Measuring Particle Concentration Through Turbid Suspension

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The objective of this project was to measure the concentration of nano polystyrene spheres in turbid distilled water. This experiment used static light scattering and Rayleigh scattering to find the concentration amount through the difference in intensity of a single scattering. The laser intensity was measured before and after scattering for many concentrations of particles. Turbidity, or cloudiness of the medium, relates to the number concentration to give a quantifiable value. Due to the unique nature of this experiment, there are no comparisons to draw with other available sources. Intensity vs. the distribution constant was graphed to check data for accuracy. Then turbidity vs. the distribution constant was graphed to find the concentration. After calculating the scattering coefficient, the concentration was found to be 4.54 x 10^{-13} 1/cm^3 with an error of 3.7% assuming a linear correlation between turbidity and diluting constant α.

I. INTRODUCTION

This experiment uses nano-scale particles to give special insight into the characterization of same-sized particles. This can be used in the food industry, environmental water studies, and multidisciplinary research, such as polymer characterization, virus sizing, and dust sample research [1].

This experiment will use static light scattering through a turbid medium to test concentration amounts in a cuvette. This experiment will use Rayleigh scattering to find the concentration amount. This can be determined when the nanospheres have a smaller diameter than the wavelength of the laser. This experiment can be replicated with new particles to give more information on those encountered on a daily basis.

II. THEORY

This experiment uses static light scattering to gauge the initial concentration from a turbid solution. Rayleigh scattering is in effect when using small spheres that are on the order of 1/20th that of the light wavelength [1]. When the particles are of the same order as the light wavelength. Kerker shows the intensity of the scattered light I must be proportional to the initial intensity I_0 modulated by some function f, which is dependent on the volume of the spherical particle V, the distance to the observer r, the wavelength of the incident light λ, and the index of refraction of the particle and medium n_1 and n_2 [2]. This is shown by

$$I = f(V, r, \lambda, n_1, n_2) I_0, \tag{1}$$

where $f(V, r, \lambda, n_1, n_2)$ is dimensionless. Deriving the intensity of a scattered wave I off a sphere we find

$$I = \frac{16\pi^4 r^6}{r^2 \lambda^4} \left(\frac{n^2-1}{n^2+2}\right)^2 \sin[\phi], \tag{2}$$

where r is the radius of the spherical particle, $n = n_1/n_2$ is the relative refractive index, and ϕ is the angle between the scattered wave and the spherical dipole [1]. Integrating Eq. 2 over the surface area of a sphere gives It is calculated by integrating Eq. 2 over the surface area of a sphere.

$$C_{sca} = \int_0^{\pi} \int_0^{2\pi} I r^2 sin[\phi] d\phi d\theta, \tag{3}$$

the coefficient of scattering. C_{sca} is defined as the total energy scattered by a particle in all directions. Since I was integrated in spherical coordinates, Eq. 3 can be reduced to

$$C_{sca} = \frac{128\pi^5 a^6}{3\lambda^4} \left(\frac{n^2-1}{n^2+2}\right). \tag{4}$$

The coefficient of scattering is important for finding the concentration, as it is related to the turbidity of the medium.

Turbidity is defined as an extinction coefficient of a liquid or other light dispersing medium. Turbidity is often referred to as “cloudiness” or “murkiness” caused by the concentration and scattering ability of the particles. Turbidity can be found using the transmitted light intensity and the incident light intensity.

As shown in Fig. 1, we can see that for each step dx some light is scattered

$$dI = -\tau I_0 dx, \tag{5}$$

where τ is defined to the the turbidity, the positive constant of interest [1]. By rearranging and integrating, the final turbidity result is

$$\tau = -\frac{1}{L} \ln\left(\frac{I_t}{I_0}\right) \tag{6}$$

The transmitted intensity is related to the turbidity coefficient using the coefficient of scattering calculated in the previous section [1]. That is,

$$\tau = -\frac{1}{L} \ln\left(\frac{I_t}{I_0}\right) = \left(\frac{N}{V}\right) C_{sca}, \tag{7}$$

