Holographic Interferometry of a Stressed Pop Can

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A series of holograms were taken of a pop can to observe the deformations that arise due to external stress. The research looked at stress due to a rubber band, carbonation, as well as the gravitational weight from a small stack of papers. The deformations were observed using double-exposure holographic interferometry in which two successive holograms were taken of the object with and without stress. Stress was successfully observed and measured in two of the samples stressed by a rubber band. The deformation of these samples were both calculated to be $\Delta y = 7.0 \pm 0.2$ μm along the surface of the can.

I. INTRODUCTION

In 1947, Dennis Gabor created a method of recording both the amplitude and phase of a scattered light wave by interfering the scattered wave with a reference wave. A three-dimensional image, or hologram, could then be reconstructed from the recorded information about the scattered light. The process of creating holograms, or holography, did not receive considerable attention until the invention of the laser. The laser provided a coherent and monochromatic light source that was able to produce clean, well-developed holograms [1]. Since then, holography has been used for microscopy, acoustical imaging, radar imaging, and interferometry. Holographic interferometry is, by far, one of the most practical uses of holography and its a technique that allows scientists to observe the microscopic distortions of an object due to stress. This technique has been applied to testing airplane parts and automobile tires, to studying the flight of a high-speed projectile [2]. In this experiment, I used holographic interferometry to measure the stress inflicted on a pop can by a rubber band, carbonation, and a stack of paper.

II. THEORY

A. Holography

A hologram is produced by interfering a reference beam of light with a beam that has been scattered off an object, as seen in Fig. 1. The xy plane is defined to be the plane of the photographic film where the two waves interefere. Light travels as a oscillating electromagnetic wave so the electric field of the reference wave can be described by

$$E_r(x,y) = E_{or}\cos[\omega t + \phi_r(x,y)] \tag{1}$$

where E_{or} is the constant amplitude of the field, ω is the angular frequency of the wave, and $\phi_r(x,y)$ is the phase of the wave. The phase depends on position because the wave does not arrive at normal incidence [3]. The wave that scatters off the object has an amplitude that depends on position because the light rays scatter at different angles and locations. Therefore, the electric field of the scattered wave can be written as

$$E_s(x,y) = E_{os}(x,y)\cos[\omega t + \phi_s(x,y)] \tag{2}$$

where E_{os} is the amplitude of the field, ω is the angular frequency, and $\phi_s(x,y)$ is the phase, also a function of position due to the irregular wavefront.

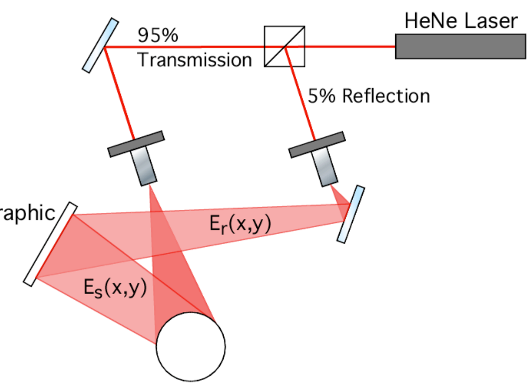

FIG. 1: The experimental setup used to create the holograms.

An irradiance distribution is created when the two beams interfere at the photographic plate. This distribution is found by

$$I(x,y) = \langle (E_r + E_s)^2 \rangle = \langle E_r^2 + E_s^2 + 2E_rE_s \rangle, \tag{3}$$

or

$$I(x,y) = \langle E_r^2 \rangle + \langle E_s^2 \rangle + \langle 2E_rE_s \rangle. \tag{4}$$

Time averaging the first two terms results in

$$\langle E_r^2 \rangle = \frac{1}{2}E_{or}^2 \tag{5}$$

and

$$\langle E_s^2 \rangle = \frac{1}{2}E_{os}^2. \tag{6}$$

The third term is the interference term of the irradiance distribution and depends on the phase difference between the two waves. If the phase of the two waves differ by an odd multiple of π, the term will be be negative resulting in destructive interference. If they differ by an even multiple of π, the term will be positive resulting in constructive interference. This can be written as

$$\langle 2E_r E_s \rangle = E_{or} E_{os} \cos(\phi_r - \phi_s). \tag{7}$$

Therefore, the photographic film records an irradiance distribution of

$$I(x, y) = \frac{1}{2}E_{or}^2 + \frac{1}{2}E_{os}^2 + E_{or}E_{os}\cos(\phi_r - \phi_s). \tag{8}$$

