Causal Dynamical Triangulation in Three Dimensions: Tiling spacetime with Tetrahedrons

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In this paper, a new approach to quantizing gravity is explored called Causal Dynamical Triangulation (CDT). This paper talks very briefly about the theory necessary to build a computer simulation to investigate Quantum Gravity with 2 spatial dimensions and 1 temporal dimension. Furthermore, we have successfully built the initial structure of the universe and filled all of vacuum spacetime in 3 dimensions with 3-simplices (tetrahedrons). The next immediate step is to investigate CDT in 2+1 dimensions by performing combinatorial moves and anti moves along the topology of spacetime.

I. INTRODUCTION

The challenge for physicists lies in developing theories to better understand the behavior of the universe. During the 20th century, the discovery of general relativity (GR) and quantum mechanics (QM) revolutionized our understanding the universe. general relativity has successfully explained the realm of the very massive. It explains that space and time is curved, and that curvature is responsible for our perception of gravity. quantum mechanics has successfully provided physicists with explanations at the small scale. QM tells us, among other things, that something may exist in an indefinite state, and that these uncertainties become very important when we try to precisely measure very small objects. Although both of these theories have been experimentally tested with incredible accuracy, the fundamental assumptions of these theories seem to contradict each other. For most regions of our universe, heavy masses are not concentrated into a small enough region to warrant an explanation from quantum mechanics and general relativity. As a result, we deal with the physics in the quantum regime separately from the gravity regime neglecting the effects of the other regime. However, regions where quantum mechanics and general relativity merge do exist and Quantum Gravity (QG) is an attempt to merge quantum mechanics to our notion of gravity.

Causal dynamical triangulation (CDT) developed by Ambjørn, Jurkiewicz and Loll is one of the most recent proposed techniques to understand and quantize gravity [1]. CDT is a conservative approach to quantizing gravity that relies solely on ideas and techniques that have already been known to physicists such as path integrals, causality, simplicial manifolds, triangulation, etc. The formulation of CDT, while being extremely straightforward, has yielded some important results- while spacetime is classical in four dimensions, it predicts a two dimensional and nonclassical spacetime at the Planck scale($\approx 10^{-35}$m) [2].

This paper introduces and briefly describes the theory of CDT and explains the initial triangulation of spacetime required for studying quantum gravity in a universe with two spatial and one temporal dimensions.

II. THEORY

To fully understand the CDT approach, it is necessary to first familiarize oneself with the basic concepts of Einstein's field equations, path integrals for an electron, and what it means to triangulate spacetime. In this section we will give a brief introduction to all these concepts and will finally explain the CDT method.

A. General Relativity

In standard geometry, we deal with surfaces with only spatial dimensions which involves applying the Riemannian metric. For example, the Euclidean metric is an example of such a metric that calculates the distance between two points given by the Pythagorean formula. However, in relativity we add a temporal dimension, setting space and time on equal footing [3]. The line element denotes an interval between two nearby points in spacetime separated by timelike intervals $d\tau$ and spacelike intervals $d\sigma$. In flat spacetime, we generalize the Pythagorus theorem to obtain the line element [4]:

$$-d\tau^2 = +d\sigma^2 = dx^2 + dy^2 + dz^2 - c^2dt^2 \tag{1a}$$

$$= \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} g_{\mu\nu}dx^\mu dx^\nu$$

$$= g_{\mu\nu}dx^\mu dx^\nu \tag{1b}$$

where the Greek indices run from 0 to 3 and the repeated indices represent an implied sum. Defining the basis for an event in space time as a four dimensional contravariant vector we have

$$x^\mu \leftrightarrow \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix} = \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

and in flat spacetime far from any stress or energy the Minkowski metric $g_{\mu\nu}$ is given by

