Proving the Coulomb's Law and Finding the Coulomb's constant from two charged spheres

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The experiment studied Coulomb's law using two charged spheres, and found from the results that the force between two charges were directly proportional to the charges squared and inversely proportional to the distance squared. This experiment was conducted using two spherical balls charged with kilovolt power supply. When the two spheres were charged equally and one was brought closer to another, and charging one of the spheres at different voltage and brought it closer to the other charged sphere, we measured the angle displacement. We used that data to plot it into several graphs, which showed that the charges obey the Coulomb's law. The experiment was then taken further to measure the experimental value of the Coulomb's constant by measuring the torsion constant on a string, calculating the force, and plotting on to a graph, which the slope yields the value of the measured Coulomb's constant. For this experiment, the experimental value of the Coulomb's constant was 3.83×10^9 N m^2/ C^2, which has a 57% error from the accepted value of 8.988×10^9 N m^2 /C^2.

I. INTRODUCTION

Charles-Augustin de Coulomb stated that a force between two charges were directly proportional to the charges squared and inversely proportional to the distance squared. This is known as the Coulomb's law of the force for charged particles[1]. This experiment tested this law using two spherical balls that were both charged with kilovolt power supply. The experiment was conducted in two ways where we observed how the charges of the sphere affect both the distance and the charge. Once we had found that the charges between two charged spherical balls obey the Coulomb' law, the experiment was taken further to find the Coulomb's constant by measuring the torsion constant on the string that was used to calculate the force.

II. THEORY

Charles-Augustin stated that a force between two charges are directly proportional to the charges squared and inversely proportional to the distance squared. This is the Coulomb's law, which is represented as,

$$F = k\frac{q_1 q_2}{d^2}, \tag{1}$$

where F is the force, q_1 and q_2 are the charges, d is the distance between the two charges, and k is the Coulomb's constant[2].

A. Force vs. Distance

For finding the relationship between the force and the distance, we take the logarithm from Eq.(1) on both sides to give

$$\log(F) = -2\log(d) + [\log(kq_1q_2)]. \tag{2}$$

The Coulomb's constant and the two charges were placed inside the square bracket because they are treated as constants since the balls were charged with equal voltage and same polarity. The Coulomb's force is proportional to the angle displacement θ[2]. Hence Eq.(2) is rewritten as

$$\log(\theta) = -2\log(d) + [\log(kq_1q_2)]. \tag{3}$$

However, it is important to note that the spherical balls does not act as point charges because the separation distance between the two balls is not large enough. Therefore the force between the balls will be less than when the charged balls were point charges. To fix this error we take the correction factor into account given as

$$B = 1 - 4\frac{a^3}{R^3}, \tag{4}$$

where B is the correction factor, a is the radius of the ball, and R is the distance between the two balls. The value we found in Eq.(4) was multiplied with the angle displacement θ, which gives a more accurate value that is represented as corrected angle displacement θ_{co}[3]. Therefore the final form of Eq.(3) is

$$\log(\theta_{co}) = -2\log(d) + [\log(kq_1q_2)]. \tag{5}$$

B. Force vs. charge

In order to find the relationship between force and charges, the logarithm was taken again in Eq.(1). Then the constants were placed in square brackets and the correction factor was taken into account, which gives

$$\log(\theta_{co}) = \log(q_1) + [\log(kq_2) - 2\log(d)]. \tag{6}$$

Since one of the balls was charged with fixed voltage and the distance between the two balls remained the same, the distance d and charge q are treated as constants in the square brackets.

C. Torsion constant and Coulomb's constant

To find the Coulomb's constant, we needed to know how to calculate the force between the two charged spheres. This was done measuring the torsion constant, where once we found the torsion constant we calculated the force using

$$F = t \times \theta_{co}, \tag{7}$$

where t represents the torsion constant and θ_{co} for this case was converted into radians to match the units [3]. The values for force and the data collected in force versus distance were plotted onto a graph, which gave a slope for the measured value of the Coulomb's constant.

III. PROCEDURE

Fig. 1 illustrates the overall setup for the first two experiments. There were two balls represented as a and b, where b can slide closer or away from a that is attached onto a torsion string. When both both balls were charged and ball b approached to ball a, ball a would either repel or attract towards ball b depending on polarity. Furthermore, ball a would pivot at an angle, which the position can be readjusted using the adjustable knob.

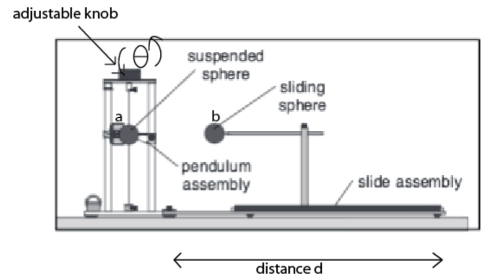

FIG. 1: The setup of the apparatus. This illustration was taken from ref.[2]. In the illustration, we see spherical balls a and b, which for this experiment were charged using a kilovolt power supply. Ball b would slide towards ball a using the slide assembly, and ball a would pivot. The adjustable knob was used to reposition ball a into its original position, in which we recorded the displacement angle.

