Coulomb Balance

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This experiment used a Coulomb balance to determine how the Coulomb force is dependent on the charge of two objects and the distance between these charges. In the first part of the experiment, the charges on the conductive spheres were kept constant, while the distance was varied. The angle at which the suspended sphere returned to its original position was recorded ten times for eight different distances. The logarithm of this angle was graphed with respect to the logarithm of the distance. The resulting fit had a slope of -2.053±0.003, an error of 3% from the expected -2. The second part of the experiment kept the distance and one sphere's applied voltage constant, while varying the voltage for the other sphere. The same angle was recorded for four different voltages. The logarithm of this angle was then plotted against the logarithm of the voltage. The linear fit produced a slope of 1.023±0.006, an error of 2% from the expected slope of 1. These results indicate that Coulomb's Law is accurate.

I. INTRODUCTION

In the 1780's, Charles Coulomb determined how electric forces were dependent upon the charges causing the forces and the distance between them. He was able to determine Coulomb's Law using a torsion balance, similar to the balance that was used in this experiment. The torsion balance involved two charged spheres, one of which was suspended. As one sphere was brought closer to the suspended sphere, the charges caused the suspended sphere to turn away from the other charge. Coulomb's Law states that the force, later known as the Coulomb Force, is proportional to the product of the two charges and inversely proportional to the square of the distance between the two charges [1]. In this lab, the dependence of the Coulomb force on charges and the distance between them was determined, then compared to this law.

II. THEORY

A. Force VS Distance

The first part of this lab looked at the relationship between the Coulomb force and the distance between two charged spheres. We start with Coulomb's Law

$$F = k\frac{q_1 q_2}{R^2},\qquad(1)$$

where F is the Coulomb force, k is a constant of proportionality, q_1 is the charge on one sphere, q_2 is the charge on the other sphere, and R is the distance between the two. In this portion, k, q_1, and q_2 are constants. Rearranging this equation

and taking the logarithm of both sides results in

$$\log(F) = -2\log(R) + \log(kq_1 q_2).\qquad(2)$$

In this lab, the Coulomb force is proportional to the angle at which one sphere swings away from the other. Therefore, Eq. 2 is rewritten as

$$\log(\theta) \sim -2\log(R) + \log(kq_1 q_2),\qquad(3)$$

where θ is this angle of interest.

For separation distances small relative to the size of the sphere, a correction factor must be incorporated, because the charges can no longer be treated as point charges. This factor B is given by

$$B = 1 - 4\left(\frac{a^3}{R^3}\right),\qquad(4)$$

where a is the radius of the sphere [2]. θ is divided by this factor, which changes Eq. 3 to

$$\log\left(\frac{\theta}{B}\right) = -2\log(R) + \log(kq_1 q_2),\qquad(5)$$

This produces a linear relationship between the force and the distance, with a slope of -2.

B. Force VS Charge

The second part of this lab looked at the relationship between the Coulomb force and the charge between two charged spheres. Similar to the first part of the lab, we start with Eq. 1: Coulomb's Law. Taking the logarithm of both sides results in

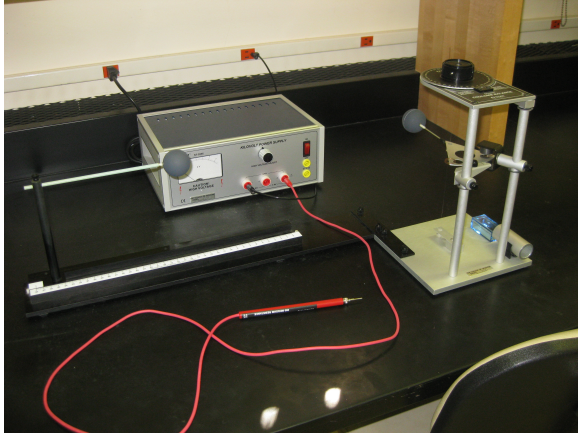$$\log(F) = \log\left(k\frac{q_1 q_2}{R^2}\right).\qquad(6)$$

FIG. 1: The system apparatus. One terminal of the kilovolt power supply is attached to the wall outlet. Another is attached to a charging probe, which was used to charge both spheres.