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{bmatrix}.$$

Using these transformation rules, we are able to define a different basis called the covariant vector with lowered indices as

$$x_\mu = g_{\mu\nu} x^\nu. \tag{2}$$

In general relativity, it is often required to change the coordinate system to understand an event from a different perspective. An example of this kind of change in coordinate system can be seen in special relativity where we apply the Lorentz tensor and perform what is called the Lorentz transformation [5]. To get from a coordinate system $x^\mu \to x^{\bar{\mu}}$ the element of infinitesimal change has the form

$$\begin{aligned} \delta x^{\bar{\mu}} &= \frac{\partial x^{\bar{\mu}}}{\partial x^\nu} \delta x^\nu \\ &= x^{\bar{\mu}}{}_{,\nu}\, \delta x^\nu \end{aligned} \tag{3}$$

where the comma represents a partial differentiation. This convention is called the Einstein's summation convention [5]. Using the tools described above, we are able to generalize the transformation. For example,

$$P_{\bar{\lambda}\bar{\mu}\bar{\nu}} = x^\theta{}_{,\bar{\lambda}}\, x^\alpha{}_{,\bar{\mu}}\, x^\beta{}_{,\bar{\nu}}\, P_{\theta\alpha\beta}. \tag{4}$$

Following Eddington's theorem [5] any indexed quantity that obeys this transformation rule described in Eq. 4 is itself a tensor.

Next, we explore the curvature of a manifold. It is necessary to develop tools to shift a vector along a manifold while making sure that the vector is always parallel to itself. For this purpose, we introduce a non tensor called the Christoffel coefficients or connection coefficients that account for the change in component for the transformed vector. This technique is called parallel transport. The details for shifting some arbitrary vector from position $x^\mu \to x^\mu + dx^\mu$ and the derivation of the connection coefficients can be found in [4]. The connection coefficients are non tensors that relate to the derivative of the metric tensor and are defined as [5]

$$\Gamma^\alpha_{\nu\mu} = \Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(g_{\beta\mu,\nu} + g_{\nu\beta,\mu} - g_{\mu\nu,\beta} \right) \tag{5a}$$

$$\Gamma_{\rho\mu\nu} = g_{\rho\alpha} \Gamma^\alpha_{\mu\nu}. \tag{5b}$$

Now, we are able to define a curvature tensor in terms of the connection coefficients known as the Riemann curvature tensor as [5]

$$R^\alpha_{\beta\mu\nu} = -\Gamma^\alpha_{\beta\mu,\nu} + \Gamma^\sigma_{\beta\nu}\Gamma^\alpha_{\sigma\mu} + \Gamma^\alpha_{\beta\nu,\mu} - \Gamma^\sigma_{\beta\mu}\Gamma^\alpha_{\sigma\nu}. \tag{6}$$

We can contract the Riemann curvature tensor on the 1^{st} and 4^{th} indices to form the Ricci curvature tensor [5]

$$R^\alpha_{\beta\mu\alpha} = R_{\beta\mu}. \tag{7}$$

Further contraction of the Ricci curvature tensor we obtain the Ricci scalar curvature [5]

$$R^\alpha_\alpha \equiv R. \tag{8}$$

In spacetime, we characterize the world line of a free particle as a time geodesic. Furthermore, the geodesic in curved spacetime is the straightest possible curve that have stationery length and is derived as [5]:

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0. \tag{9}$$

The geodesic equation is a direct consequence of spacetime being curved by matter. This forces to introduce the stress energy momentum (SEM) tensor. Details of the geodesic equation and SEM tensor can be found in [6]. The SEM tensor is a relativistic generalization of the energy density which is defined as [5]

$$T^{\mu\nu} = \frac{dP^\mu}{d^3 V_\nu} \tag{10}$$

where P^μ is the four momentum vector and V is the volume vector. With regard to the conservation of four momentum and the principle of minimal coupling, it is known that the SEM tensor is divergenceless [4]. The Einstein field equation with regard to the SEM tensor is [5]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{11}$$

where Λ is the cosmological constant to account for the accelerating size of the universe, the speed of light $c = 1$. We can write this in a more compact form by introducing the Einstein tensor, $G_{\mu\nu}$ which is also the trace reversed Ricci tensor

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{12}$$

For this paper, we assume a vacuum universe and do not take matter into account to obtain the vacuum field equation [7]

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0. \tag{13}$$

We can also derive the vacuum field equation by taking solving the Euler-Lagrange equations for the Einstein-Hilbert (EH) action. The EH action S_{EH} is the action for spacetime in general relativity and is defined by [2]

$$S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\,(\Lambda - 2R) \tag{14}$$

where $g = det[g_{\mu\nu}]$. In our case, we only deal with two spatial dimensions and one time dimension so we can rewrite the EH action as

$$S_{EH} = \frac{1}{16\pi G} \int dA\,(\Lambda - 2R) \tag{15}$$

where we diagonalize the metric over all 3-dimensional simplicial manifolds to get the invariant area element, $dA = \sqrt{-g_{tt}}\ dt \sqrt{g_{xx}}\ dx \sqrt{g_{yy}}\ dy = \sqrt{-g}\ d^3x$.

