

Observing sound waves in a one dimensional acoustic metamaterial

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A one-dimensional, acoustic metamaterial was created by drilling an array of holes into a PVC pipe. It was observed that frequencies below 500 Hz dissipate while at 500 Hz and above, the acoustic waves continue to propagate. Therefore the material exhibits a negative bulk modulus in the frequency range of 0 to 500 Hz. The intensity of sound was measured at specific distances over a range of 100 Hz to 1.5 KHz. A two inch speaker was connected to a Hewlett Packard 15 MHz waveform generator to set specific frequencies. Root mean squared amplitudes were measured with a MPLI microphone. Testing was done in various media to observe change in sound propagation.

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INTRODUCTION

In 1967, Victor Veselago proposed a theoretical material that could produce effects that are difficult or impossible to naturally produce. His work revolved around obtaining negative values for permittivity and permeability, which govern a material's interaction with electromagnetic materials, that could create materials with a negative value for the index of refraction. The theory laid dormant for decades because no materials available could physically realize Veselago's analysis.

When asked to determine the cause of a radiation-absorbing carbon, John Pendry found the property came from the physical shape of carbon fibers, not molecular or chemical structure. He realized rather than conventionally altering a material through its chemistry, the behavior of a material can be altered by changing a material's internal structure on the fine scale of microwave infrared wavelengths. This allowed for the creation of materials that produce the effects Veselago had theorized. An artificially constructed material engineered to have properties not found in nature is known as a metamaterial. Phenomena previously thought of as impossible can now be achieved with such materials, with an untold range of applications.

Since metamaterials gain their properties from structure rather than chemical composition, replacing molecules with artificial "atoms" of periodic structures on a scale much smaller than relevant wavelengths, produces small inhomogeneities that create effective macroscopic behavior. The term subwavelength describes an object having one or more dimensions smaller than the length of the wave with which the object interacts. If a metamaterial is to behave as a homogeneous material accurately described by an effective refractive index, its features must be much smaller than the wavelength.

Our understanding of electromagnetic phenomena has led to innovations in electromagnetic metamaterials that can manipulate microwaves and low frequency visible light in the electromagnetic spectrum. Much work has been invested in increasing the usable range of the spec-

trum, while new research exploring metamaterials that manipulate other types of waves has begun only recently. Commonalties between permittivity and permeability with bulk modulus and mass density allow for the creation of acoustic metamaterials that can manipulate sound waves.

The applications of metamaterials are paramount, since they bend laws of nature and exhibit new, extraordinary behavior. Super lenses with spatial resolution below a wavelength and "invisibility" cloaks operating over certain band ranges are just a few new technologies that can be achieved with the metamaterials. [1, 2, 4, 6]

THEORY

Waves are categorized by motion. Consider a stretched elastic string. Shaking one end up and down causes a disturbance to travel along the string; more generally, motion is transmitted from one particle to the next, and the disturbance propagates along a row of particles. Such a disturbance is called a transverse wave pulse. Transverse waves occur when a disturbance creates oscillations perpendicular to the propagation, or direction of energy transfer.

Alternatively, we can generate a disturbance by pushing the first particle towards the second. A perfect example would be an array of springs. When one spring is compressed it tries to restore itself compresses the next spring in the array. This kind of disturbance, where a compressional disturbance propagates along a row of particles, is called a longitudinal wave pulse. The oscillations of longitudinal waves are parallel to the direction of propagation.

Waves are often compared to the sinusoid, a mathematical function that describes a smooth repetitive oscillation. Its basic form as a function of time t is

$$y[x, t] = A \sin[kx - \omega t + \phi]. \quad (1)$$

The high points of the wave are crests, while the low points are troughs. Amplitude A the magnitude of

change in the oscillating variable with each oscillation. The distance from one crest to another is the wavelength λ , and represents the repeat difference of the wave pattern. Shifting by one wavelength produces the original wave pattern.

