Burning Up:
The Effect of Slope Upon Forest Fire Propagation

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A scaled version of a forest was created using a matchstick grid stuck onto an aluminum plate using a flame resistant putty. The setup allowed for one end of the aluminum plate to be raised, creating a constant positive slope while still keeping the spacing between matchsticks constant. By lighting one end of the matchstick grid on fire and filming the flame as it traveled across the grid, the rate of spread of the fire, R, was measured for a specific angle of elevation θ. The results from multiple angles were used to investigate whether or not this setup could be used to predict the relationship between R and θ. This analysis was done in the hopes that this data could be scaled up and used to analyze real forest fires. The setup was found to be able to analyze the relationship and the data collected supports the Rothermel model for forest fires which uses the relationship $R \propto \tan^2\theta$.

I. INTRODUCTION

Every year forest fires cause an incredible amount of damage. In North America alone 750 thousand hectares of forest are burnt down annually. For comparison, the state of Delaware is approximately 645 thousand hectares in size. This destruction costs the United States over 2 billion dollars a year [2]. As a result, the business of minimizing forest fire destruction is quite large. There is ongoing research on how forest fires behave in order to improve forest fire response. A better understanding of forest fire behavior would allow firefighters to more effectively evacuate people from dangerous areas, to better predict which areas are at risk, and to more efficiently distribute their manpower; all of which would reduce the damage caused by forest fires.

In this paper we will be building upon the research performed by Punckt *et al.* [1]. In their article they create a scale model of a forest using matchsticks and examine how the density of the matchsticks affects how fast the fire spreads through the forest after a few initial matches are lit on fire. Using a similar setup we examined whether or not this scale model of a forest can be used to examine the effect a constant uphill slope has upon the speed of the fire. An important note is that because the experiment used a scaled down model the results obtained do not directly apply to real forest fires. In order to scale the results of this experiment from describing matches to describing trees, an in-depth mathematical analysis would need to be done.

II. THEORY

One of the main reasons why slope has an effect upon the speed of fires is shown in Fig. 1. In this figure there is a grass fire spreading quickly towards unburnt grass, which we refer to as fuel for the fire. In the left frame the flames travel uphill, and as the heat from the flames rises the fuel directly in front of the flame is heated. Thus when the fire comes into contact with this fuel, the fuel is already part of the way towards catching fire. This causes the fuel to ignite more quickly meaning the fire spreads faster. In the right frame of the figure the flames travel downhill. This time as the heat from the fire rises it does not heat up the fuel. Thus when the fire comes into contact with the fuel, the fuel will take longer to ignite causing the fire to spread more slowly. This is a simplified explanation and ignores factors such as drafts being created by the fire, but gives an intuitive sense for why fire should move faster uphill and slower downhill.

There are many different mathematical models for describing forest fire behavior, each with their advantages and disadvantages. The biggest difference between the models is the scale at which the fire is viewed. For our experiment we are looking at fires whose size are the same order of magnitude as the height of our fuel, which are matches. Because of this we decided to use the model developed by Richard C. Rothermel, known as the Rothermel model [3]. In the Rothermel model, a forest fire is viewed as a series of ignitions. Through a series of interactions the ignition of one piece of fuel leads to the ignition of another piece of fuel. This occurs repeatedly over a large collection of fuel thereby modeling the full forest fire. The rate of spread of the fire R is defined as the velocity of a fire front normal to its surface [3]. In layman's terms, R is the speed with which a line of fire advances forward.

Rothermel derived the equation for the rate of spread without wind or slope,

$$R_0 = \frac{I_p}{\rho_b \epsilon Q_{ig}}, \tag{1}$$

in terms of the propagating heat flux I_p, the bulk density of fuel ρ_b, the ratio of fuel that the flames can heat to the total amount of fuel ϵ, and the heat required to ignite a single piece of fuel Q_{ig}. Rothermel then reasoned that introducing wind or slope would have an effect upon the propagating heat flux and thus the full equation for R