With the interference of the two beams recorded onto a photographic film, a hologram is produced. The information can be extracted from the hologram by illuminating the film with the original reference beam,

$$E_r(x, y) = E_{or}\cos[\omega t + \phi_r(x, y)]. \tag{9}$$

The electric field of the transmitted wave is proportional to the product of the irradiance distribution recorded in the hologram and the electric field of the reference wave, or

$$E_t(x, y) = I(x, y)E_r(x, y). \tag{10}$$

We can multiply Eq. 10 out and use the law of cosines to determine the transmitted electric field to be

$$\begin{aligned} E_t(x, y) \;=\; & \frac{1}{2}(E_{or}^2 + E_{os}^2)E_r(x, y) \\ & + \frac{1}{2}E_{or}^2 E_{os}\cos(\omega t + 2\phi_r - \phi_s) \\ & + \frac{1}{2}E_{or}^2 E_s(x, y), \end{aligned}$$

where the first term is the amplitude-modulated reference beam which was used to illuminate the hologram, the second term contains $-\phi_s < 0$ which carries information on the inverse of the object the light was scattered off, and the third term is an amplitude-modulated scattered beam [1]. This final term contains all the necessary information from the wave scattered off the object and therefore describes the virtual image of the object.

B. Holographic Interferometry

Holography can be used as a means of measuring small changes in distances or defects do to stress placed on an object. A technique known as double-exposure interferometry measures these defects by taking two successive holograms of the object. The first hologram observes the object under stress. This stress causes a point on the surface of the object to contort a distance of Δy. When the object is relieved of the stress, the small change in distance vanishes and the second hologram is taken. Therefore, two slightly different holograms are recorded onto the same photographic plate and the difference between their recorded images creates an interference pattern of light and dark fringes.

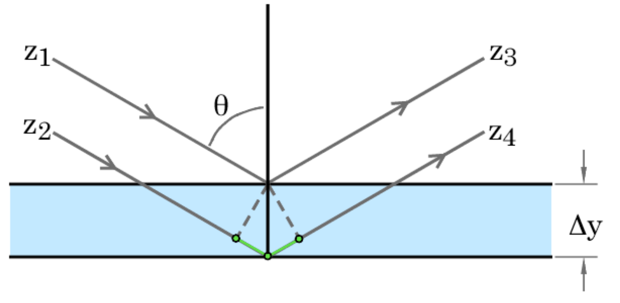

FIG. 2: A depiction of the thin film treatment of double-exposure interferometry.

We will assume the object is locally flat for a light ray hitting at any given point on the object's surface, allowing us to treat the situation as thin film interference, as shown in Fig. 2. In this scenario, we have a light ray, z_1, deflecting off the unstressed surface to z_3, followed by a light ray, z_2, deflecting off the stressed surface to z_4. Both rays deflect at an angle θ to the normal. The optical path difference between the stressed and unstressed path is given by

$$\Delta P = (z_1 + z_3) - (z_2 + z_4) \tag{11}$$

and is visually represented by the green line segments in Fig. 2. If the difference between the surfaces is Δy, we can geometrically determine the optical path difference to be [3]

$$\Delta P = 2\Delta y \cos\theta. \tag{12}$$

For the two deflected light rays to total destructive or constructive interference, the optical path difference must fulfill the requirement

$$\Delta P = \frac{n\lambda}{2}, \tag{13}$$

where λ is the wavelength of the light ray, n is the total number of fringes, odd n represent destructive interference, or dark fringes, and even n represent constructive interference, or bright fringes. By combining Eq. 12 and 13, we can derive an equation that measures the total affect of the applied stress, or

$$\Delta y = \frac{n\lambda}{4\cos\theta}. \tag{14}$$

III. EXPERIMENTAL PROCEDURE

A. Producing the Hologram

To create the holograms depicting the stress applied to a pop can, I used the set up shown in Fig. 1. A

$\lambda = 632.8$ nm HeNe laser beam was sent through a beam splitter, transmitting 95% of the laser light and reflecting 5%. The reflected light was sent through an 0.85 NA objective lens and directed onto the photographic plate. The transmitted light was passed through another 0.85 NA objective lens and directed onto the stressed pop can. The pop can was aligned to allow most of the transmitted light to reflect off its surface and onto the photographic plate without disturbing the path of the reference beam.

