Shake, Shake, Shake Tacoma

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The Tacoma Narrows Bridge collapse was a tragedy that physicists and engineers are still learning from today. Positive feedback created by vortices that formed on the bridge deck is the main theory explaining why the bridge collapsed. Engineers knew that this was a possibility and developed two ideas to try and prevent this from happening. The first was to drill holes in the railings of the bridge to allow more airflow and the second was to add more supports to the bridge deck. Neither was put into action as the bridge collapsed five days after the discoveries were made. The purpose of this experiment was to observe how these two fixes would affect the natural frequency and thus the damping coefficient of a suspension bridge. A model bridge was assembled along with three different bridge decks, one control, one with added supports, and then the control with holes drilled into it. Graphs of the three different decks amplitude vs angular frequency were created and from these graphs a Q, or quality value was discovered. The deck with the greatest Q value was found to be the deck with the holes drilled into it, followed by the control deck, and finally the deck with the lowest Q value was the deck with added supports. This Q value is inversely proportional to the damping coefficient so the deck with the holes drilled into it had the lowest damping coefficient, while the deck with the added supports had the greatest damping coefficient.

I. INTRODUCTION

Construction of the Tacoma Narrows Bridge was completed on July 1, 1940. At the time of completion the Tacoma Narrows was the third longest suspension bridge in the world. The bridge then collapsed on November 7, 1940. The only fatality in the collapse of the bridge was a dog named Tubby. There were no human fatalities. Construction workers noticed that the bridge would sway vertically in the wind; therefore the construction workers nicknamed the bridge Galloping Gertie. Safety measures were put into place, but they obviously did not prevent the collapse of the bridge. The main span of the suspension bridge was measured to be 2,800 feet long and only about 39 feet across. That is a ratio of 1:72 width to length. This ratio along with the cost-cutting design would ultimately be the downfall of this bridge. [1]

II. THEORY

The actual reason the Tacoma Narrows Bridge collapsed was due to positive feedback that was generated by vortices created on the deck of the bridge. Positive feedback occurs when an object is knocked off center and the object then tries to right itself, but by doing so actually overcompensates. An example is when a car is driving behind a truck and the wind from the truck actually pushes the car to one side or the other. The driver then tries to compensate by turning the other direction. The vortices were created on the bridge by just the right amount of wind blowing over the top of the rails on the sides of the bridge. The vortices then moved across the deck of the bridge causing the bridge to begin to oscillate. The bridge oscillated slowly at first, but due to positive feedback the oscillations of the bridge began to grow greater and greater until eventually the bridge reached its breaking point. Each time the bridge oscillated the bridge gained velocity and displaced even more than the oscillation before it.[2]

Engineers came up with two different plans to try and prevent the bridge from oscillating. The first idea they came up with involved drilling holes into the girders of the bridge. These holes would allow more airflow and would essentially eliminate the chances that the vortices could form on the bridge. The second idea was to essentially add more supports to the bridge so that it would not be able to oscillate. The plan was to attach cables from the bridge deck to concrete blocks that would be placed in the ground. The engineers decided to go with the second option because it would not be permanent and could easily be undone. The problem was the bridge collapsed five days after the decision. For this experiment the two ideas the engineers came up with would be put to the test to see how these ideas would affect the natural frequency and damping coefficient of a suspension bridge. The natural frequency is the frequency that creates the greatest amplitude or displacement of an object. The damping coefficient is a number that states how difficult it is to oscillate an object. The lower the damping coefficient, the easier it is to oscillate an object and the greater the displacement of the object.[1]

The damping coefficient could be determined by creating a trace on the graph of the amplitude of oscillation vs the resonant frequency. The trace is represented by the equation

$$D = \frac{\beta D_{max}}{\sqrt{((\omega - \omega_{max})^2 + \beta^2)}} \tag{1}$$

where D is the amplitude of oscillation, β is a constant, and ω is the angular frequency. Equation 1 was derived from the equation

$$ma + bv + kx = Fcos(\omega t), \tag{2}$$

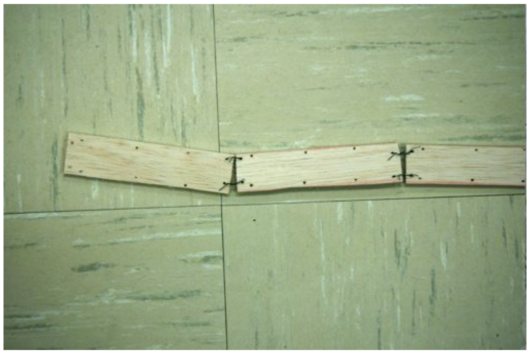

FIG. 1: Balsa wood deck with the eight holes drilled in it. Four for attaching the deck to the copper wire and four for attaching the section to other sections.

