Determining Sphere Size Through the Observation of Brownian Motion

Colm Hall
Physics Department, The College of Wooster, Wooster, Ohio 44691, USA
(Dated: May 7, 2015)

This experiment is conducted to study the Brownian Motion of spheres suspended in water. By shining a laser at the sample spheres we can determine the diameter of those spheres. This laser beam is reflected off of the spheres at an angle of 90 degrees into a photomultiplier tube or (PMT). The PMT records the intensity of light reflected off the spheres and sends the data to a program designed by Brookhaven Instruments where it is translated into an intensity correlation function. We record the intensity of light for two minutes and observe that as the time step τ increases the correlation function decays exponentially. For each run we use spheres of size 51 nm, 96 nm, and 304 nm. After performing all of the trials we found the measured size of the spheres for each run contained an average error of 18%. The 51 nm spheres were measured to be 62.2 ± 0.8 nm, the 96 nm spheres were measured to be 118.7 ± 0.8 nm, and finally the 304 nm spheres were measured to be 367 ± 1 nm.

I. INTRODUCTION

This experiment is performed to discover the size of particles suspended in water. These particles undergo random collisions with the water molecules and move in a random motion that was later named Brownian Motion. This phenomena was first observed by Robert Brown in 1827. In his famous experiment he observed the motion of tiny particles that were ejected from pollen when he immersed them in water. In 1905 Albert Einstein expanded his experiment and theorized that the particles were colliding with the water molecules, causing a random walk effect where the particles move in a random motion. He used that theory to discover the size of water molecules [3]. This experiment will be doing the opposite and measuring the size of polystyrene spheres that are immersed in the water. In this experiment the polystyrene spheres are suspended in water inside an isolated container. A laser shines into the container and interacts with the spheres inside. Then the light is reflected at an angle of 90 degrees into a photo multiplier tube (PMT), where its intensity is measured over a period of 2 minutes. The intensity of the laser light will indicate whether or not the spheres have moved. If we can understand how the spheres move we can then measure the size of the spheres inside the water and compare them to the known size of the spheres. We will use auto correlation functions that will measure the intensity of the light and a time step τ. We can plot the autocorrelation functions to extract the size of the spheres from the plot.

II. THEORY

We begin with the intensity auto correlation function. We know the form of the light coming from the laser and the probability density when measuring the position of the polystyrene spheres. Using the information about the light coming into the tube and the probability function we can expect a specific result. We can start with the equation for the incident monochromatic light by looking at the real portion of

$$E_{incident} = E_0[RE]\{e^{i(\omega t - \vec{k}*\vec{r})}\} \tag{1}$$

where E_0 is the amplitude of the electric field, ω is the angular frequency of the light, $\vec{k}$ is the wave vector, and r is the vector from where the light hits the sphere to where it impacts the PMT. This equation derived with the help of [1]. For Eqn.1 only the real part is used because the imaginary part of the light is useless information for the purposes of this lab. When examining the scattered light the equation is similar to Eqn. 1 but with an important difference in the exponential portion of the equation. The equation for light scattering can be written as

$$E_{scattered} = E_0[RE]\{e^{i(\omega t - \vec{q}*\vec{r})}\}. \tag{2}$$

The q in the scattering equation represents the wave vector change

$$\vec{q} = \vec{k'} - \vec{k}.$$

which is also known as the scattering vector. $\vec{k}$ is the vector of the scattered wave. In this case q also has magnitude

$$q = 2k \sin \frac{\theta}{2}.$$

For this experiment we can assume that the scattering is elastic. We also know that light is scattered at a right angle.

For the next portion of the theory we have to understand the basic concepts associated with Brownian Motion. As I stated earlier, the polystyrene spheres interact with the water molecules by elastically colliding with them and exchanging thermal energy. Under this assumption there should be no real probability that the particles move in any particular direction. Because the particle has no specific direction to travel, we can describe the probability density function for the position of a given polystyrene sphere with the Gaussian function. This can be written as

$$\rho[r,t] = (4\pi Dt)^{-3/2} e^{-r^2/4Dt}. \quad (3)$$

Where D is the Einstein Stokes coefficient, and r is the displacement of the particle from its original position. The Einstein Stokes coefficient was developed to measure the diffusion of matter with a consideration of friction. The coefficient can be expanded to reveal several other variables and can be written as

$$D = \frac{k_B T}{3\pi\eta d} \quad (4)$$

where k_B is the Boltzmann constant, T is the temperature of the fluid, η is the fluid's viscosity, and d is the particle's diameter. Now that I have explained the equations that describe the movement of the particles in the water, we can begin to look at the light and its effects on the experiment.

When the particles move in the water each one reflects some amount of light toward the detector. When the light is scattered the electric field changes, and the correlation function of the electric field can be written as

$$C_1[\tau] = \frac{< E_S^*[\tau] E_S[t+\tau] >}{< I[\tau] >} \quad (5)$$

This function is called the normalized electric field autocorrelation function. C_1 is the electric field function, and I is the intensity of the light hitting the PMT. This equation derived with the help of [1]. The pointed brackets indicate that the average of the function is being taken. For this equation we can think of the variable t as some time a particular particle is suspended in the water. After some time dt the particle has moved some amount. Using Eqn. 5 we can take the electric field at time t and multiply it by the electric field at τ away. Then we repeat this process at another time t when the electric field is moved to some dt, we can multiply the new electric field by τ again. Once each different time step has been calculated the values are divided by the intensity. This becomes the point that we will later use in the analysis.