The hologram was made in the dark room and the photographic plates were stored in the dark so they would not be exposed. A beam block was used to control the exposure of the plate. Before exposing the plate to the laser light, the beam block was lifted from the table and kept in the path of the laser beam for about a minute and a half to quell the vibrations caused by lifting the beam block. The block was then removed from the laser path for four seconds to expose the plate. For the holographic interferometry experiments, a second exposure of four seconds was taken after removing the stress from the pop can. As soon as the photographic plate was exposed and the hologram was recorded, the plate was removed for developing procedures.

B. Developing the Photographic Plate

The chemical solutions used for developing were prepared before the holograms were created. The procedure required 50 mL + 50 mL of an A-B developer solution, a distilled water bath, 75 mL of a bleach solution, and a container of distilled water mixed with Photoflo. The contents of the A-B developer solution and bleach were provided by the manual from Photographers Formulary Inc. [4].

With the lights remaining off, the photographic plate was submerged into the A-B developer solution with the front of the plate facing up. The photographic plate turned a cloudy black color in the developer. The solution was shaken gently for two minutes and then the plate was moved to the distilled water bath. The plate lay in the bath for two minutes and then moved to the bleach solution where it remained for less than two minutes. The bleach solution caused the plate to lose its cloudy, black color and turn transparent again. After the plate was removed from the bleach, it was placed under running tap water for three minutes to rinse off the bleach. Before it was left to dry, it was placed into the Photoflo solution for one minute to allow the plate to dry without any water marks. The lights were turned on at this point and the plate was left to stand upright, against a wall, to dry on a paper towel.

C. Viewing the Hologram

After the holographic plate was left to dry for half an hour, it was placed into a ThorLab filter holder and illuminated with the HeNe laser. The transmitted light was projected onto a white background in order to see a real image created by the hologram. This image was photographed with a digital camera and transfered to the computer for further developing work.

IV. DATA

Seven holograms were developed, HT1-HT7, using the described setup to investigate different forms of stress applied to a pop can. A rubber band was placed around the center of the can to provide the stress in HT3 through HT5. The other two forms of stress were increased pressure within the pop can due to carbonation and a stress due to the gravitational force of a stack of papers lying on top of the pop can. The stress caused by these scenarios could not be successfully seen in the holograms. Images of the holograms for HT4 and HT5 are shown in Fig. 4 and 5 respectively. The images on the left of these two figures are the actual photographs of the holograms while the images on the right were manipulated in Photoshop to increase the contrast between the red and black colors. This made the total number of fringes easier to count during analysis.

The distance between the surface of the pop can and the surface of the photographic film was measured to be $a = 14.35 \pm 0.15$ cm. The distance measured between the surface of the pop can and the tip of the objective lens was $b = 14.6 \pm 0.2$ cm. Finally, the distance measured between the tip of the objective lens and the surface of the photographic film was $c = 16.9 \pm 0.2$ cm.

V. RESULTS

The angle the light rays scatter off the pop can with respect to the normal was found through the law of cosines, or

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right). \qquad (15)$$

The error in θ was propagated using the equation

$$\sigma_\theta = \sqrt{\left(\frac{\partial\theta}{\partial a}\sigma_a\right)^2 + \left(\frac{\partial\theta}{\partial b}\sigma_b\right)^2 + \left(\frac{\partial\theta}{\partial c}\sigma_c\right)^2}. \qquad (16)$$

Plugging in the values for a, b, and c gave us $\theta = 0.62 \pm 0.01$ radians. Only the most direct angle was investigated in the analysis of the holograms. The most direct angle the light rays could scatter off the object corresponded to the most intense imaging. Therefore, the hologram produced was made primarily from light rays scattering off an angle of approximately 0.62 radians. We also desired to calculate the uncertainty in Δy. This error in Δy was found through

$$\sigma_{\Delta y} = \sqrt{\left(\frac{\partial \Delta y}{\partial n}\sigma_n\right)^2 + \left(\frac{\partial \Delta y}{\partial \theta}\sigma_\theta\right)^2}. \qquad (17)$$

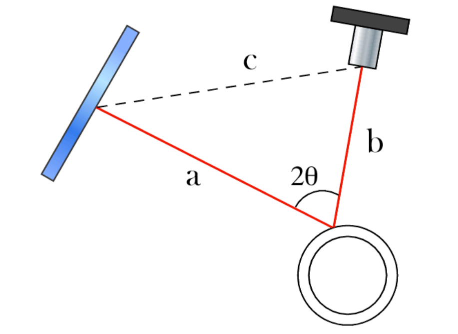

FIG. 3: The geometry used to determine the scattering angle θ.

