated slightly. Also the anemometer used to measure the wind speed was damaged so there is a high uncertainty for the recorded speed.

According to the theory behind disc flight the square of the Frisbee's velocity should be directly proportional to the sum of the aerodynamic forces. We tested this relationship using our measured wind speed and pitch angle. The wind speed is a direct measurement of the velocity and the pitch angle is proportional to the aerodynamic forces. Therefore we plotted the pitch angle versus the velocity squared shown in Fig. 9. The data show a linear relationship and we can confirm that higher velocities mean greater aerodynamic forces. When we flipped the disc upside down on our apparatus there was little to no pitch angle produced. For this reason we assume that the lift force has a greater influence on the pitch angle than drag. Thus we conclude that higher wind velocities produce a stronger lift force. The most interesting information we can pull from the graph is the difference between each of the discs. The putter, red triangles, has the smallest slope meaning the wind speed has the least

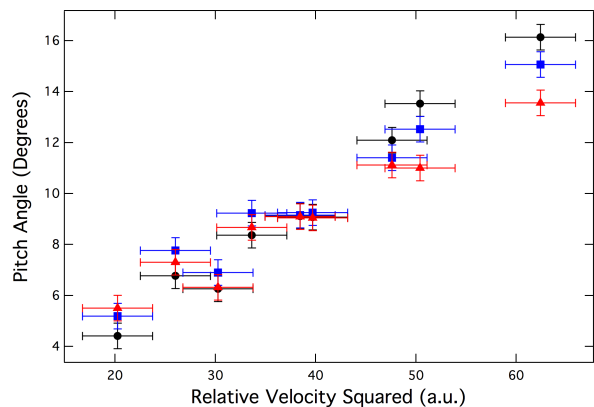


FIG. 9: The graph of the pitch angle versus the relative velocity squared for the three discs. The black circles represent the driver, the blue squares represent the mid-range, and the red triangles represent the putter.

effect on the putter's lift. The driver, black circles, has the steepest slope and the mid-range, blue squares, has a slope between the putter and driver. Furthermore the putter has the greatest lift at the lowest speed and the driver has the smallest lift. The opposite is true at the highest speed with the driver having the strongest lift and putter having the weakest. The mid-range is typically in between the the other two discs and at the middle speeds the three disc behave similarly. These trends are exactly what we expected to see. The driver is meant to be very fast to go the maximum distance and companies specifically state that it should not be thrown slowly. Our data agrees with their statement because throwing it faster generates more lift. On the other hand a putter is designed for accuracy and will typically be thrown at slower speeds. The data shows that of the three discs the putter has the maximum lift at slow speeds. The mid-range is made to be a mixture of the putter and driver allowing it to be thrown both fast and slow which is why the mid-range's lift is in between the driver and putter. Since our experiment controlled all other variables the only difference among the three discs is the shape of the leading edge and underside cavity. Therefore we have demonstrated that the design of the disc is important to lift as well as the wind speed.

2. Angular Velocity

The pitch angle versus the angular velocity for each disc is shown in Fig. 10. The angular velocity is measured by the voltage applied to the motor which we confirmed to be directly proportional to the angular velocity. As expected the spin has no effect on the lift on the disc. Any variation in the pitch angle can be accounted by the variance in video analysis or wind fluctuations. All three of the discs demonstrate that lift is unaffected by spin. Although the driver appears to have a trend with increasing angular velocity, the driver was unstable during experi-

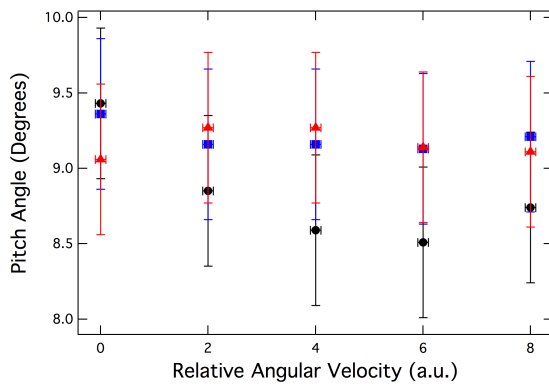


FIG. 10: The graph of the pitch angle versus the relative angular velocity for the three discs. The black circles represent the driver, the blue squares represent the mid-range, and the red triangles represent the putter.

mentation due to the low wind speed. Therefore we have reason to believe that the trend is simply experimental variation.