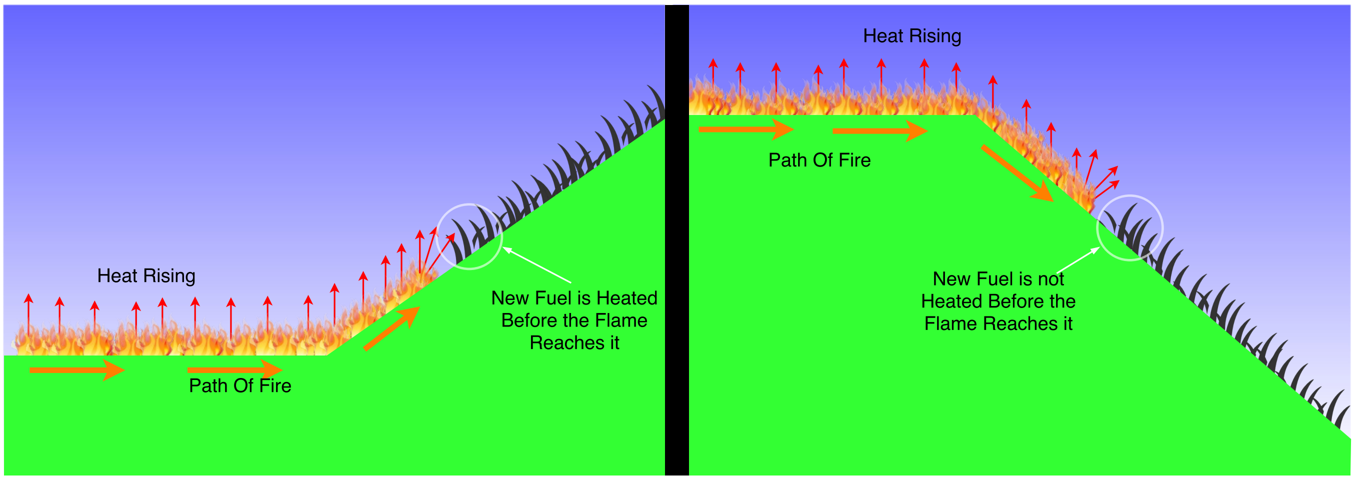

FIG. 1: Illustration of why fire burns faster uphill and slower downhill. When going uphill the fuel for the fire is heated before the flame reaches it. When going downhill the fuel is not heated before the flame reaches it.

should be

$$R = \frac{I_p(1+\phi_w+\phi_s)}{\rho_b \epsilon Q_{ig}} \tag{2}$$

$$= R_0\left(1+\phi_w+\phi_s\right), \tag{3}$$

where ϕ_w and ϕ_s are dimensionless coefficients that are functions of the properties of the wind and the slope respectively. This equation shows that the rate of spread is directly proportional to the slope coefficients,

$$R \propto \phi_s. \tag{4}$$

Rothermel did not mathematically derive the coefficient ϕ_s, stating that it must be found from experimental data. He performed a series of experiments upon model trees built out of wood and found ϕ_s to be described by

$$\phi_s = 5.275\beta^{-0.3}\tan^2\theta, \tag{5}$$

where β is a dimensionless quantity representing how much volume the fuel takes up compared to the total volume and $\tan\theta$ is the slope of the surface the fuel is on. From this equation we can see that

$$\phi_s \propto \tan^2\theta. \tag{6}$$

Combining this equation with Eq. 4 we obtain the relationship

$$R \propto \tan^2\theta, \tag{7}$$

which is the relationship we will be investigating.

The way that we will investigate this relationship is by holding constant from trial to trial all variables other than θ: R_0, ϕ_w, and β. For each trial we will have a set value for θ and record the associated R value. By graphing R versus θ for the data collected and applying different lines of best fit we will then be able to determine whether Eq. 7 properly describes the relationship between R and θ or if there is a more accurate relationship.

III. PROCEDURE

A. Apparatus

The apparatus we used is based upon the one used by Punckt *et al.* [1]. The goal of our apparatus was to create a model forest out of matchsticks upon a surface whose slope could be varied. We did this by constructing an evenly spaced grid of matches upon an aluminum plate of adjustable slope as is shown in Fig. 2. By placing aluminum blocks underneath and in front of the aluminum plate, an adjustable constant slope, $\tan\theta$, in the xz-plane could be created. On top of the plate a flame resistant putty was applied that the matches were inserted into in order to keep them in place throughout the experiment.

