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The purpose of this work was to experimentally determine whether toy cars accurately model the behavior of race cars, like NASCAR or IndyCar cars, with respect to their movement around a banked curve and the resulting drag coefficient. For a professional race car team's engineers, it is important to understand the forces that act on an individual car so that they can provide the driver with a mechanical set up that will help lead to victory. Through the use of a banked curve track and photogates, the velocity of a toy car was monitored to determine whether its behavior mimics that of an actual race car traveling around a banked curve. In addition, the drag coefficient was calculated for the toy cars to determine if a correlation exists between the toy cars and the actual race cars they model. After data were collected regarding the three cars, it was determined that there exist some correlations between the toy cars and the actual race cars; however, discrepancies still exist between the predicted and actual results. Two of the toy cars' data results match the predicted results for speed on a banked curve and there is some correlation between the drag coefficients of the toy cars and the race cars.

I. INTRODUCTION

"Three laps around the Brickyard...is an exhilarating insight into the...forces drivers experience through the 2.5-mile course's four banked turns," says USA Today writer Mark Fogarty about his ride in the IndyCar two seater driven by Al Unser Jr [1]. While traveling around the track, Fogarty notes the pressure he feels on his helmet as they drive down the straightaways along with the centripetal force as they travel through the nine degree banked turns. Although Fogarty and Unser Jr. only reach 170 mph during their ride, the Indy cars travel more than 220 mph in the Indianapolis 500 race [1]. Before every Indianapolis 500, Brickyard 500, or other race in the season run by either IndyCar or NASCAR, engineers spend hours preparing the cars so they handle properly and travel as fast as possible. Depending on the track, engineers may either work to increase the downforce of the car or decrease the drag. Thorough knowledge of the forces affecting a NASCAR or IndyCar car is obtained through tests run by both the individual teams and the league [2]. During the off season, teams use wind tunnels to study the effects of the movement of air around the car. This provides information regarding the drag force associated to the automobile and improves the teams' understanding of the forces affecting the car so that they can better prepare for the race at each track. In an attempt to understand the many forces that affect both the car and driver during a race, an analysis is performed on a toy car as it travels through a banked curve. In particular, the effect of the banking angle on the velocity of the car is analyzed. In addition, the drag coefficient associated with each of the different toy cars used is computed to determine if there is any correlation between the drag force on the toy cars and the actual cars they represent.

II. THEORY

There are many forces acting on a race car as it travels down a straightaway or turns into a corner. Engineers work and study the cars in order to prepare them so the forces act in a way that allows their driver to obtain the optimum balance between speed and handling. Two forces that play a large role in the performance of a race car are the turning force or centripetal force and the drag force. By working to maximize the ability of the car to travel through the banked turns and setting up the car to have the proper amount of drag force, a driver may find their engineers have provided them with an excellent handling car.

A. Centripetal Force

The equation of motion for a race car traveling down a straight track is given by Newton's second law,

$$F = ma, (1)$$

where F is the magnitude of the force on the car, a is the acceleration, and m is the mass of the car. When a car is going around a turn, the acceleration is given by

$$a = \frac{v^2}{r},\tag{2}$$

which means the acceleration is dependent on both the velocity of the car v and the radius of the corner r around which it is traveling [3]. Thus, for a car experiencing a centripetal force, a force directed toward the center of the curved path, the equation of motion is given by

$$F = \frac{mv^2}{r} \tag{3}$$

where m is the mass of the car, v is its velocity, and r is the radius of the corner. To understand the impact of

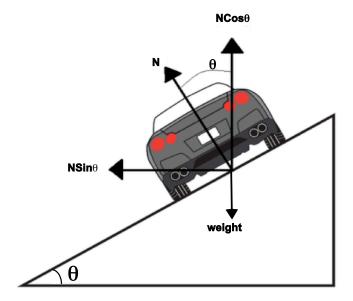


FIG. 1: A depiction of the normal force, N, and the weight of a car as it travels through a banked curve. This figure is reproduced from reference [4].

