Investigation of Fine-Tuning through the Variation of Fundamental Constants

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(Dated: May 8, 2014)

Multiverse and inflation theories occasionally require the use of fundamental constants with values that differ from what is measured in the observable universe. This potential for universes with different valued or time-dependent constants has given rise to the philosophical and physical idea of fine-tuning - that the universe was created for the purpose of creating and sustaining life. In order to investigate the claim of fine-tuning, eight parameters were varied from zero to infinity. Sets of life-permitting inequalities were imparted upon the parameter spaces to find habitable regions where all the inequalities were satisfied. The habitable region in the parameter space of α the fine structure constant, α_s the strong force coupling constant, and β the electron-proton mass ratio was found to range from $0 < \alpha < 0.107$, $0.108 < \alpha_s < 0.173$, and $0 < \beta < 0.00139$. Multiverse theories and fine-tuning itself are unfalsifiable ideas in their present stages. Results of this investigation show that a minority of parameter space is life-permitting though any value within the ranges would result in a life-permitting universe. The presence of life-permitting ranges of these parameters as opposed to one unique value challenges the philosophy of fine-tuning.

INTRODUCTION

The existence of life in our universe is a notable feature. Theories of cosmology, evolution, philosophy, theology, and physics all attempt to explain why we exist. Many theories imply that the evolution of life was nothing short of miraculous, with many aspects of the universe behaving exactly right in order to support life. This idea that the universe was designed to sanction life is known as fine-tuning; the values of constants are perfectly designed for life. The range of values these parameters may take in order to support life must then be small. This space of parameter values that can lead to universes with observers is the "habitable region" of parameter space. Fine-tuning also gives rise to the anthropic principle, or the idea that, "if observers observe anything, they will observe conditions that permit the existence of observers" [1].

Multiverse theories and universal expansion models bring about a necessity for a change in physical parameters from the measured values found in this universe. This may involve constant values that vary from universe to universe or time-variant parameters. An investigation into the fine-tuning of parameters can give insight into just how special the observable universe is. This qualitative approach will investigate eight separate parameters at every possible value to find the habitable regions that they form and ranges in which the parameters are life-permitting. This can lead to further understanding of the possibility of fine-tuning.

ASSUMPTIONS AND VARIABLES

Numerous assumptions and simplifications went into the calculations and analysis performed throughout this work.

- All forces exist regardless of parameter values. For example, changing the gravitational constant G should not alter the form of Newton's equation of gravitation, including causing the force to disappear. An exception to this is when coupling constants are varied and subsequent strength of the force is affected. Other than this strength variation, the force still functions in the same way observed in this universe.
- No new forces are created. Like the observable universe, there exist only gravity, electromagnetism, and the weak and strong forces.
- Criteria for "habitable" universes or those capable of observers assumes that "life" is defined as the carbon-based, self-replicating forms of life found on Earth. Therefore, the observable universe serves as a subtle bias upon which comparisons can be made since it is evident that life is available here with the observed set of parameters.
- Similarly, only structure formation as observed in this universe is assumed to lead to habitable conditions. For example, observed mechanisms in the creation of stars, galaxies, planets, and other celestial bodies are assumed to be the only means that can support life. Therefore, criteria based upon the observable universe are implemented.
- There is no guarantee that life must evolve in a habitable universe, only that conditions are appropriate for such evolution to happen. This implies an important but neglected factor of chance due to Darwinian evolution.
- All life-permitting criteria are order of magnitude approximations. This leaves much room for variation in the actual values and indicates trends in data rather than exact quantitative results.

TABLE I: Fundamental variables

Constant	Name	Value	Units
$\hbar$	"h-bar", Planck's constant $h/2\pi$	1.055×10^{-34}	m^2kg/s
c	speed of light in vacuum	2.998×10^{8}	m/s
G	Newton's gravitational constant, "Big G"	6.673×10^{-11}	m^3kg/s^2
m_p	mass of the proton	1.673×10^{-27}	kg
m_e	mass of the electron	9.109×10^{-31}	kg
m_u	mass of the up quark	2.4	MeV
m_d	mass of the down quark	4.8	MeV
α	fine structure constant	1/137	-
α_s	strong force coupling constant	0.1187	-
β	mass ratio of electron to proton	1836.15^{-1}	-
Ω_0	total density parameter of the universe	1.02	-
H_0	Hubble constant	71 or 67.8	Mpc s/km

Important variables with their measured values in the observable universe are presented in Table I.

