Verifying Ampere's Law and Determining the Permeability of Free Space μ_0

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The goal of this laboratory work was to experimentally verify both Ampere's equation for parallel current carrying wires and the defined constant known as the permeability of free space μ_0. This report uses a modern Ampere current balance composed of both gravitational and torsion balancing mechanisms. I verified that both the current squared and the inverse of the separation distance between wires are directly proportional to the magnetic repulsive forces acting on to parallel current carrying wires. I subsequently solved for the permeability of free space with the compilation of all of my experimental data. I found the constant to be $\mu_0 = (1.32 \pm 0.01) \times 10^{-6}$ mKg/s^{2}A^2; a 10% deviation from the accepted value of $\mu_0 = 4\pi \times 10^{-7}$ mkg/s^{2}A^2.

I. INTRODUCTION

An Ampere balance is a precise instrument invented by Lord Kelvin designed to measure current [1]. While an Ampere balance would still work as a functional ammeter, they are now typically used in instructional settings to elegantly show how the induced magnetic fields due to parallel current carrying wires exert forces on one another. Ampere's expression says specifically that the magnetic force between parallel current carrying wires is proportional to the current in both wires and the length of the wires. The force is also inversely proportional to the spacing between the wires. In this lab report I will use a modern Ampere current balance to confirm Ampere's equation. After verifying this equation, I will have experimentally determined all of the physical variables necessary to solve for the permeability of free space μ_0 relating current, separation distance and force.

II. THEORY

A. Force Between Current Carrying Wires

According to Ampere's results, the force between two parallel wires is proportional to the product of the current in each wire divided by the distance between the two wires. Explicitly, the force is given by

$$F = \frac{\mu_0 I_A I_B L}{2\pi d},\qquad(1)$$

where I_A is the current in wire A, I_B is the current in wire B, L is the length of the wire, and d is the distance between the center points of both wires as shown in Fig. 1. The current balance used for this report ran an equal and opposite current through wires A and B so $\mid I_A \mid = \mid I_B \mid$. Therefore Eq. 1 can be simplified to

$$F = \frac{\mu_0 I^2 L}{2\pi d}.\qquad(2)$$
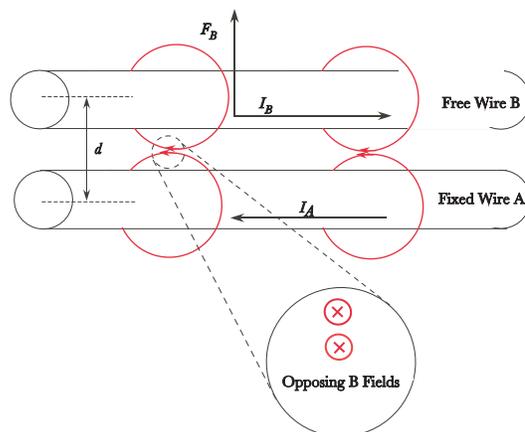
FIG. 1: Repulsive force on a free wire due to the induced magnetic field from the opposing current of the fixed wire.

B. Gravitational Balance

To determine the magnitude of the repulsive force F_B acting on a free wire due to a fixed wire, the upward magnetic force is balanced with a downward gravitational force. A schematic of the current balance used to accomplish this is shown in Fig. 2. The apparatus is a highly sensitive gravitational balance composed primarily of a rectangular frame and a counterbalance beam. The far right edge of the rectangular frame acts as wire B. The metal rod below the rectangle frame, fixed to the apparatus, acts as wire A. When current is running through the balance frame the repulsive force between parallel wires causes the rectangular frame to tilt upwards on the right side. By placing small amounts of mass m on a mass pan that sits on top of the free wire, the weight needed to bring the frame back to equilibrium can be determined. Because mass m is directly proportional to the force of gravity $F_g = mg$,

$$m = \frac{\mu_0 L I^2}{2\pi dg}\qquad(3)$$

where g is the acceleration due to gravity.
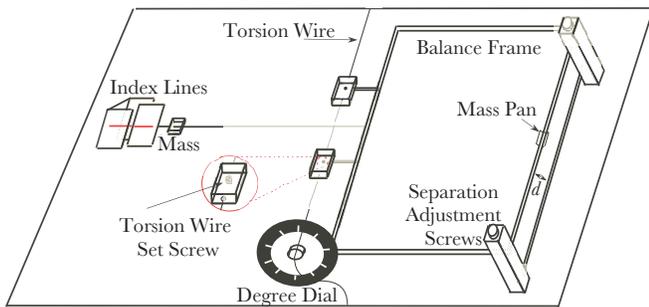
FIG. 2: A schematic of the Ampere balance. The advanced apparatus uses the restorative tendency of a torsion wire as a fine tuning mechanism.