B. Quantum Mechanics

The famous double-slit experiment first introduced by Thomas Young in 1803 involves passing a beam of light through two narrow parallel slits (as shown in Fig. 1−a) to produce an interference pattern on the screen which demonstrated the wave-nature of light, this phenomenon along with the photoelectric effect is widely known as the wave-particle duality. Now, if we keep on adding additional slits to the same screen we arrive at the situation shown in Fig. 1−b. Furthermore, if we keep on adding more screens, we arrive at the situation shown in Fig. 1−c and by adding infinitely many screens and slits we are eventually left with empty space as shown in Fig. 1−d [8].

A photon passing through a double slit from some arbitrary point a has a probability of arriving at some point b that is characterized by

$$\mathcal{P}[a \to b] = |\mathcal{E}[a \to b]|^2 \tag{16}$$

where the normalized electric field amplitude is given by

$$\mathcal{E}[a \to b] = e^{i\varphi[1]} + e^{i\varphi[2]} \tag{17}$$

where $\varphi = \int \omega\ dt$ [8]

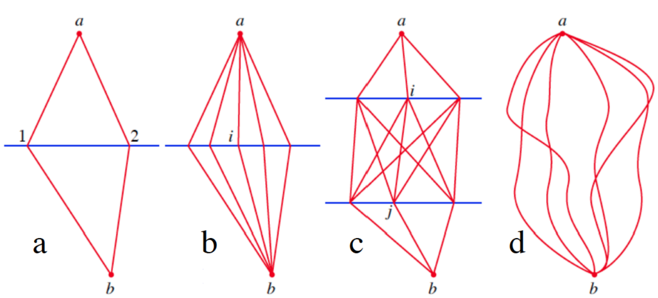

FIG. 1: Schematic of the evolution of infinitely many screens with infinitely many slits that demonstrate that an electron has infinitely many paths to reach from point a to point b. Figure adapted from [8].

for the slits, 1 and 2. With regard to waveparticle duality, we are able to replace a massless photon with a relatively massive electron. Passing the electron through the double slit, Feynman showed that the probability for a electron starting at a to reach b was

$$\mathcal{P}[a \to b] = |\mathcal{A}[a \to b]|^2 \tag{18}$$

with the probability amplitude $\mathcal{A}$ for the two slits, 1 and 2 is given by

$$\mathcal{A}[a \to b] = e^{i\varphi[1]} + e^{i\varphi[2]} \tag{19}$$

where the phase accumulated by the electron is

$$\varphi = \int \omega\ dt = \int \frac{L}{\hbar} dt = \frac{S}{\hbar}.$$

For a free particle, the potential energy V is zero and hence the Lagrangian L is equal to the kinetic energy T, $L = T - V = T$. Here, we can see that the action S in quantum mechanic is analogous to the Lagrangian L in classical mechanics [8].

As the realm of QM is a linear vector space, it follows the superposition principle. For a screen with two slits, the probability amplitude for the electron to go from point a to b is simply the sum of the two amplitudes [8]

$$\mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2, \tag{20}$$

where $\mathcal{A}_1$ and $\mathcal{A}_2$ are the probability amplitudes for slits 1 and 2 in Fig. 1−I respectively. Similarly, for a system with i screens and j slits for each screen the probability amplitude for the electron to go from point a to b is

$$\mathcal{A} = \sum_{i,j} \mathcal{A}_{i,j} = \sum_{allpaths} A_{path}. \tag{21}$$