PREVIOUS WORK

The critical analysis of fine-tuning in literature began in the 1970s with work done by B. J. Carr and M. J. Rees [6]. The concept has both physical and philosophical implications and is therefore of great interest to many scientists. Recently, attention has been given to Victor Stenger. His background in physics and current professorship of physics at the University of Hawaii and interest in philosophy and current adjunct professorship of philosophy at the University of Colorado Boulder blend together in his analysis of fine-tuning. He has published multiple books and articles of general science as well as scholarly works on the topic [7].

Stenger and *MonkeyGod*

Multiverse theory led Stenger to investigate the possibility of varied parameters yielding different habitable universes. In his program MonkeyGod, outlined in *The Unconscious Quantum* [8], he chose four constants to analyze: α, α_s, m_e, and m_p. Stenger uses order of magnitude approximations for life-permitting criteria of universes generated with different values of his chosen variable parameters [1, 8]. Among the calculated values are N_1, the age of the universe divided by the atomic light-crossing time, and N_2, the electric force between a proton and electron divided by the gravitational force between the same two particles [9]. Stenger includes an automated feature that chooses a random value for each variable within ± 5 orders of magnitude of its value in the observable universe. He noticed a trend in N_1 and N_2 from the analysis of 100 generated universes where

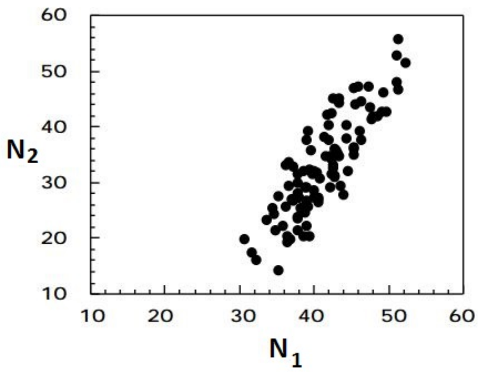

FIG. 1: Relationship between N_2 and N_1 from 100 randomly generated universes in MonkeyGod. A general linear trend can be noted, and leads Stenger to conclude that fine-tuning is inaccurate. This figure is taken from [8].

$N_1 > 10^{30}$ and $N_2 > 10^{20}$ regardless of the input parameters. The scatter plot of N_2 vs. N_1 values also trends toward a linear fit, as shown in Fig. 1. He therefore concludes that, "the conditions for the appearance of a universe with life are not so improbable as those authors enamored by the anthropic principle would have people think" [8].

Barnes' Criticism and Improvement

Critics have found numerous shortcomings in Stenger's work. Luke A. Barnes from the Institute for Astronomy in Zürich examined many of these faults in detail in his paper, "The Fine-Tuning of the Universe for Intelligent Life" [1].

To conclude the work, Barnes turns his attention to MonkeyGod. He argues that most of Stenger's criteria for a habitable universe are either inaccurate or irrelevant. Barnes also notes that three of Stenger's criteria are incorrect. Two seem to arise from incorrectly applying equations from his source, *The Anthropic Cosmological Principle* [9].

In order to begin a proper investigation of parameters, as was Stenger's intention, Barnes uses ten inequalities to create two plots that illustrate planes of the parameter space. These plots comparing α, α_s and β are shown in Fig. 2. The variables are the same as were previously investigated by Stenger through MonkeyGod, but using the dimensionless mass ratio β rather than individual m_e and m_p terms.

THEORY AND COMPUTATION

The manipulation of fundamental parameters was done computationally with Mathematica 9.

Universe and Star Lifetime Analysis

Preliminary investigation into the fine-tuning of physical constants was conducted involving the lifetime of a universe and the main sequence lifetime of its stars. In a universe that could give rise to the observers, the lifetime of the universe must exceed the lifetime of its stars so that supernovae could produce and disperse heavy elements with time to evolve into life.

A derivation for the lifetime of a universe is based upon Barbara Ryden's work in *Introduction to Cosmology* [15].

The fate of the universe is dependent upon the curvature of its spacetime. In a curved, matter-dominated universe such as our own, the Freidmann equation is

$$\frac{H(t)^2}{H_0^2} = \frac{\Omega_0}{a^3}\frac{1-\Omega_0}{a^2}, \tag{1}$$

where a is a scale factor and function of t, $H(t)$ is the Hubble parameter which defines the rate of expansion of the universe and is defined as $H \equiv \dot{a}/a$ where $\dot{a}$ is the time derivative of a, H_0 is the Hubble constant or the value of the Hubble parameter today at t_0, and Ω_0 is a density parameter for the universe measured at $a = 1$. For the universe to stop expanding, $H(t)$ must equal zero at some time. Substituting this into Eqn. 1 gives

$$0 = \left(\frac{\Omega_0}{a^3}\right)\left(\frac{1-\Omega_0}{a^2}\right) = A * B. \tag{2}$$