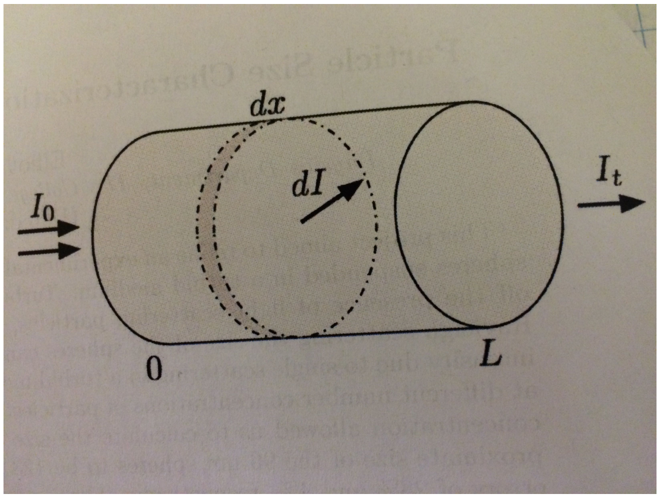

FIG. 1: Light incident on a cylinder of turbid fluid, where at each distance increment dx some light dL is scattered by the particles in the fluid, and L is the total length of the medium. Schematic created by Dr. Don Jacobs, reproduced from [1]

where N/V is the concentration of the solution at any given moment. To find the original concentration, N_0/V_0, we need to recognize that N/V will always be a fraction of the original concentration. This fraction is represented by α the diluting constant. To calculate concentration, the graph of τ vs. α will give some insight. The variables are related through

$$\tau = -\frac{1}{L}\ln\left(\frac{I_t}{I_0}\right) = \alpha\left(\frac{N_0}{V_0}\right)C_{sca}. \tag{8}$$

Rearranging, we find

$$\frac{\tau}{\alpha}\frac{1}{C_{sca}} = \left(\frac{N_0}{V_0}\right) \tag{9}$$

is able to give us the original concentration.

III. PROCEDURE

The original concentration can be calculated by focusing laser at the turbid solution and measuring the light intensities before and after the beam passes through the medium. The experiment requires very careful setup. The apparatus is composed of a Melles Griot intensity stabilizer laser, a beam expander, an optical chopper, several pinhole apertures, a photo detector, a housing for the cuvette sample, and a Stanford Research System SR830 lock-in amplifier. The entire apparatus is seen in Fig. 2.

The laser shoots a beam through each component as the incident light source which travels through the turbid medium and gets recorded as transmitted intensity. The 632.8 nm laser is intensity stabilized($I_0 \pm 0.1\%$) so that the lock-in amplifier of the photo detector's current has accurate readings [1]. The laser takes 20 minutes to stabilize intensity once turned on. Once stabilized, the laser was checked to make sure it was aligned properly. When focused properly, one will be able to see a bullseye formation leaving the beam expander. Caution was executed when adjusting the alignment as it is a time consuming process. To eliminate the room light intensity, the apparatus includes a lock-in amplifier. In addition, chopped square waves ensure that only the maximum, uniform intensity is incident to the photo detector. This provides more accurate results.

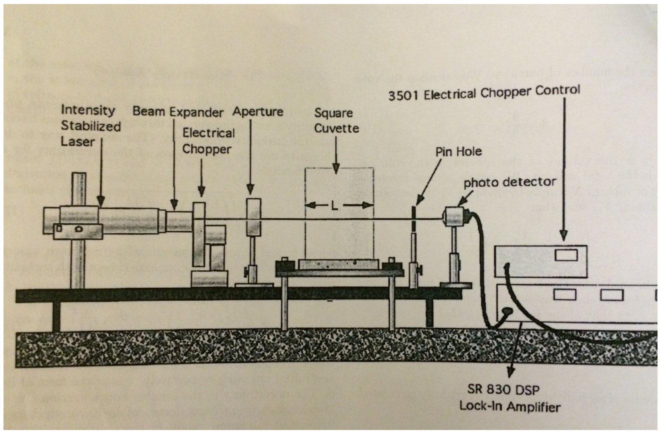

FIG. 2: A schematic of the experimental apparatus and optical arrangement. Schematic created by Dr. Don Jacobs, reproduced from [1].