The specific putty that was used was 3M$^{\text{TM}}$ Fire Barrier Moldable Putty. This putty was chosen because it was very effective at resisting high temperature flames and would not dry out nor crumble. These both were very important because they allowed us to reuse the same putty for all of our data runs. When applying the putty to the aluminum plate we found that a layer of putty 5mm to 7mm thick was enough to securely hold the matches in place for our experiment.

After the putty was applied to the aluminum plate the apparatus was propped up to the desired angle using other pieces of aluminum. To measure the slope of the aluminum plate an iPhone 5S$^{\text{TM}}$ was placed upon the

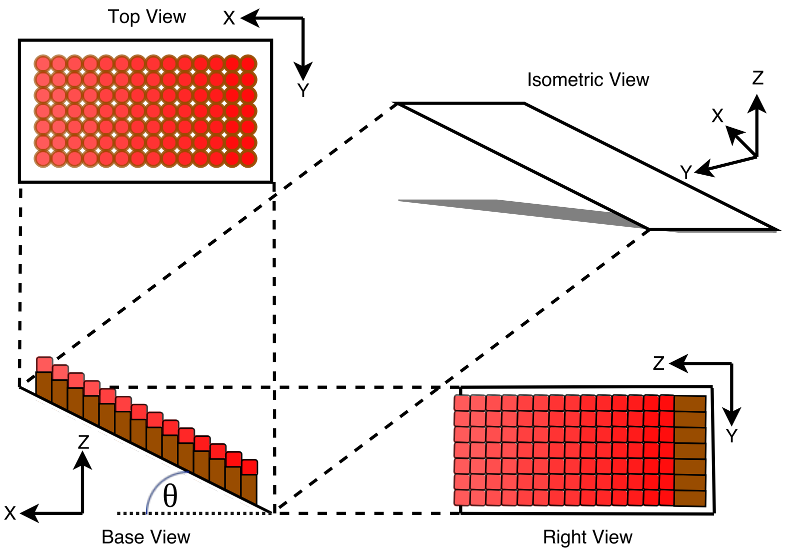

FIG. 2: Diagram of the apparatus. Note that in the isometric view the matches are not shown and that the flame resistant putty is not shown in any view.

aluminum plate, on a section that did not have the putty applied to it, and the built in level function was used to measure θ. In order to maintain a constant spacing between the matches in the xy-plane, a plastic guide was used. This guide consisted of 3 mm holes which were 5 mm apart, measured from center to center and is shown in Fig. 3. These holes were just large enough that when matches were fed through the holes they would snugly stay in place. After filling the guide with matches the guide was held parallel to the ground and temporarily inserted into the putty in order to leave impressions where the matches would be inserted into the putty. Because the desired grid of matches was larger than the guide, the guide was then shifted and reinserted into the putty to make a larger grid of matchstick impressions. In order to ensure that the new impressions were in line with the first set of impressions, there was an overlap of two rows in the two sets of impressions. When aligning the grid of matches for the second set of impressions, two rows of

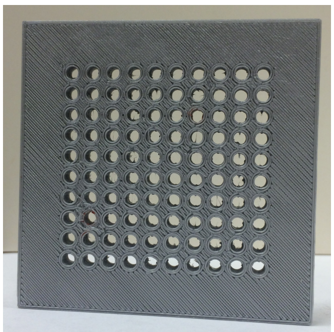

FIG. 3: The guide used to keep the spacing between the matches constant.

the matches in the grid were lined up with impressions that were already in the putty. This was repeated until the full grid of impressions was created.

Based upon the previous research of Punckt *et al.* [1] we decided not to construct our matchstick grid at 100% density because that would cause the flames to progress too rapidly and chaotically for us to reliably collect data. We chose instead to construct our matchstick grid at 50% density by making sure that every hole in our grid containing a match had all four holes adjacent to it empty. We also chose to make our matchstick grid be a 9×9 grid after the 50% density was factored in, meaning the grid would have been an 18×18 grid if we had constructed it at 100% density. We chose this size because it allowed the fire to burn for three to five seconds before covering the entire grid of matches. This ensured that the experiment was quick to set up but still produced enough data to analyze. A picture of what the apparatus looked like with 50% density with and without the matches inserted is shown in Fig. 4 and Fig. 5. This density was used for all data collections thereby ensuring that β in Eq. 5 was a constant value for all of our data.