banking on the maximum speed of a car, first consider a NASCAR car traveling around turn one at Texas Motor Speedway (TMS) if the corner was not banked. This particular turn has a 750 ft. radius [2]. Assume that driver mass plus car mass yields a total mass for the system of approximately 1633 kg. If the driver took the turn at a speed of 65 mph, the total force, given by Eq. 3, is 6031 N which, using Eq. 1, means the acceleration of the car is 0.38g [2]. If it is assumed that the weight of the car is distributed equally, then each of the tires is responsible for generating one-fourth of the necessary force which causes the car to travel around the turn. Given that race tires are manufactured to be stickier and have more grip than regular tires, studies have shown that each tire can generate approximately 1080 pounds (4804 N) of force [2]. This means the quickest speed with which a race car can travel around an unbanked version of turn one at TMS is 116 mph (52 m/s) as given by Eq. 3. NASCAR cars obtain much higher speeds through the corners. As the banking of the corners has not yet been included in the calculation, this factor will be incorporated in the calculations to determine the effect of the banking angle on velocity.

The normal force exerted by the road always acts perpendicular to the surface of the track [5]. If a race car is traveling around an unbanked corner, the weight of the car is offset by the normal force exerted by the track. But, when the track is banked, the normal force is perpendicular to the track so it has both horizontal and vertical components. Fig. 1 demonstrates the manner in which the normal force can be broken into its horizontal and vertical components when the track is banked at some angle θ . Notice that the horizontal portion of the normal force points toward the center of the curved path

and adds to the force turning the car, generated by the tires. A slight adjustment must be made with regard to the frictional force, as some of the force generated by the wheels that was used to turn the car is now used to prevent the car from sliding up the track. Although there is a loss in the turning power from the tires, the contribution from the horizontal component of the normal force more than makes up for this loss [2]. The banking of the corner allows a stock car to reach a maximum speed of 170 mph in turn one at TMS compared to the 116 mph that it can reach on the same level turn.

B. Drag Force

In addition to forces generated which cause a race car to travel around a banked curve, there are forces that act on the car as it travels through air which resist its motion. This force, known as the drag force, is given by

$$F_D = \frac{1}{2}\rho A C_D v^2,\tag{4}$$

where ρ is the density of air, A is the frontal area of the car, C_D is the drag coefficient and v is the velocity at which the car is moving [6]. The drag force opposes the motion of the car as it passes through air and, in the case of a car on a banked curve, is directly opposite to the turning force generated by the tires [7]. Notice that Eq. 4 includes the coefficient of drag, C_D associated to the car. This numerical value describes the shape of the car as it cuts through the air [2]. Although Indy cars and NASCAR cars travel faster than regular highway cars, these race cars have much larger drag coefficients. For instance, an Indy car has a drag coefficient of approximately 1.00 and a NASCAR car of 0.35 while regular highway cars have drag coefficients of approximately 0.26 [2, 7]. This may seem strange as one might assume that a car aiming to travel fast would have a small drag coefficient. However, this is not the case as a race car can generate more power than a street car and needs a higher drag coefficient as this value affects the design relating to downforce, traction, and stability [7]. Manipulating Eq. 4 it can be seen that the drag coefficient is given by

$$C_D = \frac{2F_D}{\rho A v^2} \tag{5}$$

where F_D is the drag force, ρ is the density of air, and A is the frontal area of the car [8].

III. PROCEDURE

The purpose of this work is to experimentally determine whether toy cars are a good model for race cars by analyzing their movement around a banked curve and their drag force. By analyzing the velocity of a toy car as it travels through a curve banked at one angle and then



FIG. 2: The banked curve that was made from craft sticks and covered with different colored poster board. Each different color poster board represents a different individual piece of track that was built. The black sensors are photogates which record the velocity of the car as it passes through.

changing the angle, the effect of the banking on the velocity of the car is analyzed. In addition, the drag force is computed for the three toy cars used in the experiment to determine if there is any correlation between the drag coefficients associated to the toy cars and the actual cars they represent.