Over time the wave travels in a direction with speed v and the entire wave pattern shifts. The wave pattern (but not the medium) performs a rigid translation motion. The time required for the wave pattern to travel one wavelength is called the period T . Since the wave travels one wavelength over the course of one period, the ratio of wavelength to period must equal the speed of the wave, or

$$\frac{\lambda}{T} = v = \frac{\omega}{k}. \quad (2)$$

After one period, wave crests have traveled to the previous position of the adjacent crest, repeating the same configuration it had originally. The inverse of the period is the frequency ω ; the number of crests arriving at some point in the wave path per second. The unit of frequency is cycles per second, or Hertz (Hz).

A sound wave in air consists of alternating zones of high and low density (or equivalently, zones of high and low pressure). Although these density disturbances travel, the air as a whole does not; air molecules merely oscillate back and forth along the direction of propagation. This compressional disturbance propagates along a row of particles, making sound a longitudinal wave. The restoring force that drives these oscillations is air pressure. Wherever the density of air is higher than normal, the pressure is also higher and pushes molecules apart. Wherever the density is low, the pressure is also lower than normal, and the higher pressure of adjacent regions pushes these molecules together.

The speed of a wave depends on the properties of the medium through which it propagates. If a system disturbed and moves out of equilibrium, restoring forces return the system to equilibrium. For an elastic wave in any medium, the speed depends on the relative volume change of a media as a response to a pressure change and the amount of mass, or density ρ , of the medium. In general, the speed of sound v is given by the Newton-Laplace equation

$$v = \sqrt{\frac{K}{\rho}}, \quad (3)$$

where ρ is the density, while K represents the bulk modulus, which measures the resistance of an elastic body to deformation by an applied force.

As a sound wave spreads out from its source the area of the wave front grows larger, so the energy per unit area grows smaller. The intensity I of the sound wave is inversely proportional to the square of the distance d ;

represented by the ratio

$$\frac{I_2}{I_1} = \frac{d_1^2}{d_2^2}. \quad (4)$$

In an environment without any reflecting surfaces the sound wave travels in all directions. Such a rapidly growing area causes I to decay monotonically.

Four equations collectively known as Maxwell's equations embody every aspect of electromagnetic theory, but combined they clarify the nature of light and lead to the electromagnetic wave equation. Permittivity ϵ measures the resistance encountered when forming an electric field in a medium. In other words, permittivity relates a material's ability to "permit" an electric field. Permeability μ measures a material's ability to support the formation of a magnetic field within itself. The more conductive a material is to a magnetic field, the higher its permeability, the more the magnetic field can "permeate."

TABLE I: Maxwell's equations: \mathbf{E} , \mathbf{B} , \mathbf{J} are the electric field intensity, the magnetic field density, and the current density.

$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$	Gauss's law for electric fields
$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$	Gauss's law for magnetic fields
$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$	Faraday's law
$\vec{\nabla} \times \vec{\mathbf{B}} = \mu_0(\vec{\mathbf{J}} + \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t})$	Ampère-Maxwell Law

The index of refraction n of a substance describes how waves propagate through that medium. For electromagnetic materials, n is mathematically visualized by $n = \sqrt{\epsilon\mu}$, where the positive sign by convention. Both ϵ and μ are complex, so real values are derived to define n based on behavior. Negative refraction occurs when $\epsilon, \mu < 0$ and a negative sign is used to mimic the fact that the wavevector are reversed. Since the refractive index informs on the propagation direction of the waves, the sign of n must match physical situation. Acoustic metamaterials control sound by manipulating the bulk modulus K , the mass density ρ , which are analogies of ϵ and μ . An equation for acoustic refractive index is $n^2 = p/K$. Negative values of p and K are an anomalous response derived from resonant frequencies of a fabricated medium. Negative p or K means that at certain frequencies the medium expands when experiencing compression ($-K$), and accelerates left when being pushed right ($-p$). The acoustic wave equation (Fig. 5) governs propagation and describes the evolution of p as a function of position and time.

$$\nabla^2 p - \frac{1}{v^2} \frac{\partial^2 p}{\partial t^2} = 0 \quad (5)$$

Directing waves through chambers mimics electrical circuits and these acoustic circuits manipulate acoustic properties. A Helmholtz resonator, a filter element with a single degree of freedom [8], performs as a series of inductance and capacitance. With a greater pressure gradient along the open neck than the cavity, the cavity

displays capacitive properties while the neck performs as an acoustic inductor.