Here, we have developed that there are infinitely many paths between a and b and the probability amplitude considers all possible paths. We now define the Feynman propagator for a quantum system to be the probability amplitude between two points in spacetime (t_a, a) and (t_b, b) as [7, 9]

$$\mathcal{G}[a, b; t_a, t_b] = \langle a|\,\mathcal{U}[t_a, t_b]\,|b\rangle \tag{22}$$

where $\mathcal{U}[t_a, t_b] = e^{-i\mathcal{H}(t_b - t_a)}$ is the unitary time evolution operator and $\mathcal{H}$ is the Hamiltonian of the system. A rigorous derivation for the unitary operation is beyond the scope of this paper and can be found in [7].
Combining Eq. 21 and Eq. 22 for the total amplitude we get a formal expression for the Feynman propagator to go from point a to b as [8]

$$A[a \to b] = \mathcal{G}[a, b; t_a, t_b] = \int_{t_a}^{t_b} \mathcal{D}x[t] e^{iS[x[t]]/\hbar} \tag{23}$$

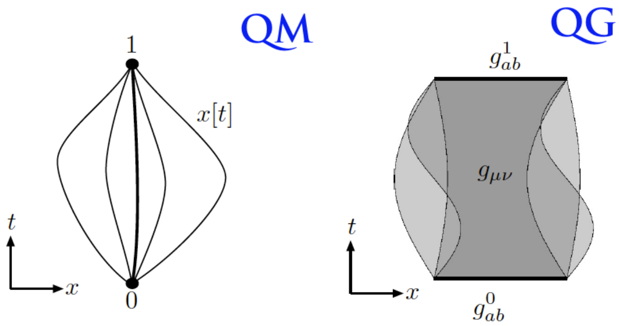

FIG. 2: Figure shows the side by side comparison of the path integral from QM with the gravitational path integral from QG in one temporal and one spatial dimension. Figure adapted from [3].

where $Dx[t]$ is a path measure that accounts for all possible paths, S is the action and $\hbar$ is the Planck's constant. The integral in Eq. 23 which gives the sum over all paths is called the path integral.

Analogous to the path integral for quantum mechanics, we introduce a gravitational path integral for quantum gravity (QG). As shown in Fig. 2, the gravitational path integral generalizes the path integral from QM based on the principle that QG should involve all metrics in spacetime, and hence is described as a sum over all possible geometries [7].

C. Wick Rotation

Wick rotation is a method that is used to solve a problem in the Minkowski spacetime by transforming it to a simpler problem in Euclidean space. This transformation involves substituting an complex-number variable for a real-number variable [10]. The concept of Wick rotation originates from the residue theorem in complex analysis that states that the integral of an analytic function $f[z]$ under a closed path in the complex plane is zero [3]. Notice that the path integral described in Eq. 23 is of the form e^{ix} which is a sinusoidal function and the integral does not converge. By rotating the contour by $\pi/2$ radians counterclockwise as shown in Fig. 3, we notice that the function goes to zero more rapidly as it gets closer to infinity, more explicitly [7]

$$0 = \oint f[z]dz = \int_{-\inf}^{\inf} f[z]dz + \int_{-i\inf}^{i\inf} f[z]dz. \tag{24}$$

Next, by changing the integral axes, $x \to z$ and $z \to iy$ we get [7]

$$\int_{-\inf}^{\inf} f[x]dx = i \int_{-i\inf}^{i\inf} f[y]dy. \tag{25}$$

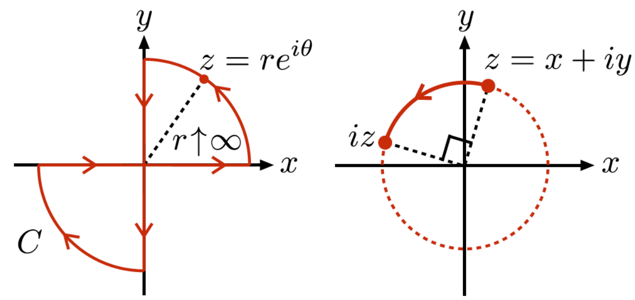

FIG. 3: The integral counter (left) and adding a complex factor (right) in the complex plane allows for Wick rotation. Figure adapted from [3].

