Rotational Analogies for Viscous Drag

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When an object moves through a fluid it will experience a drag force, the relationship for which is understood for linear motion. Experimentally we can show that these relationships can be extended to include rotational motion. This is done by spinning a steel rotor ball and allowing it to slow down as a result of the drag force created by air. Using graphical methods and applying a power law fit we can determine the power to which velocity is raised in the proportionality between the drag force and the velocity. The sphere is smooth so the flow of air around it should be laminar, and as expected the force was found to be proportional to angular velocity to the 1.03 ± 0.01 power, analogous with the translational model. By adding flags to the sphere we can try to create turbulence, doing so produced values between 1.5 and 2 for the power of angular velocity. This suggests the presence of varying degrees of turbulence analogous with the translational model. Thus we experimentally have demonstrated that there is a rotational extension for an object moving in a fluid.

I. INTRODUCTION

An object moving through a fluid will experience resistance in the form of a drag force. This force was described by both Sir George Gabriel Stokes and Sir Isaac Newton for different fluid cases. Sir George Gabriel Stokes, who is famous for his contributions to the Navier-Stokes fluid equations, formulated equations to describe the resistive force exhibited on a sphere moving in one direction through a fluid. These equations approximate the force to be proportional to the velocity of the object, but these equations assume the fluid to be laminar. Fluid flow is laminar if it is regular and smooth, not turbulent. This assumption is significant because Newton later showed that if the fluid is turbulent the drag force will be proportional to the velocity squared.

Through experimentation this project aimed to see if there was a rotational analogy to the translational equations put forward by Stokes and Newton. Would the rotational drag force behave as velocity, velocity squared, or some higher order of velocity? This was tested experimentally by using a rotating steel sphere and measuring the decay of it's angular velocity over time due to the drag force of the air surrounding it. If we plot the change in angular velocity against the angular velocity and apply a power law fit, the power of the resulting fit line will be the power velocity is raised to in the proportionality. Thus the power law fit should produce a power of one when we spin the rotor ball because the flow of the air should be laminar around the smooth ball. However if we add flags to the rotor ball the air flow may become turbulent and then the power law fit should produce a value of two. If either of these scenarios are not seen then the rotational system is not analogous with the translational models.

II. THEORY

The models describing the resistive force created by a fluid put forward by Sir George Gabriel Stoke and Sir Isaac Newton are the most generally accepted. Stokes' equation for a sphere traveling in one direction through a fluid takes the form

$$\vec{F} = -c\vec{v}, \tag{1}$$

where $\vec{F}$ is the drag force and c is a constant which is dependent on the shape of the object and the viscosity of the fluid. The force is negative because the drag force is a vector, and it opposes the motion of the object. Newton's equation for when the flow is turbulent describes the force as proportional to the square of velocity so his equation has an additional factor of v and is written as

$$\vec{F} = -cv^2\hat{v}. \tag{2}$$

Assuming that there is a rotational analogy to these equations we can find them by replacing the translational variables with their rotational counterparts. The translational force $\vec{F}$ would be replaced with $\vec{\tau}$, which is the torque or rotational force due to the fluid, and the translational velocity v would be replaced with angular velocity ω. However, we do not know what order the dependence on ω will be, so we will simply use n as a place holder and our rotational analogy becomes

$$\vec{\tau} = -c\omega^n\hat{\omega}. \tag{3}$$

For simplicity the vector notation will now be dropped and all values will be scalars since we know the direction of the drag force will be opposite the direction of motion. Now we will make Eq. 3 of more use for our experiment by substituting $I(d\omega/dt)$ for τ since $\tau = I\alpha$ and $\alpha = d\omega/dt$. Then we can solve the resulting equation for $d\omega/dt$ and find that

$$\frac{d\omega}{dt} = -\frac{c}{I}\omega^n = -\lambda\omega^n, \tag{4}$$

where $(c \;/\; I)$ is a new, rotational constant which we will call λ. The final form of this equation,

$$\frac{d\omega}{dt} = -\lambda\omega^n, \tag{5}$$

is a differential equation, but to solve it we must assign a value to n. If we assume laminar flow as Stokes did we can use $n = 1$ to form the first order differential equation

$$\frac{d\omega}{dt} = -\lambda\omega, \tag{6}$$

which can be solved easily. The only function which returns itself after a derivative is taken is an exponential function, and to acquire the factor of $-\lambda$ the exponent must be of the form $-\lambda t$ since the derivative is taken with respect to t. Thus the solution to Eq. 6 is

$$\omega = \omega_0 e^{-\lambda t}. \tag{7}$$

However if we use Newton's approximation for turbulent flow and set $n = 2$ the differential equation which results is

$$\frac{d\omega}{dt} = -\omega_0\omega^2. \tag{8}$$

This is a separable first order differential equation, but solving it is not as trivial as the previous equation. When Eq. 8 is solved the result is

$$\omega = \left(\frac{1}{\omega_0} + \lambda t\right)^{-1}. \tag{9}$$

III. PROCEDURE

This experiment was conducted using an Ealing air gyroscope, which consists of a 10cm steel rotor ball with a rod extending from it and a base to hold the ball. The base is connected to a compressed nitrogen gas source which creates a cushion of air for the rotor ball to sit on so that the only source of friction on the rotor ball is from the air. The rotor ball is positioned such that a laser will reflect off of the surface of it's upper hemisphere, be focused through a lens, and be incident on a fast photodiode. The upper hemisphere of the rotor ball is divided into four equally sized and shaped quadrants by black electrical tape. When the rotor ball rotates the laser will not reflect off of the tape and the photodiode will thus only receive a signal when the laser is incident on the shiny surface of the rotor ball and not when the laser is incident on the tape.