A. Force vs. Distance

The two balls were initially positioned 38 cm away from each other to avoid collision between the two. Then using the kilovolt power supply, both balls were charged with fixed voltage of 6 kV and with the same polarity. Immediately but carefully so that the charge would not dissipate and the balls would not collide, the distance between the charged balls were brought closer to 20 cm. After the balls were in position, ball a would pivot. It was repositioned to its origin using the adjustable know, in which how much it pivoted represents the angle of displacement and this was measured. Furthermore, the correction factor B was applied to give θ_{co}.This process was repeated several times for different distances of 14 cm, 10 cm, 9 cm, 8 cm, 7 cm, 6 cm, and 5 cm. The setup was reset by having the balls touch each other and the distance between the two were separated again to 38 cm.

As a caution of note, when the two balls were brought together very quickly, ball a would swing very wildly that it would take long time for it to calm down and reposition to its origin. The more time it takes to reposition itself caused the dissipation of the charges. This would prove to be a problem if we wanted a consistent data and measurements. Therefore this issue was fixed by having the balls brought together slowly or gently adjusting the knob.

B. Force vs. Charge

Similar to the procedure done for force vs distance, the two balls were once again charged with same polarity using the kilovolt power supply. However, the balls were positioned 7 cm away from each other and ball b was charged with fixed voltage of 6 kV, while ball a was charged with varying voltages between 1 kV and 6 kV. Ball a would pivot, and it was repositioned to its origin using the adjustable knob. The angle displacement was measured, and the correction factor was applied to the value.

C. Torsion constant and Coulomb's constant

To measure the torsion constant, the set up for ball a was detached from the main frame and lied on its side. This is shown in Fig. 2. The gravity would pivot the ball, which it would need to be repositioned to its original position using the knob. The repositioned position became the new origin. Then, masses of 20 mg, 40 mg, and 50 mg were added separately on top of the ball, which the ball would swing again and needs to be repositioned to its origin. The angle displacement was measured with the correction factor. The collected data was plotted onto a graph shown and the slope gave the value of the torsion constant t. Then Eq.(7) was used to calculate the force, and find the experimental value of the Coulomb's constant.

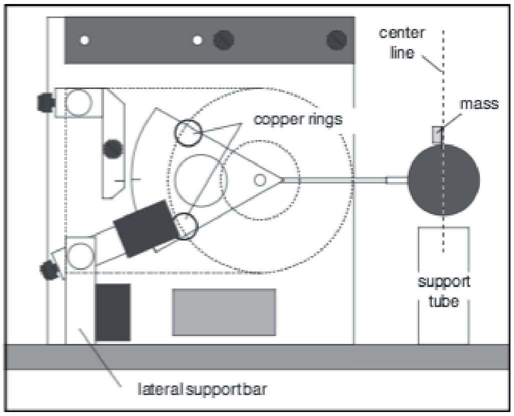

FIG. 2: The second part of the setup of the apparatus. The illustration was taken from ref.[2] that shows ball *a* detached from its main frame and lay on its side. The gravity would pull the ball to the ground, which ball *a* would pivot. The ball was readjusted again using the adjustable knob until the ball was positioned as shown in the illustration. As we increase the mass addend onto the ball, we recorded the displacement angle for each mass in order to set the ball back to its original position

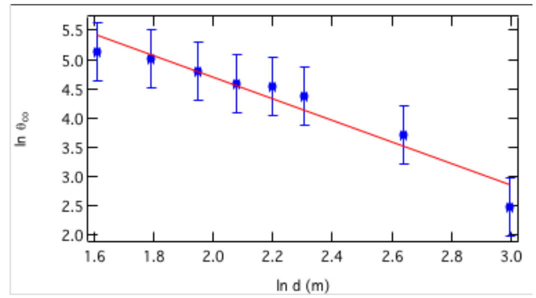

FIG. 3: The graph for $\ln(\theta_{co})$ versus ln(d) that has a slope of -1.85 ± 0.21. This graph shows that force between to charges were inversely proportional to distance squared.

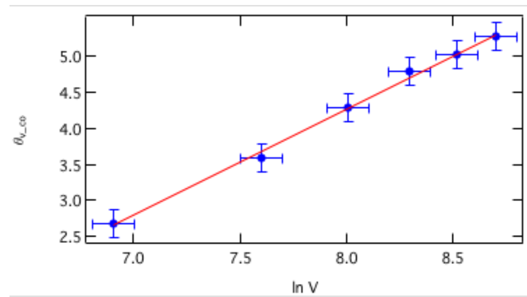

FIG. 4: The graph for $\ln(\theta_{v\_co})$ versus ln(V) that has a slope of 1.48 ± 0.04. This graph shows that the force between two charges are directly proportional to the charges, as predicted in theory.