For this part of the experiment, k, q_1, and R are constants. So Eq. 6 is rewritten as

$$\log(F) = \log(q_2) + \log\left(k\frac{q_1}{R^2}\right). \qquad (7)$$

The Coulomb force is proportional to angle θ, and q_2 is proportional to the voltage the sphere is charged to. Incorporating this into Eq. 7 results in

$$\log(\theta) \sim \log(V) + \log\left(k\frac{q_1}{R^2}\right) \qquad (8)$$

where V is the voltage. Theoretically, the force and charge are linearly related, with a slope of 1 on a log-log plot.

III. PROCEDURE

A. Apparatus

A PASCO Model ES-9070 Coulomb balance was used as the apparatus. Fig. 1 is a photograph of this apparatus. The kilovolt power supply runs from 0 kV to 6.6 kV. A cable grounds the power supply to the wall outlet. The charging probe is connected to a positive terminal of the power supply. Two conductive spheres with radii of 1.9 cm were used as the charged objects in this experiment. One of the spheres was part of the torsion balance. Torsion wire was connected to the torsion knob, passed through the counterweight vane, and connected to the torsion wire retainer. The torsion knob was attached to a degree scale, which indicated how many degrees the knob was turned. The

torsion wire retainer was used to zero the torsion balance by lining up the index mark on the counterweight vane with that of the index arm. The suspended sphere extended from the vane. Two copper rings were slid onto the vane, one on each side. These rings, along with a magnetic damping arm, prevented the vane from swinging out of control. The other sphere was part of the slide assembly, where the sphere could move anywhere between 0 cm and 38 cm. This was the distance between the center of one sphere to the center of the other when neither balls are charged, and the degree scale was set at $0°$.

B. Force VS Distance

During the first part of the experiment, the two spheres were fully discharged through skin contact. The sliding sphere was set to the furthest distance possible (38 cm), and the suspended sphere was set at $0°$. The kilovolt power supply was set to 5.5 kV. The spheres were charged to this value using the charging probe. The sliding sphere was then set to the 20 cm position. This caused the suspended sphere to rotate away from the charge. The torsion knob was adjusted to realign the index marks, and the angle was recorded. This entire process was repeated nine more times in order to obtain a consistent set of data. This procedure was then repeated for multiple distances.

The data obtained was sent to Igor Pro for analysis. In this program, the logarithm of each angle and distance was determined, then plotted with $\log(\theta)$ on the ordinate and $\log(R)$ on the abscissa. However, the data were taken at small distances relative to the size of the spheres. Therefore, the correction factor was determined with Eq. 4, and another graph with $\log(\theta/B)$ was appended to the first plot. A best fit line was fitted for both the original angles and the corrected angles. Both slopes were then compared to -2, from Eq. 5.

C. Force VS Charge

For the second part of the experiment, the two spheres were once again fully discharged in the same method. The spheres were placed at the same initial positions of 38 cm and $0°$, and the kilovolt power supply was set to 6 kV. Both spheres were charged using the charging probe. The sliding sphere was placed at a distance of 8 cm. The suspended sphere was adjusted using the torsion knob, so the index marks would be realigned. This was once again repeated nine more
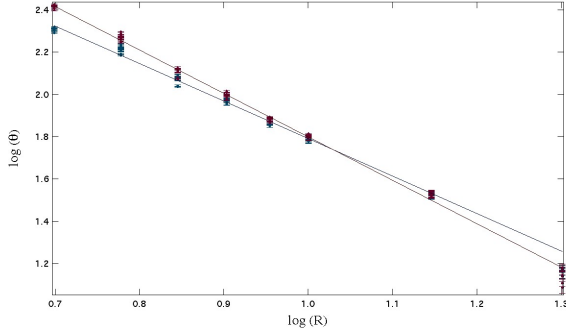
FIG. 2: The graphs of both $\log(\theta)$ and $\log(\theta/B)$ with respect to $\log(R)$. The $\log(\theta)$ plot uses the solid blue circles. The $\log(\theta/B)$ is graphed with solid red triangles.