The term A is always positive. For the equation to hold, B must be negative. This means that $\Omega_0 > 1$. This implies that after a maximum is reached, Ω_0 will cause a to collapse to zero in a reversal of $H(t)$. This universe would end in a "Big Crunch" after a finite expansion in a finite amount of time. Universes where $\Omega_0 \leq 1$ never cease expansion and their lifetime would be infinite. The $\Omega_0 > 1$ case is special in that a finite lifetime can be calculated. Since $H(t) = \dot{a}/a$, Eqn. 1 can be written as

$$\frac{\dot{a}^2}{H_0^2} = \frac{\Omega_0}{a} + (1-\Omega_0). \tag{3}$$

This is a differential equation. Solving for $\dot{a}$ gives

$$\dot{a} = \frac{da}{dt} = H_0\sqrt{\Omega_0/a + (1-\Omega_0)}. \tag{4}$$

This equation is separable. We rearranged to find

$$H_0 t = \int_0^a \frac{da}{[\Omega_0/a + (1-\Omega_0)]^{1/2}} \tag{5}$$

after integrating dt from 0 to t.

The solution to this integral assuming $\Omega_0 > 1$ can be expressed in parametric form,

$$\begin{aligned} a(\theta) &= \frac{1}{2}\frac{\Omega_0}{\Omega_0 - 1}(1 - \mathrm{Cos}[\theta]), \\ t(\theta) &= \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0-1)^{3/2}}(\theta - \mathrm{Sin}[\theta]), \end{aligned} \tag{6}$$

where $(0 \leq \theta \leq 2\pi)$. With these limits, $\theta = 0$ is the Big Bang, $\theta = \pi$ is the moment where $H(t) = 0$ and expansion ceases, and 2π would be the Big Crunch. By substituting $\theta = 2\pi$ into the parametric equation for time,

$$\begin{aligned} t(2\pi) &= \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0-1)^{3/2}}(2\pi - 0) \\ &= \frac{\pi}{H_0}\frac{\Omega_0}{(\Omega_0-1)^{3/2}}\ (\mathrm{s\ Mpc})/\mathrm{km} \\ &= t_{crunch}. \end{aligned} \tag{7}$$

Since only universes with $\Omega_0 > 1$ exist for a finite time, t_{crunch}, we will only consider such cases [15].

Life requires the presence of long-lived, stable stars. Stars are the most conducive to supporting life while they inhabit the main sequence in their evolution. Here, nuclear fusion fuses hydrogen into helium, creating thermal pressure to offset the inward pressure of gravity. In "The

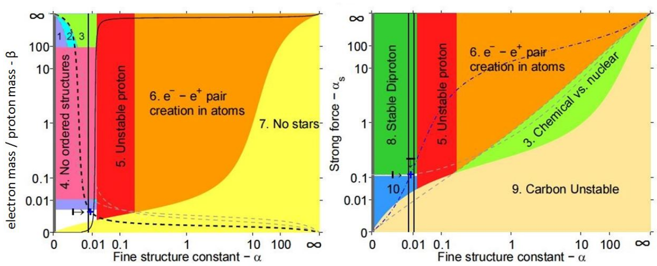

FIG. 2: Relationships between β, α, and α_s where the white regions on both plots indicate the habitable zone determined by ten inequalities and the blue crosses indicate our universe. This figure is taken from [1].

Anthropic Principle and the Structure of the Physical World" [6], Carr and Rees approach the issue of main sequence lifetime. Dirac found a dependence of the gravitational coupling constant α_G on time by

$$\alpha_G \sim \frac{\hbar}{m_e c^2 t} \sim \left(\frac{t}{t_e}\right)^{-1},$$

where m_e is the mass of the electron and t_e is the average time of a strong interaction with an electron. A similar estimate of the time the strong force can interact with a proton is $t_p \sim \hbar/(m_p c^2)$.