IV. DATA ANALYSIS AND RESULTS

A. Force vs. Distance

From Eq.(5), in theory the logarithm of angle displacement would be directly proportional to the logarithm of distance times -2. Therefore if the data were plotted onto a graph, it should show a downward linear behavior with a slope of -2; in Fig.3, it shows the achieved graph for this part of the lab. The data points were plotted in logarithm as described in Eq.(5). The linear best fit line has a slope of -1.85 ± 0.2, which is approximately close to -2 as predicted with an error of 7%.

B. Force vs. Charge

Since one of the charges and the distance was held constant, from Eq.(6) we would predict a slope of 1 if the data were plotted onto a graph in logarithm. The graph in Fig.4 shows what was achieved for this part of the experiment. The data points show a positive linear behavior that has a best fit line with a slope of 1.48 ± 0.04, which is 48% off from the predicted value of the slope.

C. Torsion constant and Coulomb's constant

The torsion constant was measured to be (8.25 ± 0.4) $\times\ 10^{-5}$ N. This information along with the correction factor for angle displacement in force versus distance part of the experiment were used to calculate the force using Eq.(7). If force was plotted against charges squared and distance squared, then the graph would show a linear behavior, which the slope would equal to the Coulomb's constant.

The graph in Fig.5 shows what was achieved. From the information we have from Fig.(5), we used that information to compute the experimental value of the Coulomb's constant, which was found to be 3.83×10^9 N $\text{m}^2/\ \text{C}^2$. This value has an error of 57% compared with the accepted value of the Coulomb's constant, which is 8.988 $\times\ 10^9$ N $\text{m}^2\ /\text{C}^2$. However, as clearly shown in the graph, the data points are behaving curvier than linearly that may indicate a systematic error in the experiment and data collection overall. Furthermore, the large uncertainty bars also indicate the possibility that during the data collection; the charges on the spheres dissipated that caused inconsistent measurement; and somewhere in the apparatus specifically the torsion string or the adjustable knob were not used correctly that caused inconsistent measurement.

V. CONCLUSION

The experiment was able to show that two charge spherical shells obeyed the Coulomb's law, which is the force between two charges are directly proportional to

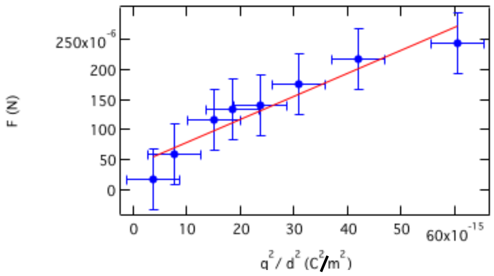

FIG. 5: This graph shows the force versus charge squared and inverse of the distance squared. From the information we collected from this graph, we calculated the experimental value of the Coulomb's constant, which was 3.83×10^9 N m^2/ C^2. Also, note that the graph shows a curvier line along the linear best fit line. This would indicate a systematic error during the data collection.

the charges squared and inversely proportional to the distance squared. This is shown in Fig. 3 and Fig. 4, where the graph shows the linear relationship. However, as we look at closely for both graphs, the data points are shown to be curvier than linear along the linear best fit. This may indicate the reason why the measured experimental value of the Coulomb's constant has a large error compared to the accepted value. From the information used from Fig. (5), we computed the experimental value of the Coulomb's constant to be 3.83×10^9 N m^2/ C^2, which has a 57% error from the accepted value of 8.988×10^9 N m^2 /C^2.

The experimental value of the Coulomb's constant may have reached closer to the actual value if we had collected more data for each experimental runs, and averaging the value to get a consistent data. Since charges dissipate over period of time, this may have caused the curvy pattern we see in all our graphs. Therefore for future work we would run the experiment multiple times under certain condition. For example getting the two charged balls closer together under a time limit or adjusting the speed of the adjustable knob as we reposition ball a, may help us observe where in the experiment would cause an error. Then we would fix the problem, and collect a more reliable and consistent data.

[1] Halliday, D., Resnick, R., & Walker, J., 2007, Extended Fundamentals of Physcis 8th Edition, John Wiley & Sons. Inc, p. 566.

[2] Lehman. S., Junior I.S., 2016, Department of Physics Spring 2016, p. 4.

[3] PASCO, COULOMB BALANCE, Instruction Manual and Experiment Guide for the PASCO scientific Model ES-9070, 1989, http://moodle-1516.wooster.edu/pluginfile.php/28438/course/section/13505/Coulomb.pdf (Feb. 26th 2016)