By applying this Wick rotation we convert the standard Einstein-Hilbert action into the Einstein-Hilbert action for a Euclidean space. This means that the action in Eq. 14 now becomes [7],

$$S_{EH} = \frac{i}{16\pi G} \int dA\,(\Lambda - 2R) \tag{26}$$

where dA is as defined above. This causes the probability amplitude from Eq. 23 to transform as follows [3]

$$A = \int \mathcal{D}g e^{iS} \to \int \mathcal{D}g e^{S_E} = Z \tag{27}$$

where Z is the partition function. Notice here that the Wick rotation transforms the Feynman phase factor e^{iS} from Minkowski spacetime to the Boltzmann weight $e^{iS_E} = e^{-|iS_E|}$.

D. Causal vs. Euclidean Dynamical Triangulation

Causal dynamical triangulation is a non-perturbative attempt to quantize gravity without adding any additional structure. It does not describe the dynamics of gravity as linear perturbation around some preferred background metric [2]. With regard to the known principles from quantum mechanics and general relativity, CDT utilizes causally triangulated geometries. In dynamical triangulation, the edge lengths are fixed and the path integral from Eq. 23 is represented as a sum over triangulations. Despite its discrete foundation, CDT does not necessarily imply that spacetime itself is discrete [1]. The CDT approach arose as a modication of its predecessor, Euclidean dynamical triangulations (EDT) where no global foliation of spacetime was imposed. In EDT, discrete geometries are made up of Euclidean simplical building blocks that only have space-like edges and allowed all possible triangulations of spacetime [11]. This lack of a foliated structure in EDT resulted in a failure to reproduce a 4-dimensional classical universe. CDT tackles this problem by implementing the Lorentzian structure of spacetime to include space-like and time-like

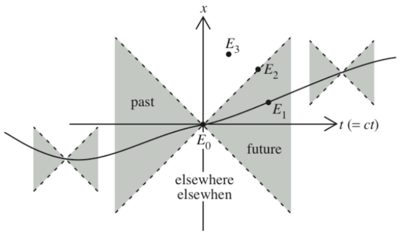

FIG. 4: Figure (adapted from [8]) of a light cone in 1+1 dimensional and 2+1 dimensional space times. The future light cone is pointing right.

edges. CDT has already had great success by predicting that the universe is $1 + 1$ dimensional at the Plank scale [3].

CDT assumes a global time slicing, where each time slice is set up such that the metric exists and the volume is finite [2]. As shown in Fig. 4, in a Minkowski spacetime, the worldline of any object enters every event via the pastlight cone and exits via the future light cone. This creates a foliation in spacetime, which is intrinsic in CDT. The foliated or sliced structure of spacetime is implemented by using Minkowski triangles. As shown in Fig. 5, unlike Euclidean triangles, the Minkowski triangles have two directed timelike edges and one spacelike edge. Each Minkowski triangle has the metric $ds^2 = -dt^2 + dx^2$. Due to the difference in the metric, the area of the Euclidean triangle with length l was noticed to be $l^2\sqrt{3}/4$ and the area of the Minkowski triangle with length l was $l^2\sqrt{5}/4$. The details of this calculation can be found in [3]. In triangulation, the Minkowski triangles are equilateral and preserve the total area. CDT uses dynamical triangulation with a foliated structure by gluing time-like edges in the same direction. The microscopic causality inherent in the resulting spacetime foliation ensures macroscopic spacetime as we know it. This implies that causality is not only a global property of spacetime, but it is also local [2].

E. Regge Calculus

Regge calculus was first developed by Tullio Regge where he introduced the idea of approximating spacetime with a piecewise linear manifold, a process called triangulation. Regge calculus offers a method to work in GR without using symmetry arguments and is ideal for numerical simulations. It is an inherently discrete formulation of complex topologies in GR [12]. Each building block of the new manifold is called a simplex, which is an $n-$dimensional generalization of the notion of a point, a line, a triangle or tetrahedron as shown in Fig. 6. In 2+1 dimensions, a manifold can be approximated using a number of 3-simplicies or tetrahedrons. This process of approximating spacetime produces a 'Connection Matrix' which contains all the data on the edges about the

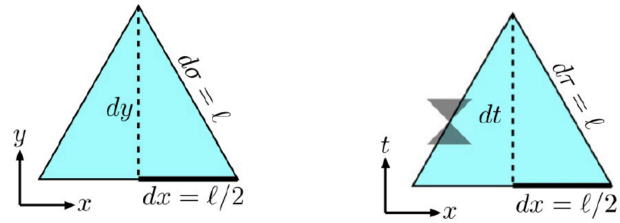

FIG. 5: Figure shows the Minkowski (right) and Euclidean (left) Triangle. Figure adapted from [3].