Using these values, Carr and Rees [6] give an equation for the time it would take a star of luminosity L to radiate away its entire mass involving α_G and t_p. This simplifies to

$$\tau_{MS} \sim \alpha_G^{-1} t_p = \frac{\hbar}{m_p c^2 \alpha_G} \text{ s}, \tag{8}$$

which simplifies with the substitution $\alpha_G = G m_p^2/\hbar c$ into

$$\tau_{MS} \sim \frac{\hbar^2}{m_p^3 c G} \text{ s}. \tag{9}$$

Many variables used within these equations do not naturally appear in SI units. For simplicity in comparison, Eqns. 7 and 9 were dimensionally converted into the SI unit of years. To do this, Eqn. 7 required two conversion factors. The natural units, seconds megaparsecs per kilometer, involve two terms of distance - parsecs and kilometers, related by 3.0857×10^{19} km/Mpc. By including conversion factors to cancel the dimensionality of length and changing seconds into years with 3.1557×10^{-7} yr/s, the final form of Eqn. 7 becomes

$$t_{crunch} = (9.7375 \times 10^{12}) \frac{\pi}{H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \text{ yrs}. \tag{10}$$

Similarly, a conversion of seconds to years and mega-electron volts into SI units of mass (from m_p, 1.783×10^{30} MeV/kg) for Eqn. 9 leaves

$$\tau_{ms} \sim (1.7888 \times 10^{84}) \frac{\hbar^2}{m_p^3 c G} \text{ yrs}. \tag{11}$$

Plot Replication of Barnes' Fig. 2

Barnes compiles a list of ten important inequalities involving his chosen parameters for the formation of Fig. 2. This list is detailed below [1]:

1. The existence of hydrogen requires that $m_e < m_n - m_p$ or $\beta < m_n/m_p - 1$, so the masses of the nucleons and electron are bound. Without this relationship, electrons in hydrogen would be captured by the proton creating neutrons and rendering stable atoms and chemistry impossible [12].

2. Barrow and Tipler state the ratio of nuclear to atomic interactions $\alpha\beta/\alpha_s \ll 1$, which is approximately 3×10^{-5} in our universe [9]. Barnes argues that this small value relates to the relative size of the radius of orbiting electrons to the nucleus. With a smaller fractional value, the electron orbit would be closer to the nucleus and create unstable atoms. Stenger, Barnes, and I all arbitrarily use 1/1000 as equivalent to the uncertainty $\ll 1$ for computational purposes.

3. By extension of the relationship above, the relative energies of nuclear constituents and atomic electrons are $\alpha^2\beta/\alpha_s^2 \ll 1 \sim 4 \times 10^5$ [9]. This shows that nuclear reactions involve much higher energies than chemical reactions. This must be the case for successful chemistry, or the identity of atoms could change in chemical reactions due to lower nuclear energy thresholds. Stenger, Barnes, and I all arbitrarily use 1/1000 as equivalent to the uncertainty $\ll 1$ for computational purposes.

4. Heisenberg's Uncertainty Principle leads to an uncertainty in the position of particles in solids, where nuclei act as ions held together in the lattice. In order for the lattice to remain rigid and prevent bonds from breaking, $\beta^{1/4} \ll 1$. This is the uncertainty in the position of the ion as its spring-like bonds expand and contract [9]. Stenger, Barnes, and I arbitrarily use 1/3 as equivalent to the uncertainty $\ll 1$ for computational purposes.

5. For the existence of stable protons, the mass-energy difference between a proton and neutron must be larger than the masses of the constituent quark masses. For this condition, $\alpha \lesssim (m_d - m_u)/141$ MeV [13].

6. The energy of an electron in the ground state of hydrogen is related to its binding energy to the atom and inversely to α. In small atoms like hydrogen, $\alpha \ll 1$ so that the motion of the electron does not create enough energy for spontaneous pair production [9]. This would lead to unstable elements and molecules. Stenger, Barnes, and I all arbitrarily use 0.2 as equivalent to the uncertainty $\ll 1$ for computational purposes.

7. Stable stars all fall within a range of masses that allow for fusion and typical stellar evolution. Stars with too little mass cannot begin fusion as they are supported through degeneracy pressure rather than thermal pressure. Especially large stars are dominated by thermal pressure, creating unpredictable pulsations leading to instability. To fall within the stable mass range, $\beta \gtrsim \alpha^2/100$ [9].

8. A sensitive range of α_s is required to form a deuteron - a stable particle comprised of one proton and one neutron. For the binding energy to equal zero, $\alpha_s/\alpha_{s,0} \lesssim 1.003 + 0.031\alpha/\alpha_0$ where $\alpha_{s,0}$ and α_0 are the constants' values in the observable universe. The existence of the deuteron is important for Big Bang nucleosynthesis theories and affects the ability of stars to fuse hydrogen, as hydrogen would have been depleted in the early universe without the ability of protons to bind with neutrons [14].

