Drip Drop: A Brief Study of the Dripping Faucet

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(Dated: May 8, 2008)

An experiment was performed which measured the time between falling drops in the leaky faucet system, and a simulation of a the leaky faucet was created by approximating the motion of the drop as a growing mass on a spring. Periodicity and chaos were found for both the simulation and the experiment. However, not enough data were collected to get conclusive results.

I. INTRODUCTION

In 1977, before chaos theory was a recognized part of physics, a graduate student at the University of California-Santa Cruz abandoned his nearly finished doctoral thesis to start exploring chaos. Robert Stetson Shaw started his study using computer simulations and the Lorentz equations to find chaos in simple systems. When the need for a physical system arose, Shaw began to experiment with the dripping faucet. The familiarity and the simplicity of the system made it ideal. As Shaw discovered, the dripping faucet provided an excellent example of a system with dynamics which start completely predictably, and then rapidly become unpredictable and chaotic [1].

II. THEORY

A water faucet is one of the most ordinary devices, familiar to almost everyone. The basic setup of the system is simple. A reservoir of water is attached to a faucet. Water drips from the faucet with some flow-rate and the time of each drop is measured. The flow-rate of the system is determined by the opening of the faucet, and by the height of the water in the reservoir.

Traditionally, the data are shown as the time between drops T_n, where T_n for the n^{th} drop is the time between of drop n and the previous drop. The data in this form can be displayed as plots of T_n vs. T_{n+1}, which are a type of Poincaré section plot. Thus, the chaotic data can be shown and analysed as chaotic attractors, or mappings of the changing dynamics of the system in phase space [1].

As well as exhibiting simple periodic behavior, the leaky faucet demonstrates a period doubling regime. The system starts with all T_n the same, which is simple periodic motion called period-1 behavior. At a slightly higher flow-rate, the system will move to having two different values of T_n so that $T_n = T_{n+2}$. When this occurs, the period is said to have doubled becoming period-2. As the flow-rate increases, the system moves to period-4, characterized by four different values of T_n. The system continues through this period doubling regime until it becomes chaotic. This period doubling path to chaos can be used to find a constant, known as Feigenbaum's delta δ, which adds predictability to the chaos of the system.

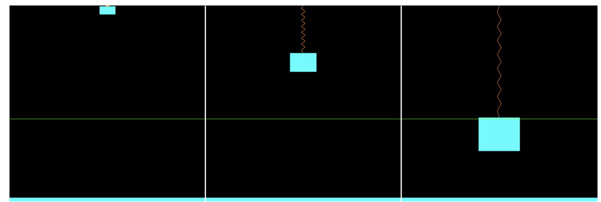


FIG. 1: The mass on a spring approximation of the drop as it begins to drop (left), continues (middle), and reaches the critical length just prior to dropping from the spring (right).

Part of the appeal of the dripping faucet as an example of chaos is that the mechanics of the dropping water can be simplified and approximated greatly.

If the shape and the internal motion of the drop are ignored, the drop can be treated as a growing mass on a spring, as shown in Fig. 1. Using this assumption, the system can be simulated as a damped harmonic oscillator with a mass varying as a function of time. Using Hooke's law, the acceleration a of the system is

$$a = -\left(\frac{bv}{m} + \frac{kx}{m}\right) + g \tag{1}$$

where x is the position of the spring from top of the spring, v is the velocity of the drop, m is the mass of the drop, k is the spring constant of the oscillator, b is the damping parameter, and g is the acceleration due to gravity [2]. Physically, the elastic constant k represents the surface tension of the fluid and the damping parameter b represents of the friction between the faucet and the fluid [3].

As the mass of the drop increases, the drop falls and oscillates according to the acceleration a given in Eq.1. When it reaches some critical distance x_c, a percentage of the mass falls off and the remaining mass rebounds and then continues to grow into the next drop. The oscillations of the spring cause the system to become chaotic. The drop falls when it reaches the critical length x_c. As shown in Fig. 2, if the drop is at the top of an oscillation, it will take slightly longer to reach x_c. Thus, the mass of the drop at the critical length M_c will be slightly larger.

The change in mass affects the amount of mass which rebounds to start the next drop[3]. Therefore, the oscillations of the spring change a significant number of the parameters for each drop, and the system becomes unpredictable and chaotic.

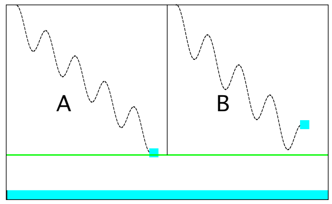


FIG. 2: The oscillation of the spring affect when the drops fall. The green line is the critical length x_c, the dashed lines represent the oscillation of the spring. Drop A is at the bottom of an oscillation as it reaches the critical length. Drop B is at the top of an oscillation at the same time. Therefore, drop A will reach the critical length and fall from the spring before drop B.