As one goal of this experiment is to change the banking angle of a turn to determine the effects on the velocity of a toy car, the first step was to acquire the necessary pieces to create a banked curve that would allow the toy cars to travel around smoothly. The turn was initially built using straight, flexible pieces of track and modeled the dimensions of Texas Motor Speedway. The first banking angle the velocity of the cars would be tested at was 24° and the turn radius of the track was 3.1 m. The initial portion of track was attached to a table so that the car could obtain speed before entering the turn, simulating a driver pushing on the gas pedal. The straight plastic pieces were connected and laid out so that they followed the proper arc to create the turn. Styrofoam wedges were created to hold the track at a 24° angle. Once this construction had been completed, test runs were performed with the different toy cars. After running many different trials, it was determined that the toy cars did not travel through the turn as desired. Instead of making a gradual turn, the cars traveled along the straight track and then turned abruptly where the pieces connected. This behavior did not model the actual movement of a race car through a banked turn.

To correct this situation it was decided that either curved pieces of track would need to be purchased or a turn would need to be made from scratch out of other materials. As the pre-made plastic curved pieces that were available all had a defined turn radius that did not match the dimension of Texas Motor Speedway, the turn was made out of craft sticks (popsicle sticks). This way the curved pieces were made to match the dimensions of the modeled track. The craft sticks were glued to one another and poster board was used to create a smooth surface on top for the cars to travel over. Craft sticks were also glued to make the wedges that held the track at the desired angle of banking. The first angle of banking was set to 24° and the cars were sent down the track for test runs to see how they would perform on the newly made track. The initial entry portion of the track was placed on a stool of height 0.64 m so the toy cars could



FIG. 3: The toy Corvette car with the 0.025 m flag.

gain speed as they entered the corner. It was also important to line the entry track up so the car could gradually round the corner. Tubing was placed on the outside of the corner as there was no driver to turn the car and the car would otherwise drive off of the track. Fig. 2 shows the banked curve track after all of these elements had been assembled. A test run was then performed and the car successfully traveled through the curve.

Once the track itself had been assembled and everything was aligned so that the cars would travel around the corner, photogates were set up so that the velocity of the car, at different points along the curve, could be monitored. The photogates, which can be seen in Fig. 2, were arranged so that a distance of 0.5 m separated them. As the cars were not tall enough to activate the photogates, flags were added to the cars so that their velocity could be determined. Large flags were made so that the velocity could easily be recorded by the photogates. These flags caused problems as they changed the aerodynamics of the cars and the cars would no longer travel around the curve; therefore, flags of width 0.025 m were attached to the cars. A picture of the toy Corvette car with the attached flag can be seen in Fig. 3. The flags did slightly affect the aerodynamics of the cars, but the cars were able to travel through the corners once more wedges were added to the track for support and the separate pieces of the track were properly aligned. Once these steps were taken, data were then collected regarding the velocities of the three separate toy cars as they traveled through the corner. Fig. 4 shows the three different cars for which velocities were recorded.

After information regarding the velocity was collected for the three cars with a banking angle of 24° , the banking was adjusted so that the angle was 14° , similar to that of Auto Club Speedway located in Fontana, California. The banking angle was changed by replacing the 24° wedges that were holding up the track with 14° wedges that were also made from craft sticks. Data were again



FIG. 4: The three toy cars used in this experiment. The toy Corvette is on the left, the toy #43 is in the center, and the toy Indy car is on the right.

collected regarding the velocity of the cars at the three different photogates along the track.