The experiment was conducted behind a large building, providing wind cover from two directions. In addition, the entire apparatus was encircled by a cardboard wall more than 10 times the height of the apparatus, acting as a wind shield. In order to ensure that our wind shields were working properly we light a single match and held in within the shielded area and watched the flame. If the flame flickered in any way or was tilting towards one direction, we knew that we needed to adjust our wind shields. Because of this we assume that we kept ϕ_w in Eq. 3 constant throughout all of our data collections. Because we used the same brand of matches and performed all of our experiments under the same conditions, we assumed that R_0 in Eq. 3 would also be constant across all data runs.

B. Data Collection

Once the matchstick grid was constructed a high speed camera was positioned at a safe distance and set to film the grid of matches. The specific camera we used was the main camera of an iPhone 5S^{TM} filming in the slow-motion mode, allowing 120 frames to be captured per second. Additional matches were added to the center of one of the edges parallel to the y-axis in order to facilitate ignition of the system. This tight cluster of ignition matches was ignited by a single hand-held match and the resulting propagation of the fire across the grid was filmed.

The video of the fire was exported onto a computer and was viewed using the program Tracker. This program allowed us to step through the video frame by frame and manually mark where the furthest advanced point of the fire was. In order to minimize the chance of error with this marking, we only marked frames where the furthest advanced point was clearly discernible. Because our videos were all at least 200 frames long, we were still able to mark a significant number of frames. In order to account for the fact that the videos were not always filmed from the exact same angle, we used the tools in Tracker to redefine the axes within each video such that the path the flame took across the matchstick grid was parallel with the x-axis and that the distance across the matchstick grid was the same distance, which we set to be 100 arbitrary units. An example of what this looked like is shown in Fig. 6.

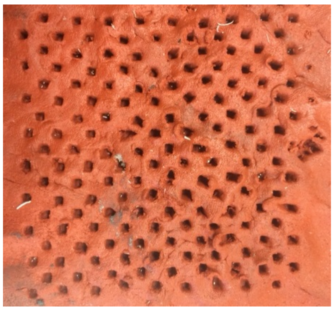

FIG. 4: Closeup of the putty with the evenly spaced impressions for where the matches should go for a 50% density fill.

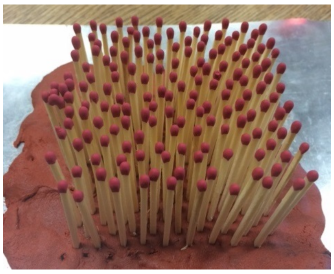

FIG. 5: Apparatus with matches inserted into the putty at 50% density. The cluster of matches on the left side are there to facilitate lighting the initial matches in the grid.

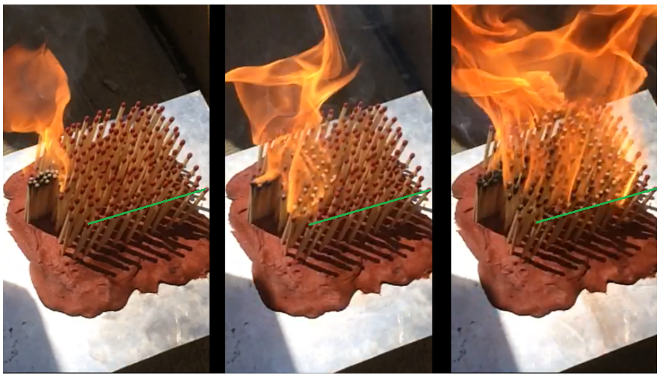

FIG. 6: Pictures of the fire spreading across the matchstick grid. The pictures are from the fire at 1 second (left), 2 seconds (middle), and 4 seconds (right) after the initial matches were lit. The green line lies demonstrates the x axis that was defined within the Tracker program and was set to be 100 units long.