FIG. 2: Bridge set up with the birch wood deck. The plastic pipe supports and copper wire can be seen.

where m is the mass, a is the acceleration, b is a constant related to the damping, v is the velocity, k is a constant, x is the position, F is the force, ω is the angular frequency, and t is the time. This equation is then divided by m to derive the equation

$$a + 2\beta v + \omega_{max}^2 x = Acos(\omega t), \tag{3}$$

where a is acceleration, β is b/2m, ω is the angular frequency, x is the position, A is the amplitude, and t is time. Equation 3 was then solved using the complementary and particular solutions. The final result is equation 1. [3]

III. PROCEDURE

First, the dimensions of the original Tacoma Narrows Bridge were found to 2,800 feet long and about 39 feet across. This yields a ratio of about 1:72 width to length. From this ratio the dimensions of the scale model were calculated to be nine feet long and one and a half inches wide. The main deck was built using one quarter inch thick balsa wood cut to an inch and a half wide and then cut into six inch segments. After the segments of the deck were cut eight holes were drilled into each of the segments. The holes were drilled as seen in Figure 1.

Four of these holes were used to connect the segments together and the other four were used to hang the deck from the main suspension cable. The segments of the deck were connected using wire. The wire was looped through the different segments of the bridge to hold them together. The segments were latched together like this instead of just using one big piece because a real suspension bridge is made in sections and attached together, not just in one big piece. Once the deck was assembled it was raised up on a platform and thread was used to attach the deck to the suspension cables. The suspension cables were copper wire and were attached to two plastic pipes on each side of the bridge. The plastic pipes were secured into a base piece of wood by dowel rods running through the pipes into the base piece of wood. The plastic pipes were also tied to supports by fishing line for extra support as shown in Figure 2. The same process was repeated for another deck made of a heavier birch wood. After this process was completed with the heavier birch wood deck, the original balsa wood deck, which had already been tested, was taken and holes were drilled into it. The holes were drilled about two inches apart and were about a quarter of an inch in diameter. The same testing process was then done to the balsa wood deck now with holes drilled into it. The first balsa wood deck served as the original Tacoma Narrows deck, while the deck made of birch wood represented the supports that were supposed to be added to the original bridge, and the holes being drilled represented the holes that were supposed to be drilled into the original Tacoma Narrows deck.

Next, after the decks were set up the natural frequency needed to be measured. The natural frequency of the bridge was measured by placing a mirror onto the bridge deck and by attaching a PASCO driver to the side of the deck as shown in Figure 3. The PASCO driver

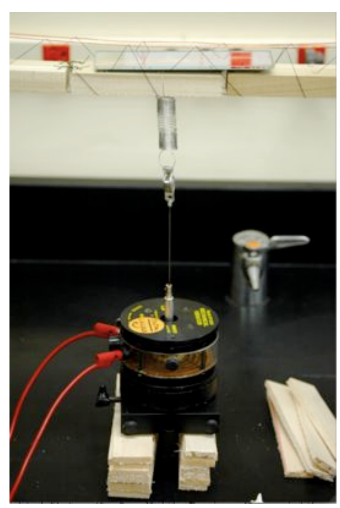

FIG. 3: PASCO driver attached to the side of the bridge deck.

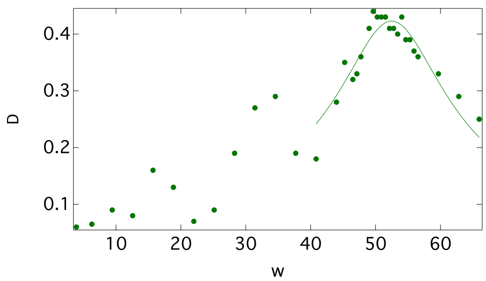

FIG. 4: Amplitude of Oscillation (meters) vs ω (Hz). Q value is ω_0/FWHM where ω_0 is the resonance frequency. This is for the bridge deck made of balsa wood.

was hooked up to a frequency generator and a laser was pointed so that it would reflect off the mirror and onto the ceiling. A meter stick was set up on the ceiling where the laser was shining. As the PASCO driver was turned it caused the bridge to oscillate side to side in turn forcing the mirror on the deck of the bridge to do the same. This caused the laser spot to actually move back and forth on the ceiling, so the amplitude could be recorded by using the meter stick on the ceiling. The frequency was then changed on the frequency generator and the amplitude was recorded. This process was repeated multiple times for many different frequencies until maximum amplitude could be discovered. This whole process was then repeated for the two other decks of the bridge.