Once all of the data were collected regarding the velocities of the cars as they traveled around the banked curve, data were collected regarding the retarding forces on the car so the drag coefficient associated to each toy car could be calculated. To calculate the total retarding forces on the car, a straight track was created using the pre-made plastic pieces [9]. Again, the beginning of the track was placed on a stool of height 0.64 m to give the car an initial velocity. Three photogates were placed a distance of 0.50 m from one another along the straight track so that the velocity of the car could be recorded in multiple places. The straight track used to collect this data can be seen in Fig. 5. Flags with a width of 0.025 m were again used on the cars to trigger the photogates. Data were taken regarding the velocity of the car at each photogate so the acceleration associated with each car could be determined [9]. Once this was completed, the frontal area of each of the three toy cars was calculated using Logger Pro to first outline the figure of the cars, with their attached flags, and then compute the area [9].

IV. RESULTS & ANALYSIS

The first goal of this experiment was to determine whether the toy cars accurately model the behavior of race cars with respect to their movement around a banked curve. From the information provided previously about a NASCAR car traveling around turn one of Texas Motor Speedway, it is expected that the cars will obtain a larger maximum velocity when the banking angle is higher. If the toy car accurately models a race car, a larger banking angle implies a larger normal force which results in a higher velocity for the toy car. The aver-

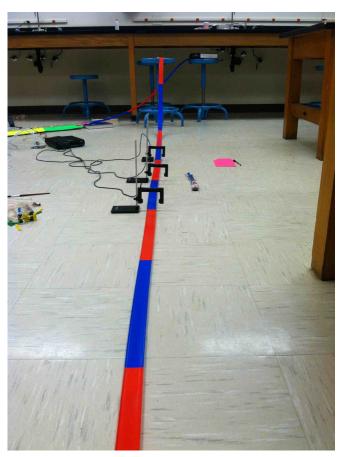


FIG. 5: The straight track that was used to calculate the aerodynamic retarding force on the cars.

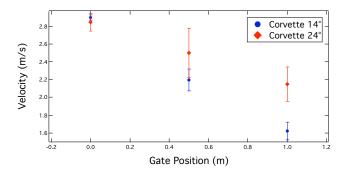


FIG. 6: The graph of the velocity of the toy Corvette car as it travels around a curve banked at 14° (blue) or 24° (red).

age velocity of the toy Corvette car is graphed versus the position of the photogates in Fig. 6. Note that the velocity of the car at the initial photogate is similar for both banking angles. This is expected as the car has not yet traveled around the curve by the time it has only reached the first photogate. The two values should be similar as the car is released from a similar position for each trial. The discrepancy may exist because of inconsistencies in the way the car is initially released or differences in the support and alignment of the track pieces that exist be-

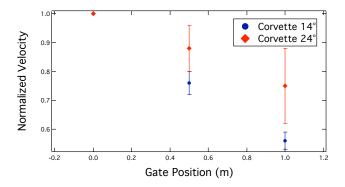


FIG. 7: The graph of the normalized velocity of the toy Corvette car as it travels around a curve banked at 14° (blue) or 24° (red). Even with the reduction in initial starting error, the toy Corvette car travels faster around the 24° banked curve compared to the 14° banked curve.

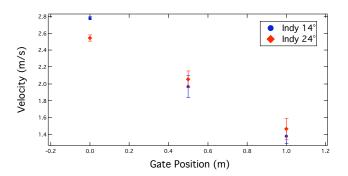


FIG. 8: The graph of the velocity of the toy Indy car as it travels around a curve banked at 14° (blue) or 24° (red).

tween the 24° and the 14° banking angle. The velocity of the car in the other two gates is higher for the 24° banking compared to the 14° banking angle. In order to reduce the error from the initial start of the car, the velocity was normalized, based on the velocity value of the first gate. The graph of the normalized velocity of the toy Corvette car can be seen in Fig. 7. From this graph, it can be seen that the velocity of the toy Corvette decreases more when traveling around the 14° banked corner than for the 24° banking.

Similar results are seen when the toy Indy car is analyzed. The graph of the velocity versus the photogate position for this particular toy car can be seen in Fig. 8. Notice that the velocity of the car is similar as it travels through the first photogate and, then, the velocities at the other two photogates are higher for the 24° banking angle compared to the 14° banking angle. The graph of the normalized velocity, seen in Fig. 9 for the toy Indy car, also shows that the car travels faster through the 24° bankd curve compared to the 14° banked curve even with the reduction in the starting error.