3. Angle of Attack

The pitch angle versus the angle of attack was plotted for each disc as shown in Fig. 11. Interpreting this plot the same way as the wind speed experiment, we would conclude that as the angle of attack increases the lift decreases. However previous wind tunnel experiments had demonstrated that increasing the angle of attack increases the lift. There are two possible causes for this deviation from previous experiments. First the center of pressure is changing as we change the angle of attack. Theoretically the center of pressure is dependent on the angle of attack. In our wind speed experiment, we were using a zero angle of attack which has a center of pressure towards the trailing edge of the disc. The lift acting on the back end of the disc would cause it to tilt nose down as we observed experimentally. Changing the angle of attack could move the center of pressure closer to the middle of the disc. Based on the design of our apparatus, the lift force acting in the center of the disc would produce a small pitch angle. Therefore it is possible that the angle of attack is not decreasing the lift but changing where the lift is acting on the disc. As the angle of attack increases the center of pressure moves towards the center of the disc leading to a smaller pitch angle. However the relationship between the angle of attack, center of pressure, and pitch angle is extremely complicated so we can not make a definitive conclusion that the center of pressure is moving.

The second explanation is that our wind stream was too small. When the disc rotates it may rotate outside of the wind stream leading to a smaller pitch angle. In our plot we see that the pitch angle increases at the final data point which could have been due to the disc stay-

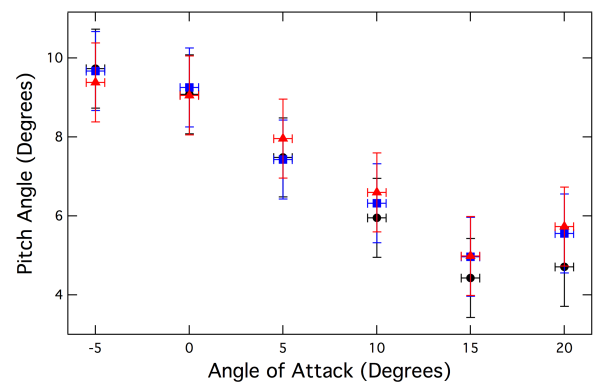


FIG. 11: The graph of the pitch angle versus the angle of attack for the three discs. The black circles represent the driver, the blue squares represent the mid-range, and the red triangles represent the putter.

ing in the wind stream. For future experimentation, a full wind tunnel should be built around our apparatus to ensure consistent airflow when the disc rotates. More experimentation needs to be done to determine the relationship between angle of attack and the pitch angle.

B. Angular Momentum

We plotted the roll angle versus the angular velocity as shown in Fig. 12. The theory states that a higher angular velocity would cause a decrease in precession because the angular momentum of the disc resists the effects of torques on the disc. Therefore a higher angular velocity would decrease the roll angle induced in our experiment. According to our data, as the angular velocity increases the roll angle produced by the moment increases. While these results seem contradictory to theory, they demonstrate a typically neglected effect: the Robins-Magnus effect. When a circular object, such as a Frisbee, rotates the two sides of the disc are traveling in opposite directions. As that disc travels through air, one side of the disc opposes or slows the air motion and the other assists or speeds up the motion. The Bernoulli principle tells us that the faster moving air has a lower pressure than the slow moving air. The higher pressure region pushes on the disc creating a side force perpendicular to the direction of the motion. The Robins-Magnus effect is illustrated by Fig. 13. In Section II B we stated that the Robins-Magnus effect could be ignored but only for low spin parameters. Our experiment did not have a low spin parameter because the angular velocity of the disc was much greater than the translational velocity of the disc. Therefore increasing the angular velocity increased the Robins-Magnus effect making the roll angle larger as we observed.

Despite multiple attempts no experiment could be conducted to demonstrate precessional dampening by the

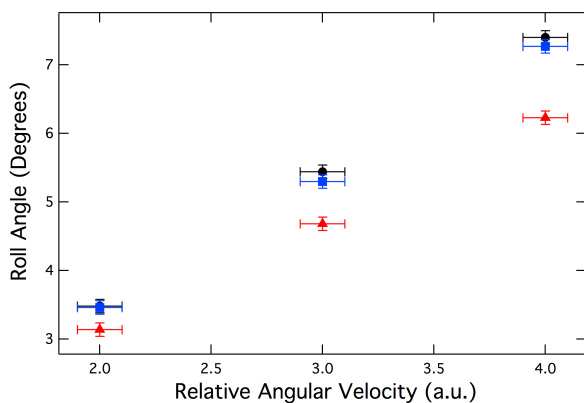


FIG. 12: The graph of the roll angle versus the relative angular velocity for the three discs. The black circles represent the driver, the blue squares represent the mid-range, and the red triangles represent the putter.