IV. RESULTS AND ANALYSIS

The frequency and amplitude were recorded for each of the three decks of the bridges. After that a graph of the amplitude versus ω was created. ω is just the frequency multiplied by 2π. The amplitude of these graphs is known as the amplitude of the motion and is represented by D. These graphs were then used to to solve for Q, otherwise known as the quality value. The equation for Q is $Q = \omega_0/\Delta\omega$ where Q is the quality value, ω_0 is the resonant frequency associated with the greatest amplitude, and $\Delta\omega$ is the width of the resonance. The frequency response

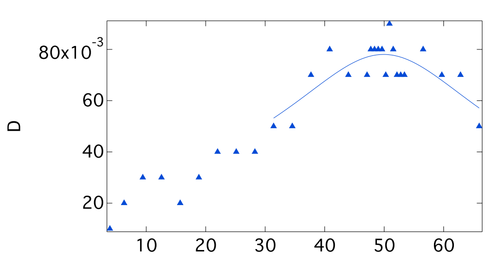

FIG. 5: Amplitude of Oscillation (meters) vs ω (Hz). Q value is ω_0/FWHM where ω_0 is the resonance frequency. This is for the bridge deck made of birch wood.

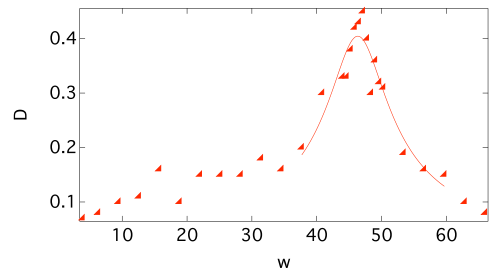

FIG. 6: Amplitude of Oscillation (meters) vs ω (Hz). Q value is ω_0/FWHM where ω_0 is the resonance frequency. This is for the bridge deck made of balsa wood with holes drilled into it.

of the bridge made of just balsa wood can be seen in Figure 4. The Q value was determined by a trace of the graph using equation 1.

Two more graphs were created by graphing the driving force for the birch and for the balsa with holes drilled into vs angular frequency. Figure 5 shows the frequency response for the bridge deck made out of birch wood and Figure 6 shows the resonance for the bridge deck made of balsa wood with holes drilled into it. Finally, Figure 7 shows all three of the graphs on one set of axis. It is easy to see that the width of the resonance peak is greatest for the birch wood and smallest for the balsa wood with holes drilled into it. This means that the birch wood has a low Q value followed by the balsa wood, and then finally the balsa wood with holes drilled into it. Since Q is inversely proportional to the damping coefficient this means that the bridge deck made of birch wood has the highest damping coefficient followed by the balsa wood, and finally the balsa wood with holes drilled into it has the lowest damping coefficient.

V. CONCLUSION

The Tacoma Narrows Bridge collapsed due to positive feedback created by vortices that formed on the bridge deck. Engineers came up with two ideas to try to attempt this from happening. One was to drill holes into the gird-

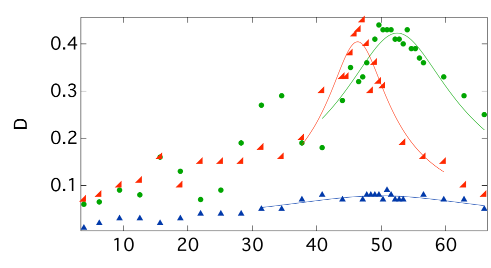

FIG. 7: Amplitude of Oscillation (meters) vs ω (Hz). Q value is ω_0/FWHM where ω_0 is the resonance frequency. The colors and shapes are the same as before with the other graphs.

ers of the bridge and the other was to add supports to the bridge deck. According to experimental results drilling holes into the girders may have allowed for more airflow, but it would have also lowered the damping coefficient of the bridge itself meaning that it would have been easier to cause the bridge to oscillate. On the other hand adding supports to the bridge deck would have greatly increased the damping coefficient allowing the bridge deck to withstand a lot more force. Although both of these methods would have proven to be effective adding the supports would have been the better option. Neither one of these methods had the chance to prove itself as the bridge collapsed five days after discovering these two methods.

There were many problems encountered in the construction of the model bridge. The original design and materials had to be changed a couple times. Future work would include putting up the rails on the bridge and attempting to recreated the vortices on the bridge deck. Dry ice or liquid nitrogen could be used to create a fog so that the air from fans could actually be visible and allow for the researcher to witness the vortices forming. After this was accomplished, holes could be drilled into the railings to see if this method would have an effective way of preventing the collapse.

VI. ACKNOWLEDGEMENTS

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[1] Wikipedia, *Tacoma Narrows Bridge 1940*, January (2011). ($http://en.wikipedia.org/wiki/Tacoma_Narrows_Bridge_(1940)$)

[2] D. Green and W. Unruh, *Am. J. Phys.,*, Vol. 74, No. 8, (2006).

[3] M. Thornton *Classical Dynamics of Particles and Systems.*, Harcourt Brace and Company. Philadelphia, 1995.