METHODOLOGY AND RESULTS



FIG. 1: An view of the acoustic metamaterial with microphone, speaker, and waveform generator.

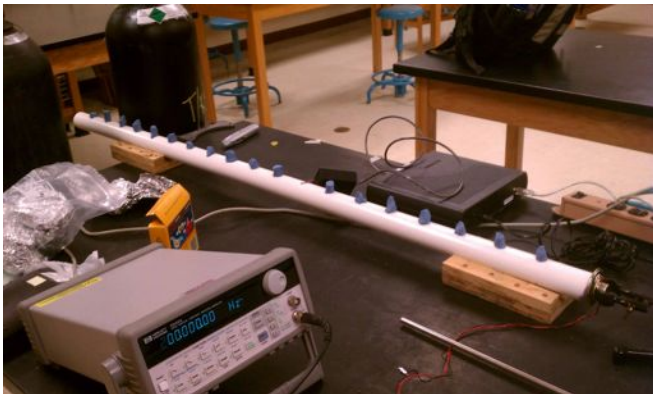


FIG. 2: A view of the metamaterial when its holes are plugged with crafting putty.

PVC piping with an inner diameter of 31.75 millimeters was used to create an acoustical metamaterial 1405 mm long as seen in Fig. 1; holes 10 mm in diameter were drilled at regular intervals of 70 mm, with a total of 19 holes. The array of holes in the pipe perform similarly to an array of Helmholtz resonators. A 2 inch speaker was connected to a Hewlett Packard 15 MHz waveform generator and placed at one end. The MPLI microphone from Vernier Software was opened and a plastic ring was removed, and microphone slipped out of its casing into the holes. The wires of the microphone were made of a fragile wire mesh that often broke, requiring soldering.

From 100 Hz to 1.5 KHz, sound intensity in Volts was measured at each hole. The amplitude of the generator was set to 500 mVpp (peak to peak voltage). As quiet a space as possible was used. The standard deviations

in data studio were considered to be the V_{rms} values of the waves. Data was also taken with the holes plugged with crayola crafting putty (Fig. 2) to see the effects of a hollow cylinder on sound waves. To get an idea of typical propagation without interference, the intensity was also recorded each hole position in free space.

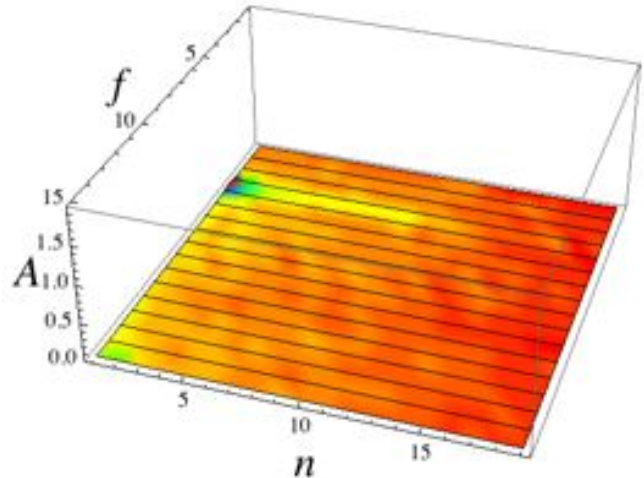


FIG. 3: Sound propagating without interference.

A three dimensional plot oriented with position number along the y axis, frequency along the x axis, and Amplitude along the z axis is shown in Fig. 3. The intensity of the sound waves deteriorates exponentially as the surface area of the wavefront increase so fast, as predicted from Eq. 4. At 400 Hz there is some increased intensity at the first position number, possible because of natural resonance with microphone. The microphone had difficulty recording frequencies in the low range of 100 to 300 Hz. The results for the closed pipe are shown in Fig. 4. Operating like a primitive megaphone, the sound waves are limited in the directions they can travel in and are Basically moving in one dimension. Without the wavefront spreading out, the sound waves experience less loss in intensity over a greater distance and continue to propagate. Around the 400 Hz range there was evidence of resonance, now with much greater exaggeration. Figure 5 shows the results of testing sound through the metamaterial. Below a certain critical frequency ω_c , sound quickly dissipated. A much higher reading was taken at the first 400 Hz position, but the intensity leveled off shortly after. At 500 Hz and above there is continued propagation. The material has a negative K under ω_c and the medium expands. This expansion changes the intensity ratio and amplitude decays.