The data that Tracker produced was exported into Igor Pro, and the distance the fire had covered was graphed versus time as is shown in Fig. 7. A linear fit was applied to the data, and the slope of this line, with the uncertainty Igor Pro calculated for it, was used as the rate of spread R for that data run. The units of R were arbitrary but because we were examining the relationship

$$R \propto \tan^2 \theta, \tag{7}$$

the fact that the units were arbitrary was not a problem. After obtaining an R value for each of our data runs, we graphed R versus θ in Igor Pro and applied different lines of best fit in order to see whether or not the relationship in Eq. 7 accurately described the relationship between θ and R.

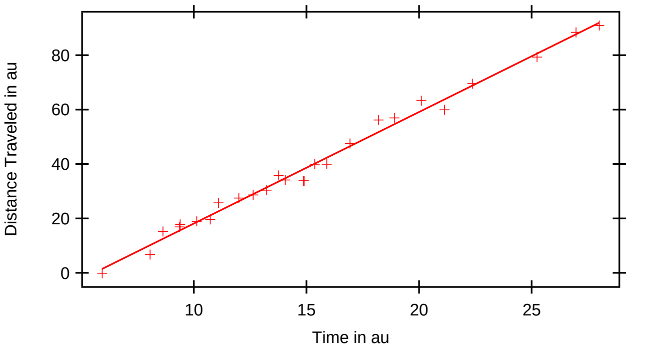

FIG. 7: Graph of distance traveled versus time for $\theta = 7°$ with the line of best fit for the data. The equation for this line is $y = a + bx$ with $a = -22 \pm 1$ and $b = 4.1 \pm 0.08$. We thus use 4.1 ± 0.08 as the R value associated with $\theta = 7°$.

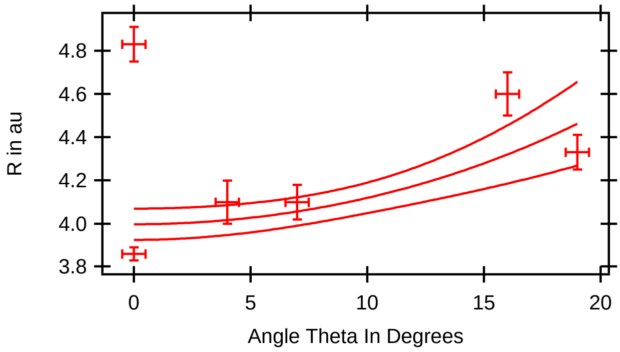

FIG. 8: Graph of θ versus R with a line of best fit of the form $R = a(1 + b\tan^2\theta)$ and the 95% confidence bands applied. Igor Pro found that for 95% confidence $a = 4 \pm 0.08$ and $b = 1 \pm 0.5$.

IV. RESULTS

The angles θ that we investigated were 0°, 4°, 7°, 16°, and 19°. Because of the limitations of the level function of the iPhone 5S™, these θ values could have been ±0.5° off. Because Igor Pro calculated R using a series of points over time, each R value also had an uncertainty associated with it.

The data were graphed and a line of best fit of the form $R = a(1+b\tan^2\theta)$ was applied with a weighting inversely proportional to the uncertainty associated with each data point producing the graph shown in Fig. 8. This graph includes the line of best fit and its 95% confidence bands, which Igor Pro found to be $a = 4 \pm 0.08$ and $b = 1 \pm 0.5$. Looking at the graph we can see that five of our data points are consistent with the trend and that the one outlier of the trend also appears to be an outlier of the data set in general. As such we conclude that not only does our apparatus allow us to see a relationship between R and θ, but that our data supports the relationship in Eq. 7.

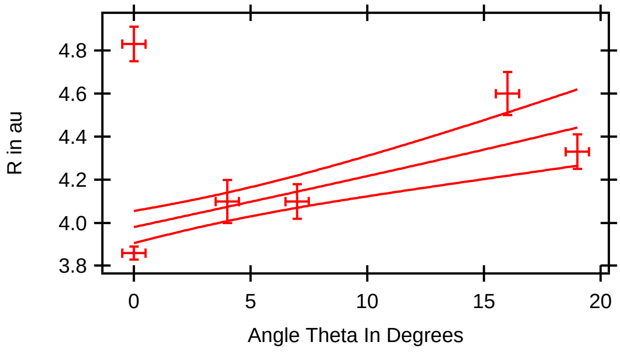

FIG. 9: Graph of data collected with a line of best fit of the form $R = a(1+b\tan\theta)$ and the 95% confidence bands applied. Igor Pro found that for 95% confidence $a = 3.98 \pm 0.07$ and $b = 0.3 \pm 0.1$.