geometry and topology of our spacetime [12]. Following this information stored triangulation of a smooth manifold can be obtained by gluing together these simplices in an appropriate manner. These tetrahedrons are locally flat and indeed this kind of discretization leaves gaps in the approximation of the spacetime; Regge calls these gaps 'hinges' or 'bones'. The angle of curvature in the hinge, called the deficit angle ϵ is illustrated in Fig. 7. A positive decit angle represents positive curvature and a negative decit angle stands for negative curvature [12]. The sum over deficit angles, weighted by the volume V each hinge is proportional to the integral of the Ricci scalar over a surface. We are now able to express the Einstein-Hilbert action as a function of just the edge lengths of making appropriate substitutions [2]

$$\int d^n x\sqrt{-g}R \to 2\sum_{i\in T}\epsilon_i, \tag{28a}$$

$$\int d^n x\sqrt{-g} \to 2\sum_{i\in T}V_i \tag{28b}$$

where T is the triangulation where the curvature is located. Using the tools recently developed, we can simplify the Wick rotated Euclidean action from Eq. 26 as

$$S_{EH} \to S_R = \frac{1}{8\pi G}\sum_i V_i\epsilon_i - \frac{\Lambda}{8\pi G}\sum_{simplicies} V_{simplex} \tag{29}$$

where i is the number of hinges in the curvature. The path integral in Eq. 23 over all unique field configurations is then taken to be a sum over all triangulations T weighted by S_R from above to get

$$Z = \int \mathcal{D}g e^{iS_E} \to \sum_T \frac{e^{iS_E}}{C_T} \tag{30}$$

where $1/C_T$ is the symmetry factor with C_T begin the order of the homomorphism group for a triangulation T [2].

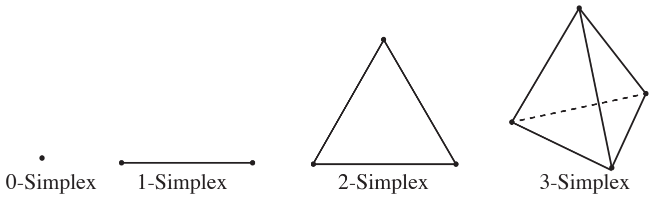

FIG. 6: The n-simplices used in the triangulation of spcae-time.

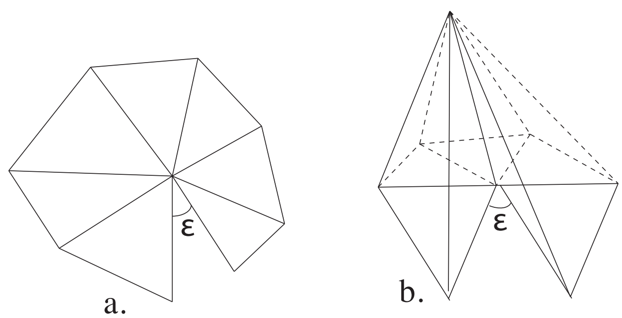

FIG. 7: Schematics of 2D and 3D triangulated manifold (a) has a curvature that is situated on vertices while on a 3D triangulated manifold (b) the curvature is concentrated on edges (figures not to scale).

III. FILLING SPACETIME WITH TETRAHEDRONS

The building blocks of a 3 dimensional CDT are 3-simplices that span adjacent time slices. A timelike (p, q) $n-$simplex, where $p+q = n+1$, has p points on the earlier time slice and q points the adjacent later time slice. However in a spacelike simplex all points must be on the same slice [2]. The tetrahedrons in CDT inherit causality as a result of time slicing. To preserve the causal structure, CDT does not allow topology changes of the time slices. The global causal structure for any triangulated geometry with local causality is obtained by gluing these 3-simplicies in an appropriate way such that their spacelike edges are all of length squared a^2 and timelike edges have same length $-\gamma a^2$ where γ is an asymmetry factor that accounts for the distance between timelike and spacelike geodesics. Following these rules, there can only be three types of basic tetrahedrons, namely $(1, 3)$, $(2, 2)$ and $(3, 1)$ as illustrated by Fig. 8. As mentioned earlier, these tetrahedrons are classified by the number of points it has in the adjacent spatial slices. For example, a $(3, 1)$ tetrahedron has 3 vertices at time t_0 and 1 vertex at time t_1.