9. Barrow and Tipler [9] offer the limit of $Z^2/A \lesssim 49(\alpha_s/10^{-1})^2((1/137)/\alpha)$ where Z is the atomic number or number of protons and A is the total number of nucleons, for stable elements. This explains why very heavy elements do not appear in nature. It also relates the stability of atoms to the coupling constants, such that $\alpha_s \lesssim 0.3\alpha^{1/2}$. This in particular allows for the existence of carbon and carbon-based organisms [9].

10. The deuteron is only stable when $\alpha_s/\alpha_{s,0} \gtrsim 0.91$. When unmet, the fusion strategy of stars would be inhibited as energetically unfavorable [14].

These inequalities were used to replicate Barnes' plots of Fig. 2. They were also used to create an additional two-dimensional plot of β vs. α_s. The unused parameter was set equal to its measured value in our universe.

Scaling

In order to analyze the entirety of parameter space, the axes were scaled so that (0, ∞) fit on a finite axis. To do this, two sets of nonlinear functions,

$$N[x] = \text{Arctan}[\text{Log}[x]/s]$$

and its inverse

$$N^{-1}[x] = \text{Exp}[s\ \text{Tan}[x]]$$

and

$$M[x] = \text{Arctan}[\text{Log}[x]/s]/\pi + 1/2$$

and its inverse

$$M^{-1}[x] = \text{Exp}[\text{-}s\ \text{Cot}[\pi\ x]],$$

were defined (where Log[x] uses base e and s is a scale factor). The relationship between N and N^{-1} is shown in Fig. 3, where the linear nature and slope of one proves that the functions are inverses of each other. A similar graph can be made with M and M^{-1}.

The inequalities were then plotted in the form $N^{-1}[f[N[x]]]$. The axes themselves have manually set

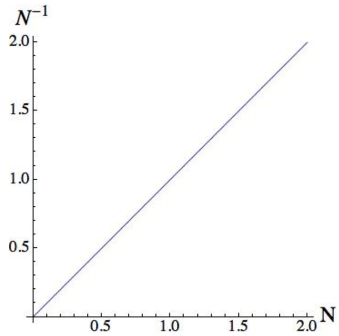

FIG. 3: The relationship of the nonlinear scaling functions $N[x]$ and $N^{-1}[x]$. The slope of one indicates that the used functions are inverses of each other.

tick marks to reflect the scaling. In order to span the entirety of parameter space, x values of $-\pi/2$ to $\pi/2$ for the N set and 0 to 1 for the M set were used. These extremes, when used with the scaling $N^{-1}[x]$ or $M^{-1}[x]$, yield the desired range of 0 to ∞. For example,

$$N^{-1}[-\pi/2] = \text{Exp}[\text{Tan}[-\pi/2]] = \text{Exp}[-\infty] = 0$$

and

$$N^{-1}[\pi/2] = \text{Exp}[\text{Tan}[\pi/2]] = \text{Exp}[\infty] = \infty.$$

A similar method was used to set the tick marks, where the desired scaled value was inserted as x into $N^{-1}[x]$. A log-like axes scale was chosen to appropriately segment the region. The scale factor term s was inserted so that the "zoom" of a region could be increased. Increasing s shifts the tick marks closer to the center, allowing the viewer to see the extreme regions in more detail. Analysis of the plots were all made with $s = 3$.

The presence of three independent parameters lent itself to an analysis in three dimensions. The scaling was generalized into three dimensions by $N^{-1}[f[N[x], N[y]]]$, so that all three parameters were affected by the scaling functions. A three-dimensional parametric plot was made to investigate the behavior of each inequality in the parameter space. A region plot was also made, taking the inequalities into account, to find a three-dimensional surface of the region that satisfied every inequality - the habitable region.

RESULTS

Dimensional Constants $H_0, \Omega_0, m_p, \hbar, \&\ c$

Equation 10 relates the parameters H_0 and Ω_0. Figure 4 shows a contour plot with nonlinear scaling of t_{crunch} times as H_0 and Ω_0 are varied. The labeled contours are in years. Experimentally found values are represented by the red and green points. Both are shown with

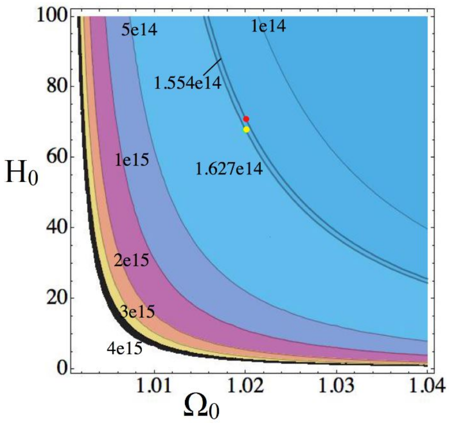

FIG. 4: Contour plot of t_{crunch} values as H_0 and Ω_0 range from $(0, \infty)$. The red and yellow dots represent measured H_0 values in the observable universe by supernova analysis and with the Planck mission, respectively.