The photodiode will turn the laser signal into high and low voltages depending on if the laser is incident on the shiny surface of the rotor ball or the black tape respectively. This voltage signal is then sent to a Schmitt Trigger which shapes the signal into a sharp square wave. This square wave can be read by the Hewlett Packard 5385A Frequency Counter which is also connected to a computer. The computer then records the data using LabView. LabView will take the frequency data and calculate the angular velocity ω of the rotor ball. LabView averages the frequency over a 10 second period and multiplies it by $\pi/2$ since $\omega = f \times 2\pi$ for each data point.

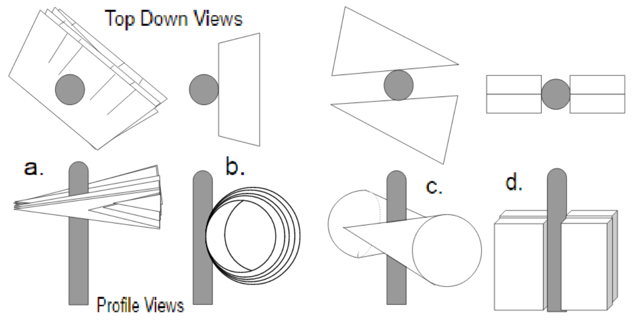

FIG. 1: The Top down and profile schematics for each flag a. TailFeatherFlag b. OnionSliceFlag c. DoubleWindsockFlag d. MassiveFlag

The reason why LabView multiplies by $\pi/2$ instead of 2π is because the frequency sent from the frequency counter will be four times larger than the actual frequency since the tape is seen four times per revolution. This process was repeated with the addition of multiple different flags, shown in Fig. 1, to the rod of the rotor ball.

IV. DATA PRESENTATION & ANALYSIS

The angular velocity was measured for the rotor ball spinning with and without the addition of multiple different flags. Fig. 2 displays the data collected for seven runs with four different flags plotted as the angular velocity ω versus time on a semi-log plot. In this graph the general trends can be seen but the graph is not conducive to further analysis because the paths all appear to be somewhat linear. To gain further insight into the system we can plot $d\omega/dt$ versus ω on a log-log plot. Graphing the data in this way is useful because we can apply power law fits to the data, and the exponent of the fit will be the value for n in Eq. 5. Thus the power law fit has given us the power for relationship between the rotational drag force τ and angular velocity ω for each run. Fig. 3 and Fig. 4 below are plotted in this manner with the power law fits shown in black.

The runs displayed in Fig. 3 are all of the clockwise runs, and they can be seen to have similar trends. The n value calculated from the power law fit for each run is shown in Table I below and range from 1.52 to 1.90. Fig. 4 shows the clockwise and counterclockwise runs for the OnionSlice and the DoubleWindSock flags, a total of four runs. The values found for these runs were somewhat surprising. Both flags produced a value of $n \approx 1.8$ when spun clockwise and $n \approx 1.6$ when spun counterclockwise. We would expect to see a small change in the n value since the flags are asymmetrical, but it is difficult to judge how large of a change 0.2 is. Also the fact that the values are so similar for two very different flags suggests that there could be something else controlling or affecting the

TABLE I: Graph Identifiers and the calculated n values with one standard deviation for each data run. An * denotes counterclockwise spin

Flag Used	Graphical Identifier	Fit Slope (n)
NoFlag	Pink Rhombus (Horizontal)	1.03 ± 0.01
TailFeatherFlag	Orange Rhombus (Vertical)	1.52 ± 0.03
OnionSliceFlag	Blue Square (Outline)	1.86 ± 0.06
OnionSliceFlag*	Blue Square (Filled)	1.64 ± 0.03
DoubleWindSockFlag	Red Triangle (Outline)	1.85 ± 0.05
DoubleWindSockFlag*	Red Triangle (Filled)	1.60 ± 0.05
MassiveFlag	Green Circle	1.9 ± 0.3

motion, but to draw a definitive conclusion far more data would need to be recorded.

When observing the data from Table I we see that the NoFlag run produced a value of 1.03 ± 0.01 for n. This result aligns with the theory since laminar flow should produce a linear relationship between drag and velocity. This suggests that our rotational analogy to Stokes' translational model is true, and if the analogy continues we should see values of $n \approx 2$ if the presence of the flags creates turbulence. Looking at the remaining calculated n values in Table I we see that all of the values fall between $n = 1$ and $n = 2$. The most likely explanation for the values which are in this range but are neither 1 or 2 is that the flag for that run did not cause consistent turbulence or strong enough turbulence to match Newton's ideal model. However these results are still supportive of our rotational analogy for the translational models.