This autocorrelation function is used to describe the electric field over different time steps. If the time step is on the order of microseconds (μs) then the particle will have not moved very much so the reflection of light will be similar, but if the time step is on the order of seconds, then the particle will have moved enough to change the electric filed by a larger amount. The change in electric field will be displayed in the data as a change in intensity. As the time step increases, the intensity drops more and more until the correlation between t and $(t+\tau)$ becomes effectively random. The intensity correlation function looks similar to equation 5, but it can be written as

$$C_1[\tau] = \frac{< I[\tau] I[t+\tau] >}{< I[\tau] >^2}. \quad (6)$$

Now that these two equations have been explained, we can move to the most important equation that is associated with the this experiment: the Siegert relationship, which can be expressed as

$$C_2(\tau) = 1 + |C_1|^2 = 1 + ke^{-2Dq^2\tau} \quad (7)$$

The D in this equation is the same D that was mentioned before in equation 4. This equation is powerful because it relates the electric field to intensity by multiplying the electric field by its complex conjugate. It was originally thought that when the electric field was multiplied by the complex conjugate the phase information would be lost, but Siegert proved that the phase information is retained[2]. The k in this equation will allow us to find the best fit for the data provided. This equation will also allow us to fit the data and analyze the necessary information to be able to identify the size of the particles that were of interest to us. Once the data has been plotted in relation to the Siegert relationship, a fit line can be constructed. Using that fit line we can confirm the value of D. Using some simple calculations we can calculate D using Eqn. 4 to obtain the diameter of the spheres.

III. PROCEDURE

This experiment consists of several parts, the He-Ne laser, an isolated test chamber, and a PMT which can be seen in Fig. 1. The test chamber is similar to a black box. The only thing that can be seen is the laser moving through the liquid inside the sample and the oil that mimics the refractive index of glass. We know that the laser fires a beam into the bottom of the sample, which can be seen in Fig. 2. The laser then interacts with the polystyrene spheres inside the sample and reflects light at an angle of 90 degrees into the PMT which measures the intensity of the light that was reflected off the spheres. Because the laser only interacts with the bottom of the containers, the containers don't need to be filled more than halfway.

The laser that is shown into the test chamber is mounted on a rail to ensure that the beam is shining at the sample in a consistent and straight path into the test chamber. There are small glass openings to let the laser light in and out of the test chamber to ensure that no excess light enters the sample and interferes with the data collection. The second glass opening ensures that the reflected light moves into the collection area of the PMT. Each vial needed to be wiped down thoroughly to remove any foreign contaminants before being placed in the test chamber. Once the vial was placed in the

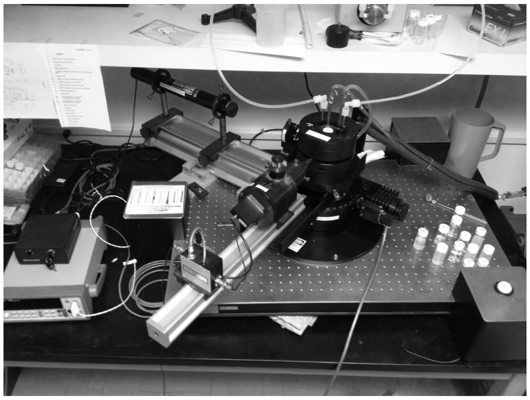

FIG. 1: The full apparatus consisting of a He-Ne laser, central test chamber, and the PMT.

chamber the lens on the PMT was opened and the laser was turned on. After the apparatus was set correctly the data collection could begin. This was done by starting the collection process on the software produced by Brookhaven Instruments. This software was designed to use the autocorrelation function that was mentioned earlier to correlate the intensity of light that was being collected in the PMT. Data was taken on the software for two minutes. Once the program was finished running it produced several important pieces of information. The first piece of information is the autocorrelation function $G(\Delta t)$, the second is the time steps that the program used in-between taking intensity measurements. The program also gives the calculated size of the particles that were being measured. The program was especially good at factoring in the initial conditions that need to be considered. The temperature and the refractive index of water as well as its viscosity are all taken into account. I made sure to take two runs for each size of particle. This was to ensure that the data was consistent and there were no outlying points. After finishing the trials the raw data was exported into Igor Pro to be analyzed.