In 2+1 CDT we take our standard x and y axes to be our spatial axes and the standard z axis as our temporal axis. In the experimental section in three dimensions, it

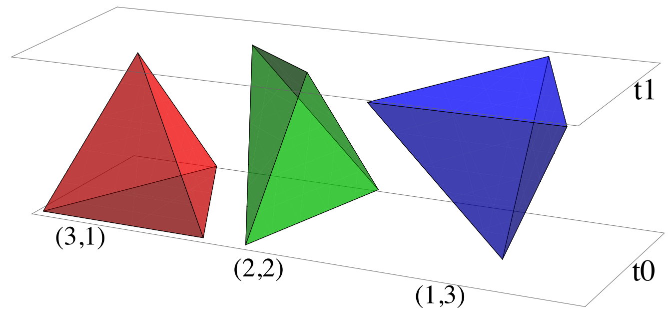

FIG. 8: Figure shows the three types of basic building blocks in 3 dimensions.

is necessary to carefully tile tetrahedrons to fill spacetime in such a way that there are no gaps, and that all edges of the tetrahedrons are aligned correctly to the adjacent one. To make sure that none of these rules are violated, we followed a face centered cubic (FCC) lattice structure where we mapped vectors from the center of one lattice point to another to define the initial basis vectors. The three basis vectors used are of unit length and were taken to be

$$\begin{aligned} \hat{u}_1 &= \langle 1,\ 0,\ 0 \rangle \\ \hat{u}_2 &= \left\langle \frac{1}{2},\ \sqrt{3}2,\ 0 \right\rangle \\ \hat{u}_3 &= \left\langle \frac{1}{2},\ \frac{1}{2\sqrt{3}},\ \sqrt{\frac{2}{3}} \right\rangle . \end{aligned} \tag{31}$$

The tetrahedron where the edges are the basis vectors is shown in Fig. 9. Once the basis vectors are defined, we merely have to translate vectors in order to fill each time slice.

To make sure that the tetrahedrons are glued along the edges, we followed the FCC lattice ABA structure to fill adjacent time slices. Once the first layer was completed, the mirror image of the layer was constructed in the $+t$ direction. The initial lattice structure was drawn in $Mathematica$ [13].

For numerical simulations of the discrete Lorentzian model it is required to make 'moves' and 'anti-moves', that are a set of basic manipulation of a triangulation which results in a different triangulation [14]. The details of the moves in three dimensions are beyond the scope of this paper and is fully explained in [14]. Keeping in mind that these moves have to be made in the future, it is required that we build a computer simulation to perform these numerical simulations. In this paper, we only deal with the initial structure of our universe. The simplices in the initial universe was drawn using Objective-C in Xcode 4.4.1. Since each triangulated spacetime is glued to a set of tetrahedra in a specific manner, it was necessary to store the data for each tetrahedron. For this reason, we initialized a hash table to store information about the current state for each tetrahedron. Each tetra-

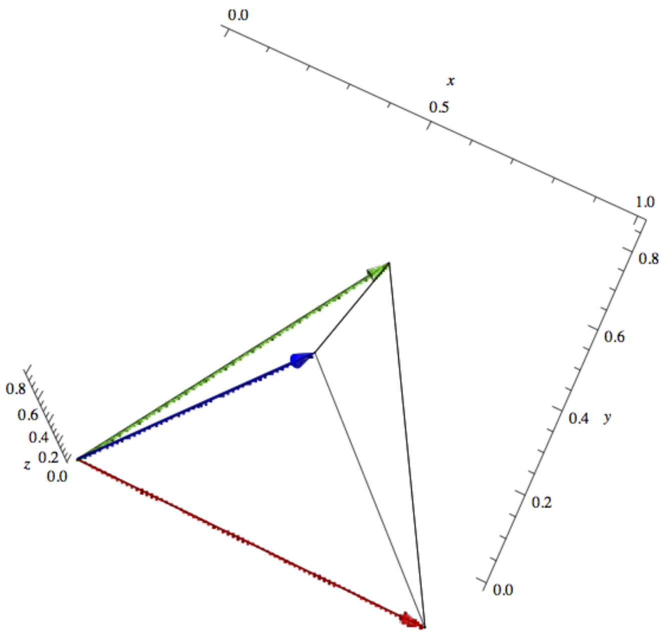

FIG. 9: Figure shows a Type 3 tetrahedron with the basis vectors. The basis vectors, $\hat{u}_1$ is the red arrow, $\hat{u}_2$ is the green arrow and $\hat{u}_3$ is the blue arrow.