400 and 600 Hz were graphed in Fig. 6. For free sound waves, the intensity decays. Waves in pipe show evidence of standing waves, where waves remain in a constant position. In a stationary medium like the pipe, standing waves are a result of interference between two waves trav-

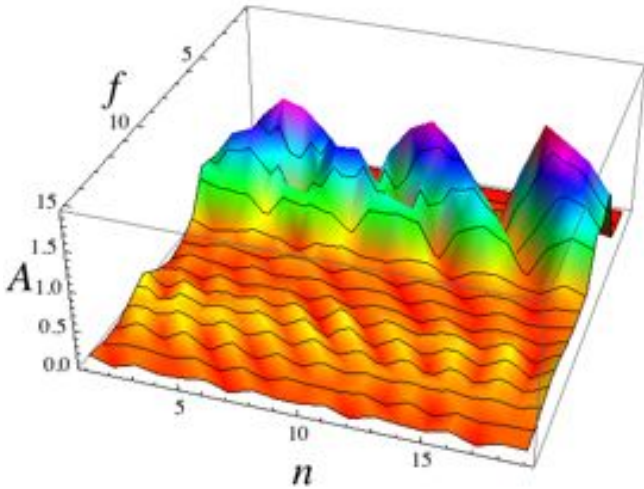


FIG. 4: Sound propagating through a tube.

eling in opposite directions and in a resonator, standing waves occur during resonance. This suggests the pipe has resonance at 400 Hz, and would explain of the incredible magnitudes recorded. The metamaterial at 400 Hz best shows the sound wave dissipating exponentially and it can be seen that at 600 Hz the waves propagate with minimal intensity loss.

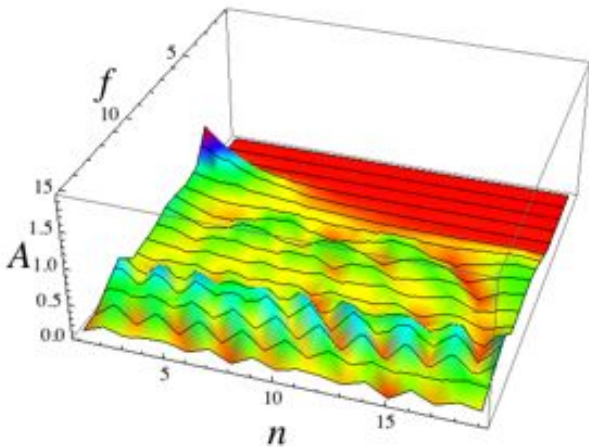


FIG. 5: Sound propagating through the metamaterial. Below ω_c the sound decays; above it continues to propagate.

Further work could be done to create metamaterials with negative refractive index. Obtaining a material with more dimensions or an adjustable index of refraction could have many applications.

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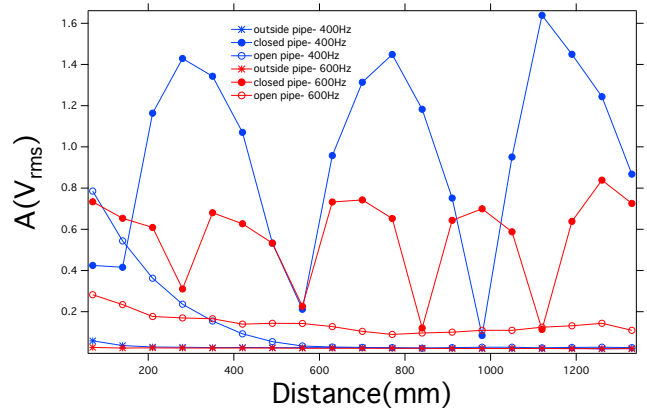


FIG. 6: All the results recorded at 400 and 600 Hz.

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