For the #43, different results are observed. From Fig. 10, it can be noted that the velocity of the car is always larger when it is traveling around the 14° banked

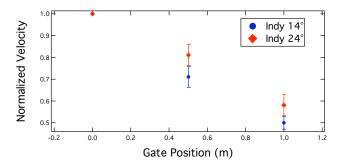


FIG. 9: The graph of the normalized velocity of the toy Indy car as it travels around a curve banked at 14° (blue) or 24° (red). Even with the reduction in initial starting error, the toy Indy car travels faster around the 24° banked curve compared to the 14° banked curve.

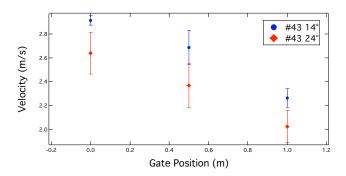


FIG. 10: The graph of the velocity of the toy #43 car as it travels around a curve banked at 14° (blue) or 24° (red). Notice, in this graph, that the velocity of the toy car is always higher for the 14° banking compared to the 24° banking.

curve compared to the 24° banked curve. Also, in the graph of the normalized velocity of the toy #43, it can be seen that the cars travel at very similar speeds around the corners. This means, even with a reduction in the error from the initial start, the toy #43 car, on average, travels slightly faster around the 14° banked curve. From Fig. 10, it can be seen that the velocities are different even at the first photogate, this indicates that something different occurs to this car that does not affect the other two toy cars that were used. One possible explanation for this discrepancy is that the track pieces are better aligned during the 14° trial for this particular car, compared to the runs that were taken with the 24° banking.

One way to determine if the physical track setup is the source of the discrepancy in the velocity seen in Fig. 10, is to create a track out of plastic so it would be consistent, there would be no gaps in the pieces of track and no transition from one material to another. A second potential cause to explain this discrepancy is the setup of the initial plastic track that led the car onto the banked curve. If this initial track was angled so that the impulse force between the tubing lining the outside of the track and the car was larger for the 24° banking compared to the 14° then the car would have lost more of its kinetic

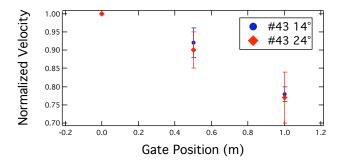


FIG. 11: The graph of the normalized velocity of the toy #43 car as it travels around a curve banked at 14° (blue) or 24° (red). Notice, in this graph, that it is hard to determine which curve the car travels faster around. With the reduction in the initial starting error, the velocity of the toy #43 car is similar for both the 14° banking and the 24° banking.

energy before even reaching the first photogate. If this misalignment was corrected before the the 14° banking trials, then the car would have had more speed during this trial as its kinetic energy would have been higher, thus explaining the results seen in Fig. 10. To determine whether the impulse force between the tubing and the toy car led to the discrepancy seen in the results, the entire experiment could again be run with a track that was made of a uniform material. This would prevent problems that might exist regarding the angle at which the toy car enters the banked curve. In addition, other features of the actual car could contribute to this discrepancy. If data were taken with more cars, it may become clear that a given physical aspect of the car affects the velocity the car can reach as it travels around a corner.

The second goal of this experiment was to determine if there exists a correlation between the drag coefficients of the toy cars compared to the actual cars they model. To calculate the total retarding force (drag force), data were collected regarding the velocity of the toy cars as they passed through three photogates on a straight track. The average acceleration of each car was then computed by dividing the change in velocity of the toy car from the first photogate to the third photogate by the time it took the car to travel from the first photogate to the third photogate. For instance, the toy Indy car was traveling at 2.31 m/s at the first photogate and 1.67 m/s at the third photogate and it took the toy car 0.53 seconds to travel this distance. The change in the velocity is 0.64 m/s and, dividing this by the elapsed time, produces an acceleration of 1.21 m/s² for this one trial. The average acceleration was calculated in this manner for each trial run by each car and then averaged to determine the acceleration for the car. Table I shows the different acceleration and mass values for each toy car. The frontal area of the car was calculated using Logger Pro. In Fig. 12, the calculation of the area for the toy Indy car can be seen. Notice that the car was first outlined and then the software filled in the outlined area to determine the frontal area of the toy car. By using Eq. 1, the total