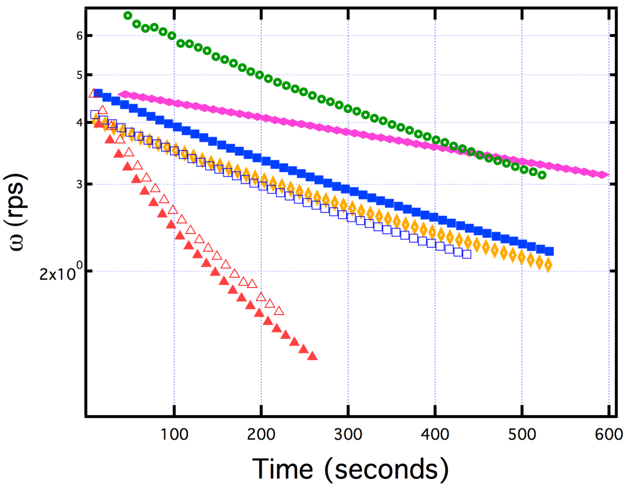

FIG. 2: Angular Velocity ω versus time. See Table I for a list of flag identifiers.

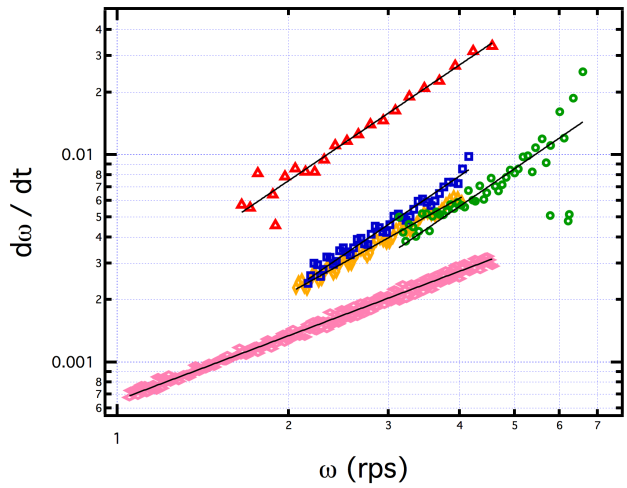

FIG. 3: Angular Acceleration $d\omega/dt$ versus ω. See Table I for a list of flag identifiers. This graph only displays the clockwise runs for the OnionSliceFlag and DoubleWindSockFlag.

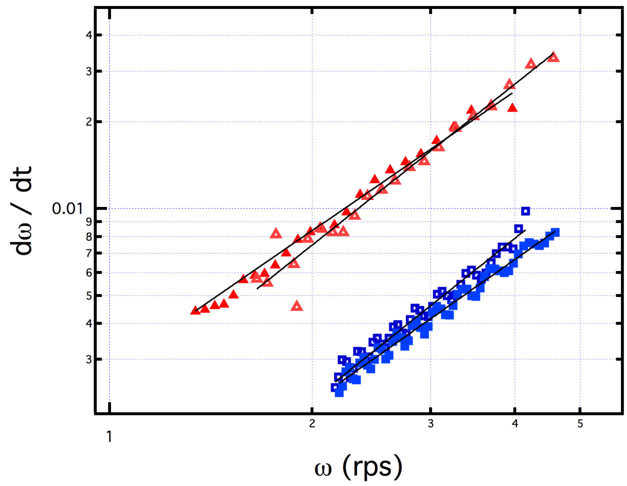

FIG. 4: Angular Acceleration $d\omega/dt$ versus ω for the OnionSliceFlag and DoubleWindSockFlag. The solid identifiers correspond to counter clockwise rotation as specified in Table I.

V. CONCLUSION

The viscous drag force on an object moving through a fluid is typically modeled by either Newton or Stokes' equations. If the flow is laminar it is modeled by Stokes' equation which predicts the drag force to be proportional to the velocity. If the flow is turbulent then it is modeled by Newton's equations which predict the drag force to be proportional to velocity squared. However both of these models are for translational movement, and we have shown through this lab that there are rotational analogies.

Using a rotating steel rotor ball with no flags the drag force was found to be proportional to the angular velocity, analogous to Stokes' model, as we predicted since the flow off of the smooth ball and rod would be laminar. By adding flags to the rod we were able to disturb the air flow, and by varying the size and shape of the flags we saw the drag force become proportionate to the angular velocity raised to higher powers. These powers fell between 1 and 2, which represent the classical laminar and turbulent regimes respectively. This range can be accounted for by considering that the turbulence created by the flags will be subject to fluctuations, but the presence of the turbulence undeniably causes an increase in n towards $n = 2$. Thus we have correctly predicted and shown the existence of the rotational analogy for viscous drag from the translational equations put forward by Newton and Stokes.