FIG. 3: The relationship between $\log(\theta)$ and $\log(V)$ at a fixed distance.

times. This was repeated with the power supply set to 5 kV, 4 kV, and 3 kV. Note that the suspended sphere was kept at 6 kV; only the sliding sphere's charge was changed with the power supply.

The data was analyzed in Igor Pro. Here, the logarithm of each voltage and angle was calculated, then graphed with $\log(\theta)$ on the ordinate and $\log(V)$ on the abscissa. Distances did not play a part in this part of the experiment, so a correction factor was not used on the angle. A best fit line was fitted to the points, and the slope of the linear curve was compared to 1, from Eq. 8.

IV. RESULTS AND ANALYSIS

A. Force VS Distance

In this portion of the experiment, a linear relationship was expected between the logarithm of the force and the logarithm of the distance between the two spheres, as described in Eq. 5. Fig. 2 is the graph with both the pure data and the corrected factor. The pure data is shown with solid blue circles. The corrected factor is shown with solid red triangles. Both of the best fit curves are linear functions. The theoretical slope value is -2. The plot of $\log(\theta)$ produced a slope of -1.773±0.003, which is off by 11%. The best fit line from the $\log(\theta/B)$ data produced a slope of -2.053±0.003, an error of 3%. These values confirm that the Coulomb Force is proportional to $1/R^2$, as indicated by Coulomb's Law. In addition, the correction factor results in a more accurate slope value. This confirms that this factor is necessary when the distance between the charges is small relative to their sizes.
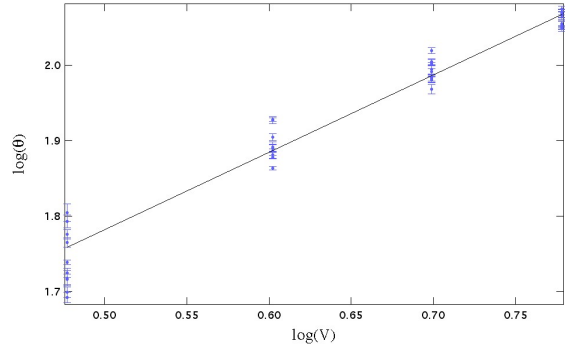
B. Force VS Charge

In the second part of the experiment, a linear relationship was expected between the logarithm of the force and the logarithm of the charge on the sphere, as shown in Eq. 8. Fig. 3 shows the relation between the Coulomb force and the changing charge on the sphere. The data produced a linear function as the best fit curve. Theoretically, the slope of this line is 1. This plot produced a slope of 1.023±0.006, which is an error of 2%. This indicates that the Coulomb Force is proportional to the charge of the sphere, as stated in Coulomb's Law.

V. CONCLUSION

In this lab, a Coulomb balance was used to determine the dependence of the Coulomb force on the distance between two charges and the value of the charge. Two conductive spheres were charged from a kilovolt power supply using a charging probe. The first part of the experiment kept the voltage constant while varying the distance between the two spheres. The Coulomb Law states that the force is proportional to $1/R^2$. The logarithm of the deflected angle was plotted against the logarithm of the distance, where the deflected angle is proportional to the Coulomb force. The best fit curve was a line with a slope of -2.053±0.003, an error of 3%.

The second part of the experiment kept the distance between the spheres constant and the charge of the suspended sphere constant while varying that of the sliding sphere. Theoretically, the Coulomb force is proportional to the charge. The logarithm of the deflected angle was graphed with respect to the logarithm of the voltage, which

is proportional to the charge on the sphere. The best fit curve resulted in a linear function with a slope of 1.023±0.006, which is an error of 2% from the expected slope of 1. From these results, the relation

$$F \propto \frac{q}{R^2} \tag{9}$$

is produced, which is indicated with Coulomb's Law. Therefore, this law is proven to be accurate.

VI. ACKNOWLEDGMENTS

[1] D. Giancoli, *Physics -Principles with Applications-* (Prentice Hall, 1998), 5th ed.

[2] *Coulomb Balance*, PASCO Scientific (1989).