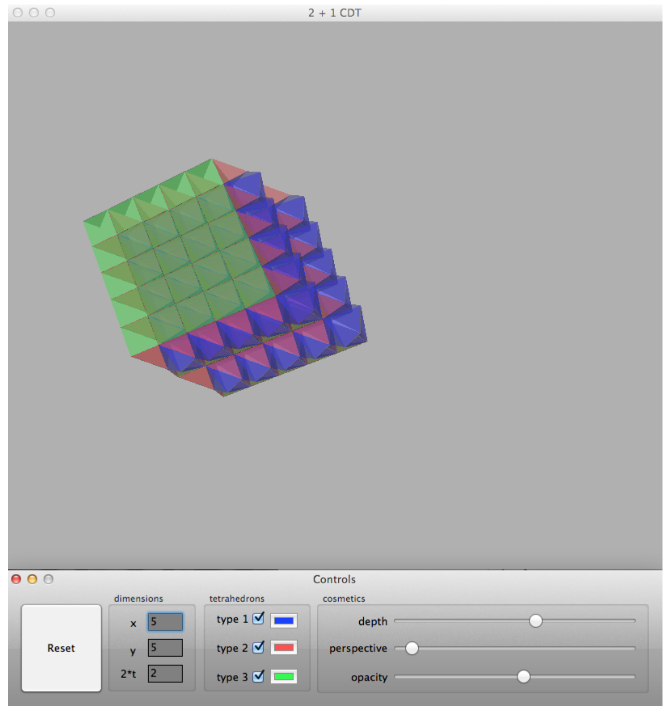

FIG. 10: Screenshot of the program written in Xcode that draws tetrahedrons to fill spacetime. The program allows the user to change the dimension of the lattice structure, the depth and perspective of the view as well as the opacity. The user also has the option to draw only type 1, type 2 or type 3 tetrahedrons as well as change the color for each.

hedron has a unique key, and the hash table stores information about its vertices and the key for the neighboring tetrahedra. The structure of the hash table is

$$\langle\ key,\ type,\ t,\ v1,\ v2,\ v3,\ v4\rangle\ ,$$

where key is the unique id for the tetrahedron,$type$ is the type of tetrahedron (declared as integer to store either 1, 2 or 3), t is the earlier time slice ($\min[t_a, t_b]$), $v1, v2, v3$ and $v4$ are declared as a structure to store the x, y and z coordinates for each vertex. For example, the list $\langle 21, 2, 4, 0.0, 0.0, 0.0, 1.0, 0.0, 0.0, 0.5, 0.8, 0.0, 0.5, 0.3, 0.8\rangle$ represents a type 2 tetrahedron with id 21 resting on the time slice 4 with vertices $v1 = (0.0,\ 0.0,\ 0.0)$, $v2 = (1.0,\ 0.0,\ 0.0)$, $v3 = (0.5,\ 0.8,\ 0.0)$ and $v4 = (0.5,\ 0.3,\ 0.8)$.

IV. CONCLUSION AND FUTURE WORK

Here we have just set up the initial structure of the universe. The next immediate step in this project is to write an algorithm to perform the combinatorial moves and anti-moves in 2+1 dimensions along the topology of spacetime. While writing an algorithm to perform these moves, it is required to account for the time foliated structure of our universe. Furthermore, it could be worth minimizing the deficit angle to get more accurate results by either increasing the number of tetrahedrons or decreasing the length of the edges. These are just a few things that we have yet to build and explore. Once the program is complete, the next step would be to plug in values for the cosmological constant and the Planck constant to study how much fluctuations would be seen in order to understand higher dimensions.

V. ACKNOWLEDGEMENTS

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