Using a 10 mm, 3.4 mL rectangular optical cuvette, nanoscale polystyrene spheres from Duke Scientific of diameter 96 nm were suspended into a deionized (distilled) water solution [1]. Initial measurements of the light intensity without the cuvette and with the cuvette filled only with 1.4 mL distilled water were taken for I_0. Next, 1 drop of the nanospheres was dropped into the cuvette. Intensity data was taken. Then 0.25 mL distilled water was added to the solution and data was recorded. This last step continued until the cuvette reached 3.4 mL. At that point, the solution was heavily mixed to avoid separation. immediately after, 2 mL were removed from the solution, leaving a smaller volume of the same concentration. This way, more water could be added without restarting the concentration process. Following this, several 0.25 mL doses of water were added as data was recorded. Once the cuvette reached 3.4 mL again, the dosage stopped.

IV. RESULTS AND ANALYSIS

The intensity verses the diluting constant was plotted to confirm that the data matched the theory of Eq. 10. Displaying the first set of data points yielded FIG. 3. FIG. 3 maps the Intensity vs. the distribution constant α. This data was plotted because it was expected to be

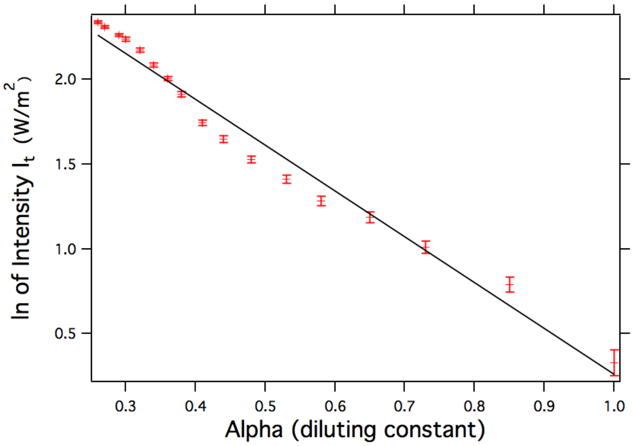

FIG. 3: Transmitted intensity data for 96 nm polystyrene spheres. The graph is fit on a log scale with a linear slope, where $b = -2.70 \pm 0.11$ cm ^{3}. The initial intensity is 13.09 W/m^2 for the 96 nm spheres.

a straight line. From Eq. 10, one sees how I_t and α are related on a log scale. With this in mind, the slope should be linear. This was done as a precaution. If the data was not linear, an issue would be exposed. As seen in FIG. 3, the fit line is roughly linear. The fact that not all data points fall on the line can be accounted for by a few factors. These include systematic error recorded by the lock-in, random error from the amount of liquid in the cuvette, random error from particles clumping rather than evenly distributing, and systematic error from particles with inaccurate diameters. The error bars on FIG. 3 account for the laser's systematic error.

Now that the results are useable, a second graph was made to find the concentration. The turbidity is calculated using Eq. 8 and then plotted verses α, as shown in Fig. 4. The slope is important because, as seen in equation 11, it can be used to find the concentration. The uncertainty from these results are given from the systematic error of the machines reading the intensities. The fit line was expected to be linear because Eq. 11 shows the relationship between the variables. The points plotted are arguably not best fit linearly, so a second version of this graph was included with a fit line by a power law. This is seen in FIG. 5. Based on Eq. 11, the relationship is expected to be linear, so FIG. 4 will be used to calculate error. With that in mind, the results appear to be following a non-linear trend. For the purpose of calculating error, the information from FIG. 4 will be used to find the scattering constant can be calculated.

After finding the laser's wavelength, the wavelength in the distilled water, the index of refraction for water and polystyrene, and the radius of the particles the scattering coefficient can be calculated. These values were found as presented in the table below.

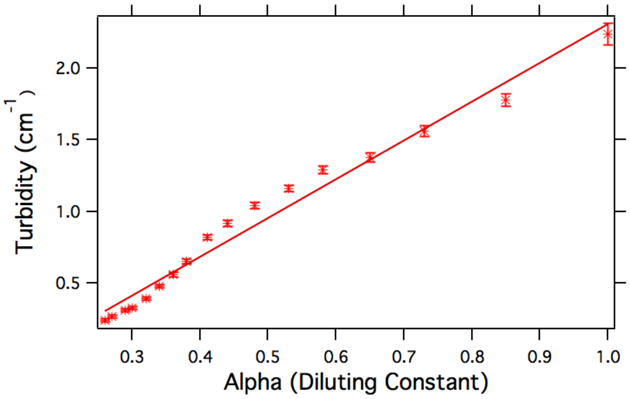

FIG. 4: Turbidity data for 96 nm polystyrene spheres. The graph is linearly fit with a slope in the form $y = a + bx$, where $b = -2.7 \pm 0.1 \text{cm}^{-1}$. The initial intensity is 13.09 W/m^2 for the 96 nm spheres.