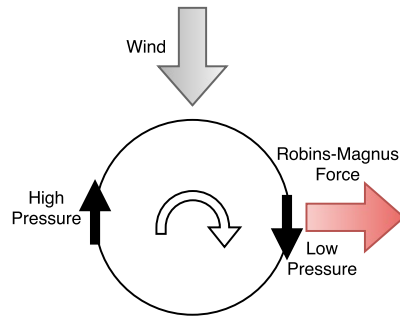


FIG. 13: A visual representation of the Robins-Magnus effect on a disc.

angular velocity. We believe our apparatus has this limitation because of the relative strength of the angular momentum to the other forces acting on the disc. If the disc has a measurable rotation along the axis of rotation the force of gravity that brings it back to equilibrium will far out weigh the angular momentum. Increasing the angular velocity to combat the force of gravity does not work because we then introduce the Robins-Magnus effect. Our apparatus does not have the sensitivity to measure the effects of angular momentum which is disappointing considering how important it is to Frisbee flight.

V. CONCLUSION

Our experiment results, along with theory, provide a basis for analyzing disc flight and describe the effects of multiple factors. The two most important components of disc flight are the aerodynamic forces and angular momentum. The aerodynamic forces, mostly lift, keeps the disc aloft for extended periods of time and is influenced by the angle of attack, wind speed, and design of the disc. The wind speed results confirmed that the lift is directly proportional to velocity squared and demonstrated that the disc shape effects how much lift is produced at each speed. The angle of attack results were inconclusive however previous experiments have shown that the angle of attack affects Frisbee flight. The angular momentum provides the necessary stability by reducing precession and resisting external torques. The angular momentum also induces a roll moment when there is an external pitch moment and visa versa. Although we could not prove these effects experimentally, these two phenomena have been widely studied in gyroscopes and can still be applied to the Frisbee. Bringing all these concepts together we have the knowledge required to begin analyzing the complicated dynamics of disc flight.

VI. APPENDIX

In order to prove the relationship between velocity squared and the pitch angle we must consider the sta-

ble point of our apparatus. The apparatus reaches a stable point when the torque produced by the aerodynamic forces, τ_L for lift and τ_D for drag, equals the torque generated by gravity τ_G . From the geometry in Fig. 7 in Section III C we have the equation

$$\begin{aligned}\tau_G &= \tau_L + \tau_D \\ F_G D \sin\theta &= F_L R \cos x + F_D R \sin x.\end{aligned}\quad (7)$$

with the variables defined in Fig. 7. We need the components of F_D and F_L perpendicular to R so we find x to be $90 - \theta - a$. For simplicity we define $b = 90 - \theta$. Applying trigonometric identities, we can rearrange the right side of Eq. 7 as follows

$$\begin{aligned}&F_L R \cos x + F_D R \sin x \\&F_L R \cos(b - a) + F_D R \sin(b - a) \\&F_L R [\cos b \cos a + \sin b \sin a] + F_D R [\sin b \cos a - \cos b \sin a].\end{aligned}$$

By the definition of sine and cosine, $\sin a = C/R$ and $\cos a = D/R$. Furthermore $\sin b = \cos\theta$ and visa versa. Substituting these relationships into the previous expression and reducing we get

$$\begin{aligned}&F_L R [\sin\theta \frac{D}{R} + \cos\theta \frac{C}{R}] + F_D R [\cos\theta \frac{D}{R} - \sin\theta \frac{C}{R}] \\&F_L [\sin\theta D + \cos\theta C] + F_D [\cos\theta D - \sin\theta C]\end{aligned}$$

Substituting this new expression in for the right side of Eq. 7 and simplifying we get

$$\begin{aligned}F_G D \sin\theta &= F_L [\sin\theta D + \cos\theta C] + F_D [\cos\theta D - \sin\theta C] \\F_G &= F_L [1 + \cotan\theta \frac{C}{D}] + F_D [\cotan\theta - \frac{C}{D}] \\F_G &= F_L [1 + \cotan\theta \tan a] + F_D [\cotan\theta - \tan a]\end{aligned}$$

If we assume that a is approximately constant then we can state that $F_G \propto (F_L + F_D) \cotan\theta$ and since F_G is constant we have $F_L + F_D \propto \tan\theta$. Since the pitch angles are all 16° or below we can use the small angle approximation for tangent that states $\tan\theta = \theta$. Also both F_L and F_D are proportional to velocity squared so our final relationship is $v^2 \propto \theta$. This derivation only applies to our apparatus when the fan has a zero angle of attack. The geometry becomes more complicated when we change α altering both the aerodynamic forces and center of pressure.

[1] R. Lorenz, *Spinning Flight: Dynamics of Frisbees, Boomerangs, Samaras, and Skipping Stones* (Springer, New York ; London, 2006).

[2] S.A. Hummel, *Frisbee Flight Simulation and Throw Biomechanics*, University of California Davis, 2003.