We were surprised that the Rothermel model used the relationship $R \propto \tan^2\theta$ because we would have intuitively guessed that $R \propto \tan\theta$ would have been the relationship between R and θ. As such we decided to test out the relationship $R \propto \tan\theta$ with our data. We graphed R versus θ and applied a line of best fit of the form $R = a(1 + b\tan\theta)$, again weighting each point inversely proportional to the uncertainty associated with the data point. The graph of the line of best fit and the 95% confidence bands are shown in Fig. 9. Igor Pro found these confidence bands to be $a = 3.98 \pm 0.07$ and $b = 0.3 \pm 0.1$. From the graph we can see that once again five out of the six data points are consistent with the trend.

What is interesting about these two graphs is that the fit based off of our intuition both fits the data better and is more precise than the fit based off of Rothermel's findings. Both fits had an uncertainty of 2% for the value of a, but the fit for $R \propto \tan\theta$ had an uncertainty of 33% for the value of b while the fit for $R \propto \tan^2\theta$ had an uncertainty of 50% for the value of b. These uncertainties are too large for us to conclusively say that our fit is better than Rothermel's fit, but it does give support to the idea that R may be proportional to $\tan\theta$ and not to $\tan^2\theta$ for the matchstick simulation of a forest fire.

V. FUTURE WORK

When we were collecting data we purposefully chose to examine a small range of the possible values of θ. We made this choice because we wanted to obtain data for the values of θ most similar to real hills, and then use that data to extrapolate for more extreme hills. After analyzing the data however, we realized that by doing so we confined our possible $\tan^2\theta$ values to an extremely small region. By only looking at θ values between 0° and 20° we confined our possible $\tan^2\theta$ values to the interval [0, 0.37]. If we would have increased our range of θ values to 50° we would have nearly quadrupled the possible values of $\tan^2\theta$ and had the interval [0, 3.84]. While the larger range of θ values would include unrealistic values for a hill, the larger range would have helped make the deviation of data points from the line of best fit more noticeable and increased our confidence in determining which data points were not consistent with the trend line. This would also make it easier to see the difference between $\tan\theta$ and $\tan^2\theta$, thereby making it easier to discern which line of best fit is more accurate. Another way to increase the range of θ values that would be beneficial would be to include negative θ values, representing forest fires traveling downhill.

Collecting more data points at each θ value would be beneficial because it could allow us to rule out the outlier data point that we collected. If it turns out that the R value for a given θ is consistent between trials, then the outlying point can be ruled out as the result of some sort of error. If however, the R value is not consistent between different trials of the same θ value, then either there is a flaw in our experimental method, or there is a parameter that we did not consider that is playing a role in how fast the fire propagates.

VI. CONCLUSION

Every year forest fires cause billions of dollars in damages to countries around the globe. As a result lots of research is being done into how forest fires behave, in order to improve forest fire fighting efficiency. In this experiment we used an apparatus consisting of matches attached with flame resistant putty to an aluminum plate to simulate forest fires traveling uphill. By varying the slope of the aluminum plate, $\tan\theta$, and measuring the rate of spread of the fire R we attempted to verify the relationship $R \propto \tan^2\theta$. Our data did indeed support this relationship but supported more strongly the relationship $R \propto \tan\theta$. Both of these show that in our system R depends upon θ, just as it does in a real forest fire. With data from a larger range of θ values and more data points per θ value the exact relationship should be able to be determined. With further mathematical analysis these results should be able to be scaled up to represent actual trees burning in a forest fire.

VII. ACKNOWLEDGMENTS

I would like to thank the College of Wooster Physics Department for funding this project and for assisting me in many ways throughout the experiment. I would also like to thank Michael Bush, Zane Thornburg, and everyone else who helped with the safety procedures for the experimentation.

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