C. Torsion Balance

An alternative to using very small mass increments for fine tuning the Ampere balance is using small angular deviations of the torsion wire illustrated in Fig. 2. A twisted wire exerts a torque in the direction resisting the twist. The more it is twisted, the greater restoring force it exerts. The torque on the torsion wire of the current balance can be modified by rotating the degree dial shown in the apparatus schematic. The angular change in the torsion wire, given by the number of degrees dialed, corresponds to the torque on the balance.

D. Permeability of Free Space μ_0

Permeability is the relative increase or decrease in the magnetic field inside a material compared with the magnetic field in which the material is located [2]. The permeability of free space is also known as the magnetic constant of a vacuum. It is defined to be $4\pi \times 10^{-7}$ H m^{-1} [3]. Eq. 3 can be rearranged to solve for the permeability of free space as

$$\mu_0 = \frac{2\pi dmg}{I^2 L}. \quad (4)$$

Experimental values for the current I, the distance between the parallel wires d, and the mass m will be used to verify this constant.

III. PROCEDURE

A. Wiring the Apparatus

Before making measurements with the current balance the apparatus was wired and calibrated. To wire the apparatus, a variable DC power supply with the capacity of supplying a range of 0-12A of current was attached to the current balance via an ammeter with long leads. The long leads were important to ensure that the power supply was held far enough away from the apparatus that

it did not affect the magnetic fields of the wires. Additional magnetic effects due to the Earth's magnetic field were minimized by orienting the balance such that the parallel wires were directed along the N-S axis of a compass [4]. Lastly, 9V of power was supplied to pots with gallium. The melted gallium provided both a frictionless pivot and a circuit connectivity for the balance. Detailed instructions on wiring the current balance made by PASCO Scientific can be found in the Instruction Manual and Experiment Guide for the PASCO scientific Model EM-8623 [4].

B. Calibrating the Torsion Wire

Using the gravitational balance to determine the repulsive magnetic force on the rectangular frame did not require a torsion wire. The torsion wire is an additional feature of PASCO Scientific's current balance that aids with both the speed and accuracy of the data collection.

The torsion wire was held at the mandatory tension of 18 N so that the calibrations listed in the instruction manual were correct. To get the tension in the torsion wire to the desired level the first step was loosening the set screws shown in Fig. 2 with the allen wrench that came with the apparatus. The next step was to attach the excess torsion wire to a spring scale. Once the scale was attached, the wire clamp on the back of the degree dial was loosened so the tension of the torsion wire could be changed. After the spring scale was pulled taut to 18 N, the wire clamp and set screws were retightened. Care was taken so that the set screws were not over-tightened. If the screws were over-tightened the torsion wire could have snapped, however if the screws were under-tightened the angular rotations in the torsion wire would not affect the balance frame at all. I found it easiest to perform the calibration process with two people. When the torsion wire was properly calibrated, it was confirmed that a three degree rotation of the degree dial shown in Fig. 2 equated to a one milligram change in force.

C. Zeroing the Balance

Before zeroing the balance the gallium pots were heated so that the rectangular conducting frame was supported by only the torsion wire, and the pivots of the frame were immersed in liquid gallium. Zeroing the balance required moving from coarse adjustments to fine adjustments. The counterbalance mass labeled in Fig. 2 was adjusted first. When the balance was level, the separation adjustment screws were dialed counter-clockwise to move the fixed parallel wire up to meet the rectangular frame. This position is considered the zero position for all measurements. The zero position was marked by lining up the movable index with the reference line on the counterbalance beam. The rear rear thumb screw is located opposite the degree dial along the torsion wire. It has a

TABLE I: Sample calculation with a variable current and fixed separation distance of 4.2 mm.