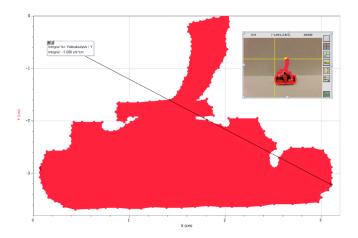


FIG. 12: A figure of the calculation of the frontal area of the toy Indy car in Logger Pro. The car was first outlined with points and a measurement was given to ensure proper scaling. Logger Pro then filled in the outline area to provide a value for the frontal area of the toy Indy car. The frontal area for the toy Indy car is $0.00051\,\mathrm{m}^2$.

TABLE I: Total Retarding Forces

Car	Mass (kg)	Average Acceleration(m/s ^s)
Corvette	0.0398	0.9 ± 0.1
#43	0.0248	1.0 ± 0.2
Indy	0.0299	1.2 ± 0.1

retarding force can be calculated.

Because the total retarding force was calculated for each car, the individual drag coefficient associated with each toy car was computed using Eq. 4. Table II displays the total retarding force, the frontal area, the average velocity through the corner, and the drag coefficient associated to each of the three toy cars used in this experiment. The toy Indy car has the highest drag coefficient of the three toy cars which matches the expected results as an Indy car has a higher drag coefficient compared to a NASCAR car and a street car. The toy Corvette car has a higher drag coefficient than the toy NASCAR car which does not match the predicted results. The fact that there are discrepancies in the masses of each car compared to the actual cars they represent could explain the unexpected data results. The toy NASCAR car should weigh more than the toy Corvette. This discrepancy may account for the difference in results from the expected values. In order to determine if the mass of the car is the cause of this discrepancy, the trials would need to be performed again with toy cars that have the same mass ratios as the cars they model.

TABLE II: Drag Coefficient Calculation Values

Car	Retarding	Frontal	Average	Drag
	Force (N)	Area	Velocity	Coefficient
		(m^2)	(m/s)	
Corvette	0.035 ± 0.004	0.00054	2.8 ± 0.2	14 ± 2
#43	0.025 ± 0.005	0.00053	2.7 ± 0.2	11 ± 3
Indy	0.036 ± 0.003	0.00051	2.0 ± 0.3	29 ± 3

V. CONCLUSIONS

The goal of the experiment was to determine if toy cars accurately model the behavior of race cars with regard to their velocity as they travel through a banked corner and their associated drag coefficient. As the toy Indy car serves as a model for a real Indy car, it can be seen that this car does reflect many of the qualities of an Indy car. For instance, the velocity of the car is seen to be higher when the banking angle of the track is higher, which models speed changes seen in a real Indy car. In addition, the toy Indy car had the highest drag coefficient of the three cars tested. This is also reflective of an actual Indy car which, when compared to a NASCAR car or a non-race car, has the highest drag coefficient. The toy Corvette accurately reflects the velocity of a car in

a banked corner as it obtains a higher maximum speed when the banking angle is greater. With regard to the drag coefficient, the value associated to the toy Corvette is lower than the Indy car, as expected, but higher than the drag coefficient for the NASCAR car which is not expected. Finally, the toy NASCAR car, #43, did not display results that correlate to an actual NASCAR car. The velocity of the car decreases when the banking angle of the curve increases. In addition, the toy NASCAR car has a lower drag coefficient than the toy Corvette which is not expected. Thus, some of the toy cars accurately model the results expected of their associated race cars while other cars do not display the anticipated results. The most likely reason for the discrepancy in the anticipated and received results stems from the design of the track and the inconsistencies between the toy cars and the actual cars they model.

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