$\Omega_0 = 1.02$. The red point is $H_0 = 71$ Mpc s/km, a value calculated through the analysis of supernovae. The yellow point at $H_0 = 67.8$ Mpc s/km is the value calculated by the 2013 Planck full-sky survey. These conflicting values lead to different predictions for the total potential lifetime of our universe (though other data leads one to believe that the measured Ω_0 is closer to 1 implying flat spacetime and infinite expansion). The disagreement between these values is a currently debated issue in cosmology.

For a habitable universe, $t_{crunch} > \tau_{MS}$ so that the heavy elements created in supernovae have time to disperse and allow for the evolution of life. Combining Eqns. 7 and 9 with the conversion factor 3.0857×10^{-7} to account for the Mpc and km values in H_0 and using the SI versions of all constants gives

$$(3.0857 \times 10^{-7}) \frac{\pi}{H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} > \frac{\hbar^2}{m_p^3 c G}. \tag{12}$$

By solving for $\hbar$,

$$\hbar < \sqrt{(3.0857 \times 10^{-7}) \frac{\pi m_p^3 c G}{H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}}. \tag{13}$$

The variables c and m_p are related by the same equation, so separate inequalities need not be calculated. The gravitational constant G is more complex since $\Omega_0 = 8\pi\rho G/3H^2$, however the $\hbar$ inequality should serve to represent the entire inequality of Eqn. 13. The *3DRegionPlot* of this inequality is shown in Fig. 5. The inequality appears as one sheet rather than a closed surface due to the singular nature of the inequality used. Therefore, the habitable region is the entire area under the curve. The red dot represents the constants' values in the observable universe. This dot falls within the constraint of

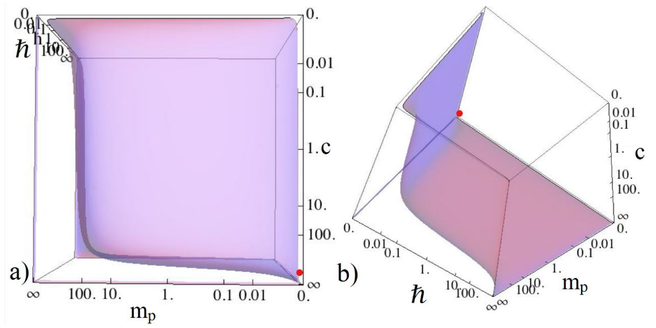

FIG. 5: 3D plot of Eqn. 13. The region under the curve satisfies the inequality. The red dot represents the observable universe, and falls within this range of habitable universes.

Eqn. 13 since it is on the side of the sheet that satisfies the inequality.

Dimensionless Parameters

Though often considered fundamental, dimensional constants such as those investigated above typically serve as dimensional conversion factors in equations and it is common practice in many fields to take them to unity by using the proper units. Paul Wesson [11] argues that their existence arises from a human shortcoming in viewing the universe and our innate compartmentalization of concepts such as mass, length, and time. Therefore, Wesson concludes that these constants should not be considered fundamental at all - they are merely man-made tools for computation [11]. Tegmark was accurate in excluding them from his tables of fundamental physical constants.

Since these values are easily removable from equations with no loss of accuracy, they seem to be inadequate parameters to investigate. Wesson [11] and Tegmark [10] both suggest the use of dimensionless parameters instead. Many scientists investigating fine-tuning,including Barnes, also focus on these dimensionless parameters. The freedom from dimension in all these parameters implies a step toward discovering fundamental properties of our universe without the error of human dimensionality.

Dimensionless Constants α, α_s, & β

Replications of Barnes' plots from Fig. 2 are shown in Fig. 6. In order to calculate 2D representations of inequalities involving three variables, the unused parameter was set to its observed value in this universe. The nonlinear scaling factors were used to scale the input constant parameter, functions, and axes. The highlighted yellow region is the area that satisfies every inequality therefore representing the habitable region, and the red squares