IV. DATA AND ANALYSIS

We first need to plot the data that we acquired in Igor. We plot the correlation function on the y-axis and time on the x-axis. The values must fall between one and two. The correlation function on the y-axis needs to be normalized. This is done by finding the minimum value in the specific data set and dividing every point by that value. This will ensure that every point will fall between one and two. Now that the data has been normalized, the information that we need is contained in the exponential of Eqn. 7. Once the fit line is constructed the value of D will become clear. With some simple calculations using Eqn. 4 the diameter of the spheres can be found. Fig. 3 shows the comparison between the data and the fit for spheres of 51 nm. This fit line was constructed in Igor

FIG. 2: An example of a generic vial containing polystyrene spheres for reference

Pro to find D which is the Einstein Stokes coefficient. This value is contained in the information given to us by the fit line. After calculating the size of the spheres I arrived at $62.2 \pm 0.8 nm$. This value has an error of 17.8% for the next two plots. I will calculate the size of the spheres in the same way.

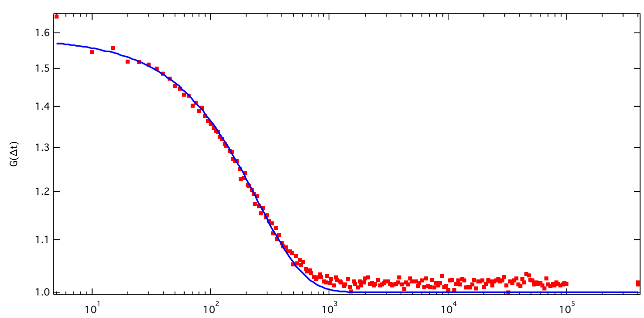

FIG. 3: A plot containing the exponential fit of data collected for spheres of 51 nm in diameter. The autocorrelation is plotted on the y-axis against the time step on the x-axis. The data is plotted as red points and the fit line is plotted as a blue line.

In Fig. 3 we see the plot for the spheres of 96 nm. The fit function for this graph was used the same way. First yielding the value of D and then using the relationship from Eqn. 4, the size of the spheres was calculated. The graph displays the correlation function on the y-axis and the time step on the x-axis. For the 96 nm spheres I calculated their diameter to be 118.7 ± 0.8 nm. This value yields an error of 19.1%.

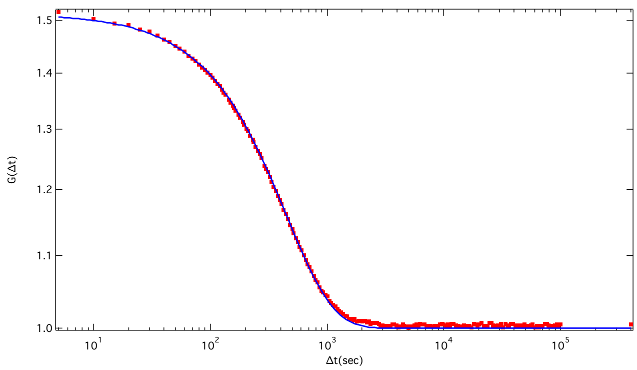

FIG. 4: A plot containing the exponential fit of data collected for spheres of 96 nm in diameter. The autocorrelation is plotted on the y-axis against the time step on the x-axis. The data is plotted as red points and the fit line is plotted as a blue line.

In Fig. 5 we see the plot for the spheres of 304 nm. We repeat the process from before by taking the fit of the data and using the value of D to calculate the diameter. For this trial we found the diameter to be 367 ± 0.8 nm. This value is larger than the real value of the spheres and contains an error of 17.2%.

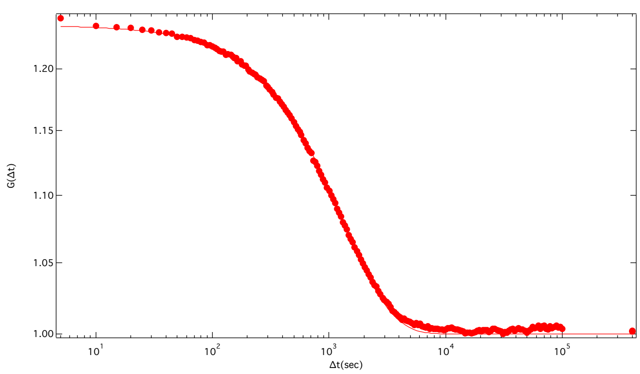

FIG. 5: A plot containing the exponential fit of data collected for spheres of 304 nm in diameter. The autocorrelation is plotted on the y-axis against the time step on the x-axis. The data is plotted as red points and the fit line is plotted as a blue line.

V. CONCLUSION

Our purpose in performing this experiment was to experimentally measure the size of unknown particulates inside a vial full of water. We were successful in measuring the sizes of the spheres that were suspended in the water. We used an exponential function inside Igor Pro to model the correlation of the spheres at several different time steps. We find that as the time step increases the correlation decreases in an exponential way. There were some possible sources of error within this experiment. Initially all of the data that I had taken was wrong because of problems with the apparatus itself. Also, the system as a whole is very fine-tuned to account for small variables that would affect data collection. If anything was touched in the process of fixing the apparatus that could have contributed to some error in the data collection process.

[1] Understanding Autocorrelation in Time, 2/13/2015 (FCS Expert Solutions)
[2] Junior Independent Study, 5/6/15 (College of Wooster Physics Department), 2015
[3] http: //en.wikipedia.org, 5/6/15 (Brownian Motion), 2015