Current	Mass	Degrees Dialed	Total Mass Equiv.
7.00 A	55 mg	39°	68 mg

small range of rotation, but was helpful for fine tuning the alignment of the movable index with the reference line on the counterbalance beam. With no separation between the wires, the center to center distance between the wires is 3.2 mm [4]. One whole clockwise turn of both separation adjustment screws moved the fixed wire one millimeter away from the free wire.

D. Data Collection Processes

Two different kinds of data collecting processes were used. The first data collecting process varied the current I through the range of 11 A while holding a fixed separation distance d. The second data collection process varied the separation distance d through a range of 5 mm while holding the current fixed. Two different trials with a range of eleven data points were taken for each process. Each constant current trial was held at a different set current. Each constant separation distance trial was held at a different set distance.

I combined the gravitational balance and the torsion balance to obtain the most accurate possible measurements of the counteractive force required to balance the magnetic force acting on the free wire. For each data point I estimated how much mass was needed to balance the scale from previous data points. After waiting for the scale to settle I fine tuned my estimated amount with the degree dial. Table I shows a sample calculation of the total mass that it took to balance out the repulsive force of the parallel current carrying wires.

IV. RESULTS AND ANALYSIS

A. Verification of Ampere's Equation

One of the goals of this lab report was to verify Eq. 2, Ampere's force equation. This was accomplished by looking at the effect both a variable current and a variable separation distance had on the mass needed to bring the balance to its equilibrium position.

For Ampere's force equation to be confirmed, the square of the current I should be directly related to the mass m as shown in Eq. 3 . In order to fit the variable current data to a linear function, I graphed the logarithm of the mass m versus the logarithm of the current I. I expected to see a slope of approximately 2. As shown in Fig. 3(a) my results show a slope of 2.00 ± 0.02 for the trial with a separation distance of 4.2 mm, and a slope
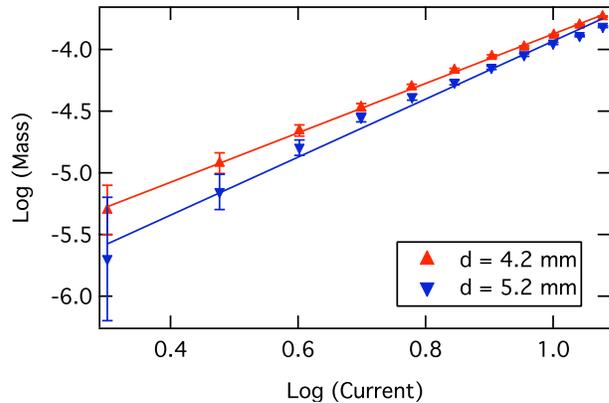
(a)
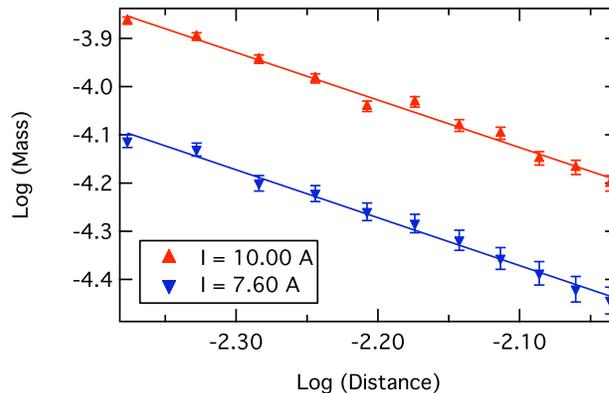
(b)

FIG. 3: (a) A log-log plot of the mass m vs. the current I for separation distances of 4.2 mm and 5.2 mm. The linear fit for the separation distance of 4.2 mm has a slope of 2.00 ± 0.02. The linear fit for the separation distance of 5.2 mm has a slope of 2.35 ± 0.09. (b) A log-log plot of the mass m vs. the separation distance d for currents of 10.00 A and 7.60 A. The linear fit for the current of 10.00 A has a slope of -0.99 $\pm$ 0.04. The linear fit for the current of 7.60 A has a slope of -0.99 $\pm$ 0.03.

of 2.35 ± 0.09 for the trial with a separation distance of 5.2 mm.