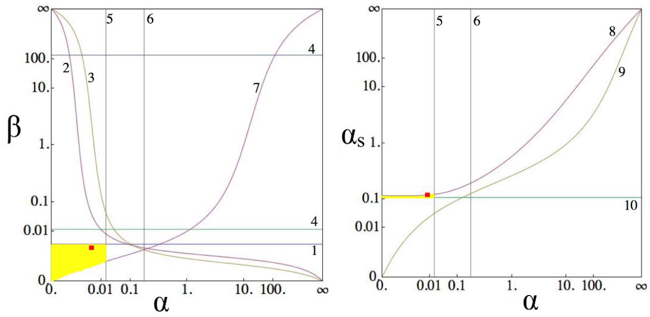

FIG. 6: Replication of Fig. 2 using MATHEMATICA. The yellow highlighted areas satisfy every inequality and represent the habitable region, and the red squares are values of the observable universe. The unused parameter α_s or β was set to its value in the observable universe ($\alpha_s = 0.1187$ on the left and $\beta = 1836.15^{-1}$ on the right). The numbers refer back to the inequalities the curves represent from *Plot Replication of Barnes' Fig. 2.*

represent parameter values of the observable universe. The graph on the right of Fig. 6 lacks Barnes' third inequality which is present in Fig. 2. When solved for α_s, the original inequality becomes

$$\alpha_s = 100\alpha\sqrt{\beta}$$

where $\beta = 1836.15^{-1}$. For scaling purposes, $N[\beta]$ was used. Recall $N[x]$ involves the natural log nested in the inverse tangent. Since $\beta < 1$, the natural log is negative yielding a negative arctangent. This gives imaginary values from the square root, so the function is not visible on the graph. Scaling the constant was necessary for the computation of all the other inequalities, and the unused constant was scaled as such in all other 2D plots. This inequality was therefore excluded from Fig 6.

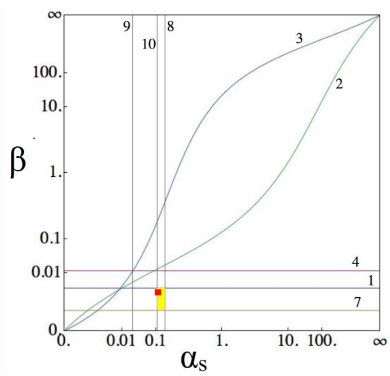

FIG. 7: 2D plot of the relationship between β and α_s when $\alpha = 1/137$, its measured value in the observable universe. The habitable region is shown highlighted in yellow, and the observable universe as a red square. The numbers refer back to the inequalities the curves represent from *Plot Replication of Barnes' Fig. 2.*

Using the same inequalities from Barnes [1], an additional 2D plot of β vs. α_s was created in an identical manner. This is shown in Fig. 7. Again, the yellow highlighted region is the habitable region when the invariable α is set to its observed value in this universe, 1/137.

The analysis of three independent parameters naturally led to the creation of a three-dimensional plot. In

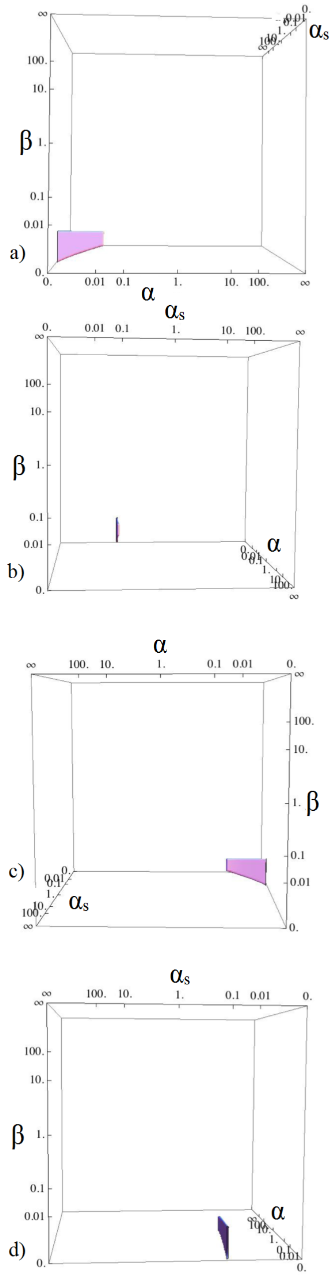

FIG. 8: Example viewing angles of a 3D region plot of the habitable region in all of parameter space. a) is shown from $\alpha_s = 0$ and the cube is rotated 90° clockwise along the β axis for each sequential image.

order to visualize the habitable region, a *RegionPlot3D* was created. This command creates a 3D surface that satisfies each inequality. Multiple views of this plot are shown in Fig. 8. Figure. 8 a) is shown from $\alpha_s = 0$ and the cube is rotated 90° clockwise along the β axis for each sequential image. The 3D habitable region can now be seen independently. Some views, such as from where $\alpha_s = 0$, represent identically the regions found in the 2D plots such as Fig. 6. Those that do not disagree due to the view of the 2D plot. For example, in the β vs. α_s plot of Fig. 7, $\alpha = 1/137$. The β vs. α_s view of Fig. 8 is shown from the edge of parameter space where $\alpha = \infty$. This difference in α explains the discrepancy between shapes of the habitable region in two dimensions. A slice of the 3D region at $\alpha = 1/137$ would yield similar results to Fig. 7.