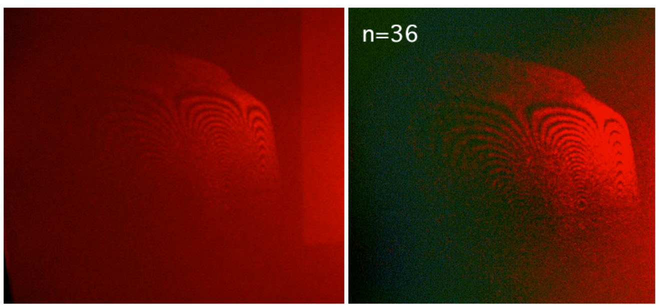

FIG. 4: The HT4 hologram depicting the pop can under the stress of a rubber band.

Out of the three trials performed with the rubber band as the agitator, HT4 and HT5 produced clear three-dimensional holograms with interference patterns corresponding to the deformations. Both holograms, shown in Fig. 4 and 5 respectively, had $n = 36 \pm 1$ total fringes, resulting in a deformation of $\Delta y = 7.0 \pm 0.2$ μm. An uncertainty of $\sigma_n = 1$ was chosen because it was fairly ambiguous where to begin counting the total number of fringes. It is interesting to note that the rubber band created different regions of strain. In HT4, the fringes for each region were distinct, separate, and concentric, however, in HT5, the regions began to overlap causing outer fringes to merge. This can be seen in between the two fringe patterns in Fig. 5

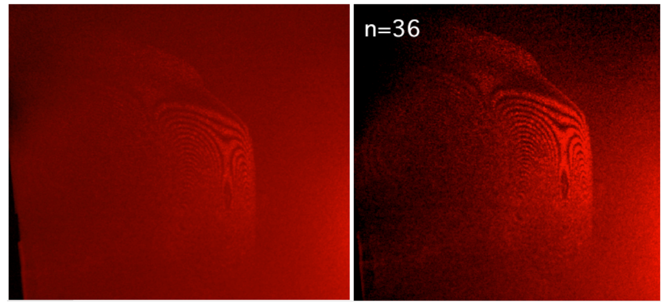

FIG. 5: The HT5 hologram depicting the pop can under the stress of a rubber band.

VI. CONCLUSION

The results of this experiment demonstrated that the stress inflicted on a pop can can be experimentally quantified but defects do remain in the setup. From HT4 and HT5, it was calculated that a rubber band can apply enough stress on an object to distort it by $\Delta y = 7.0 \pm 0.2$ μm. While the rubber band applied a stress along the total circumference of the pop can, the stress was unevenly distributed, resulting in the various regions of strain observed in Fig. 4 and 5. Therefore, the value of $\Delta y = 7.0 \pm 0.2$ μm can only be attributed to one point along the circumference of the pop can. The unequal deformation of the can was most likely due to the variable strength along the length of the rubber band, the defects of the pop can (eg. minor dents, imperfect spherical shape), as well as the imperfect placement of the rubber band around the can. If the rubber band was tilted slightly with respect to the ground, it would apply stress to the can in an elliptical form.

The stress caused by carbonation and the weight of a stack of papers should be further investigated. For this investigation, a better set up needs to be constructed that can image the entire can and not just the upper half. Kasper and Feller [2] describe a technique in which two beams are used to illuminate different portions of the object being imaged. This would improve the holographic image as well as give a broader view of the pop can, allowing more forms of stress to be investigated.

[1] E. Hecht, Optics, *2nd ed*, (Reading, Addison-Wesley Publishing Company, 1987), p. 593-610.

[2] J.E. Kasper & S.A. Feller, The Complete Book of Holograms, (Mineola, Dover Publications, Inc., 2001).

[3] *Holographic Interferometry: Physics 332 Lab Manual*, http://www.ruf.rice.edu/ dodds/Files332/holography.pdf (Rice University, February 2003), January 31^{st}, 2008.

[4] Tung Jeong, "JD-2 Holography Film Developer," *Photographers' Formulary*, Cat. No. 04-3010, 2000.