Additionally, for Ampere's force equation to be confirmed, the inverse of the separation distance $1/d$ should be directly related to the mass m as shown in Eq. 3. To fit the constant separation distance data to a linear function, I graphed the log of the mass m versus the log of the separation distance d. I expected to see a slope of approximately -1. As shown in Fig. 3(b) my results show a slope of -0.99 $\pm$ 0.04 for the trial with a current of 10.00 A, and a slope of -0.99 $\pm$ 0.03 for the trial with a current of 7.60 A.

I achieved a high level of precision while collecting data from the current balance. I am confident that I found the necessary mass to balance each level of current to the nearest one milligram. The error bars in Fig. 3(a) and Fig. 3(b) reflect this uncertainty.

The excellent correlation between the theoretical re-
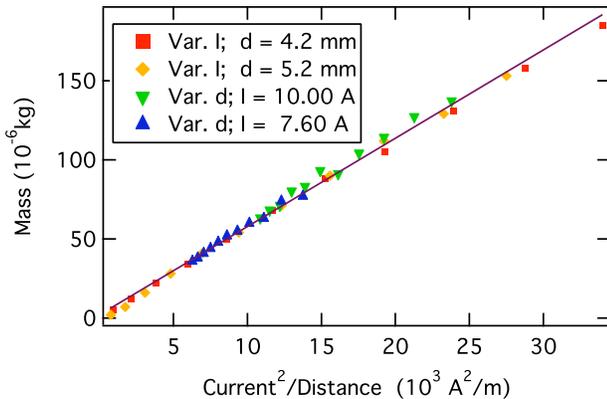
FIG. 4: A plot of the mass m vs. the square of the current over the separation distance I^2/d for all data taken. The linear fit has a slope of $(5.58 \pm 0.06) \times 10^{-9}$ kgm/A^2. The error bars denoting the uncertainty in measurements are smaller than the point markers.

lationships and experimental relationships of current I, separation distance d, and mass m are sufficient confirmation of Ampere's equation.

B. Determination of the Permeability of Free Space μ_0

Because Ampere's equation Eq. 2 was confirmed, the permeability of free space μ_0 can be determined using Eq. 4. In order to account for all of the various data sets that were performed experimentally, a ratio of current squared over separation distance I^2/d was deter-

mined for each data point. The total counterbalance mass needed for each data point was graphed against its corresponding I^2/d value. The compilation of all of these points was graphed in Fig. 4. All of the data points were fit to a linear function. The resulting slope of the fitting function was $(5.58 \pm 0.06) \times 10^{-9}$ kgm/A^2. Eq. 3 shows that this slope must be multiplied by a constant factor of $L/2\pi g$ to solve for μ_0. I found μ_0 to be $(1.32 \pm 0.01) \times 10^{-6}$ mKg/s^{2}A^2; a 10% difference from the accepted value of $\mu_0 = 1.26 \times 10^{-6}$ mKg/s^{2}A^2.

V. CONCLUSION

The two goals of this report were to verify Ampere's equation for parallel current carrying wires and to experimentally determine a value for the permeability of free space μ_0. Ampere's equation was successfully verified by proving the direct relationship between the magnetic force on a current carrying wire and both the current squared and the inverse of the separation distance of the wires. The permeability of free space was found to be $\mu_0 = (1.32 \pm 0.01) \times 10^{-6}$ mKg/s^{2}A^2; a 10% difference from the accepted value of $\mu_0 = 1.26 \times 10^{-6}$ mKg/s^{2}A^2.

The results obtained by using the gravitational balance and the torsion balance of the current balance were exceptional. Though the determined value of μ_0 was 10% off from the accepted value, the experimental slopes of the log(mass) vs. log(current) and log(mass) vs. log(separation distance) were less than 1 % away from the theoretical slopes in 3/4 of the trials. In the last trial the slope error was only 15%.

[1] "Ampere Balance." Wikipedia. 20 Feb. 2009.
[2] "Magnetic Permeability." Britannica. 19 Feb. 2009 http://www.britannica.com.
[3] "Unit of Electric Current (Ampere)." Historical Context of the SI. National Institution of Standards and Technology. 19 Feb. 2009 http://physics.nist.gov/cuu/Units/ampere.html.
[4] Instruction Manual and Experiment Guide for the PASCO Scientific Model EM-8623: Current Balance. Publication. Roseville, CA: PASCO Scientific, 1990.