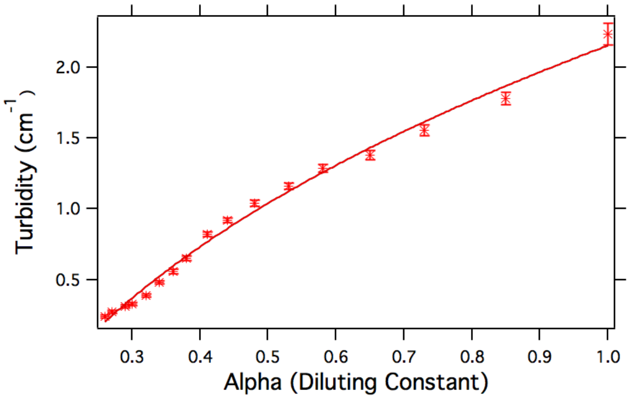

FIG. 5: Turbidity data for 96 nm polystyrene spheres. The graph is fit by the power law with a slope in the form $y = y_0 + Ax^k$, where $k = 0.34 \pm 0.13 \text{cm}^{-1}$. The initial intensity is 13.09 W/m^2 for the 96 nm spheres.

The values from Table. I were converted to centimeters and then used to calculate

$$C_{sca} = \frac{128\pi^5 r^6}{3\lambda^4}\left(\frac{n^2-1}{n^2+2}\right), \tag{10}$$

the scattering coefficient. This coefficient was found to be 1.68 x 10−13. Due to the unique nature of this experiment, this value could not be compared to other scattering coefficients. For this reason, calculations continued as if this is accurate. The relation in equation 11 is used to find the initial concentration of the solution. This gives concentration amounts of (2.7)(1.68 x 10^{-13} = 4.54 x 10^{-13} 1/cm^3. Using the error from Fig. 4, there was an error of 3.7% in the concentration.

V. CONCLUSIONS

The experiment demonstrated the calculation of the initial concentration of polystyrene nanospheres in the

TABLE I: The laser's wavelength, the wavelength in the distilled water, the index of refraction for water, the index of refraction for polystyrene, and the radius of the particles were calculated and recorded in this table.

variable	shorthand	numerical value
laser's wavelength	λ_0	632.8 nm
wavelength in distilled water	λ	474.7 nm
index of refraction of water	n_{fl}	1.333
index of refraction of polystyrene	n_p	1.60
relative reflective index	n	1.2
radius of particles	r	48 nm

solution. From the initial concentration, easily manipulation of the diluting constant can find the concentration for any amount of dilution. This is only true for when the particle size is known and is smaller than the wavelength of light being scattered. The initial concentration amount was found to be 4.54 x 10^{-13} 1/cm^3 with an error of 3.7% assuming a linear correlation between turbidity and diluting constant α. This error is primarily from the systematic error of the laser, although the random error from the diluting constant contributed significantly. To ensure further accuracy, attention is needed when adding in 0.25 mL of distilled water to the solution. A funnel is suggested, as some of the water remained on the rim of the cuvette rather than in it. In addition, the type of syringe used plays a significant role. Furthermore, time allowing, it would bring more accurate results to repeat the experiment and have more data. Adding smaller amounts that 0.25mL is suggested to collect a more full range of data points. If executed correctly, one could determine whether or not the fit line from FIG. 4 in this experiment was due to error or some other phenomena. Overall, this experiment was able to calculate the initial concentration of polystyrene spheres in the solution assuming a linear correlation between turbidity and diluting constant α.

[1] E. Wainright, *The College of Wooster, Particle Size Characterization in Turbid Colloidal Suspensions, unpublished* (2014), p. 1-7.

[2] M. Kerker, *The Scattering of Light and other Electromegnetic Radiation* (Academic Press, New York, 1969).