This plot shows a habitable range of 0-0.107 for α, $0.108 - 0.173$ for α_s, and $0 - 0.00139$ for β. This range is better seen in Fig. 9 where the axes ranges are decreased to these ranges in order to zoom view into the habitable region. The fourth image in Fig. 9 makes the shape appear hollow. This lack of an edge is because the region continues to the $\alpha = 0$ axis.

This enlarged region is also shown in Fig. 10 with linear axes instead of the nonlinear scaling from above. This represents more realistically what the region looks like without being able to compare it to all of parameter space.

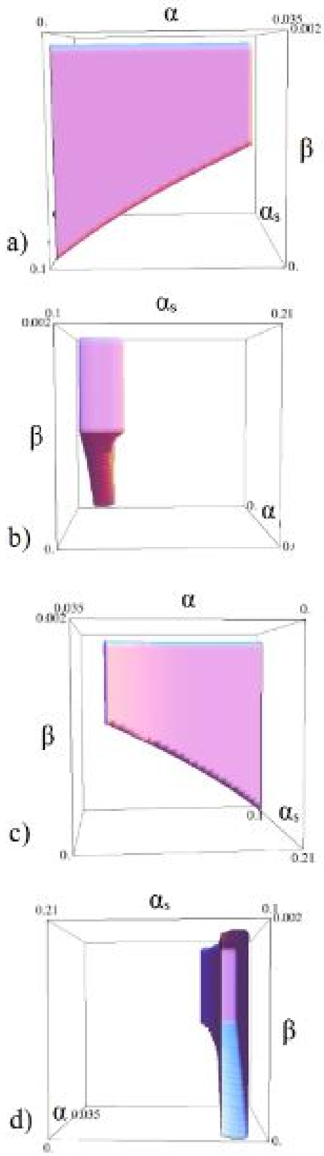

FIG. 9: Example viewing angles of a 3D region plot of the habitable region in all of parameter space zoomed into a smaller range to see the shape. a) is shown from $\alpha_s = 0$ and the cube is rotated 90° clockwise along the β axis for each sequential image

CONCLUSION

Scholars of different fields argue as to the merits and likelihood of the fine-tuning of our universe. An analysis of eight physical parameters was conducted using inequalities that all must simultaneously be met for universes to support life. Axes were scaled so the range of zero to infinity could be plotted on a finite axis, and the entirety of parameter space was investigated. More common dimensional parameters $\hbar, c$ and m_p resulted in a function that cut through parameter space creating a habitable region less than the function. A comparison of the dimensionless parameters α, α_s and β gave a three-dimensional habitable region with ranges of three orders of magnitude or less. The habitable region is $0 < \alpha < 0.107$, $0.108 < \alpha_s < 0.173$, and $0 < \beta < 0.00139$. In general, the habitable region of investigated parameter spaces was small compared to the parameter space. Though shows that a specific range of these parameters is essential for the development of life. Due to the habitable ranges found for each parameter, I am not yet convinced by the fine-tuning of our universe as any combination of these parameters could produce

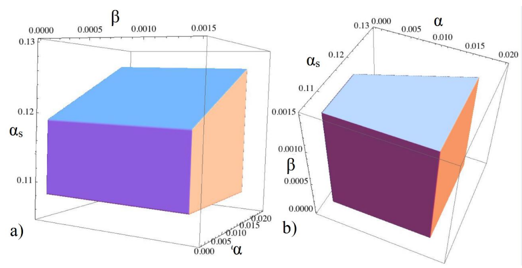

FIG. 10: Example viewing angles of a 3D region plot of the habitable region in all of parameter space with linear axes.

a life-permitting universe provided they fall within the appropriate range. Additionally, the applications of the work in multiverse theory are exciting but these results cannot be used in any way to refute or deny such unfalsifiable theories.

ACKNOWLEDGMENTS

I would like to thank Dr. Lehman and the College of Wooster for making this research possible, Dr. Lindner for his extra help, guidance, and patience, and the rest of the Junior Independent Study class for their support and encouragement.

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