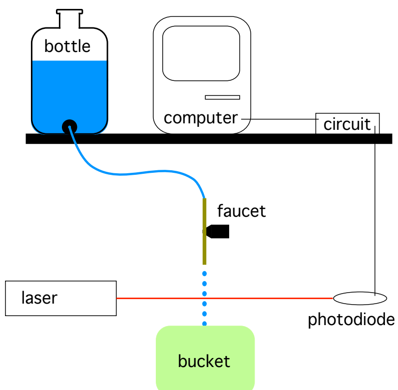


FIG. 3: A glass bottle was used as a reservoir for the water. This bottle was connected to the Nupro faucet, which could be finely adjusted to control the flow rate. The water dripped into a bucket set on the floor. To record the times of the drops, a laser was shone through the path of the drops, and its signal was recorded by a photodiode connected through a circuit to a computer.

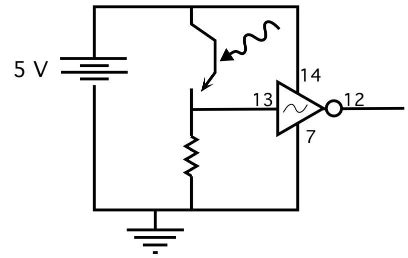


FIG. 4: To collect the data from the dripping water, the photodiode (squiggly arrow and notched arrow) was connected to the computer and an oscilloscope through this circuit. The power for the photodiode was provided by a 5 volt power source. The power supply also powered a Schmitt trigger(triangle with wave) with an inverter (small circle). The signal from the photodiode passed through the Schmitt trigger to the computer. The circuit served not only to connect the photodiode to the computer, but also to reduce the noise of the data by sending it though the Schmitt trigger.

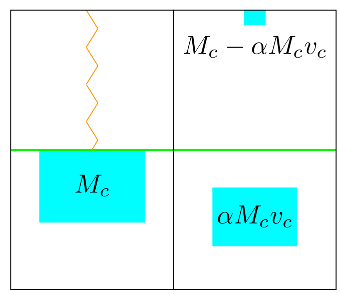


FIG. 5: The drop as it reaches the critical length and a moment later when it has been split into a falling drop and a rebounded mass.

III. PROCEDURE

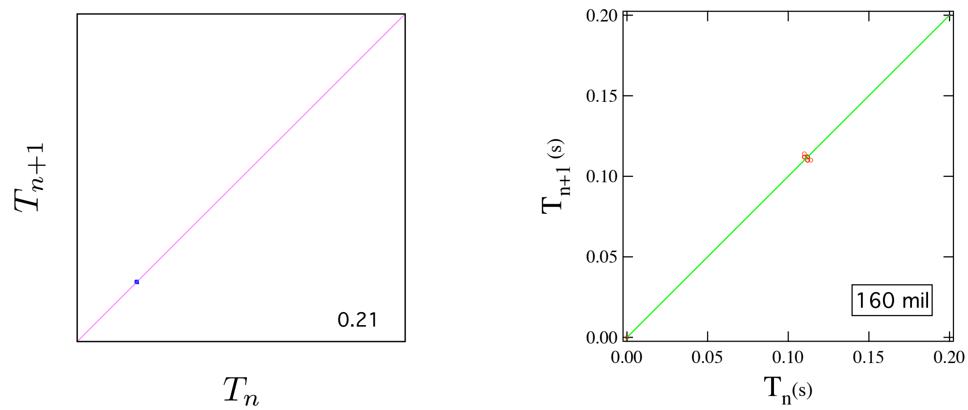
The experimental apparatus was setup as shown in Fig. 3. To count and measure the time of the drops, a HeNe laser and a photodiode were used. The laser was directed so that the dripping water interrupted its path. Thus, anytime the photodiode was not sensing the laser, a drop was passing.

To record the data, the photodiode was connected to a computer and an oscilloscope through a breadboard circuit. The circuit, explained fully in Fig. 4, reduced the noise of the signal using a Schmitt trigger and then sent the signal to a computer using a National Instruments USB-6009 device. Data was collected at a sampling rate of 500 Hz, which ensured that each drop was detected.

Once the data was collected, IgorPro was used to find the time between the drops $T_n = time\ of\ drop\ n - time\ of\ drop\ (n-1)$ from the raw data. Using this manipulated data, I was able to produce Poincaré sections of T_n vs. T_{n+1}.

IV. IMPLEMENTATION

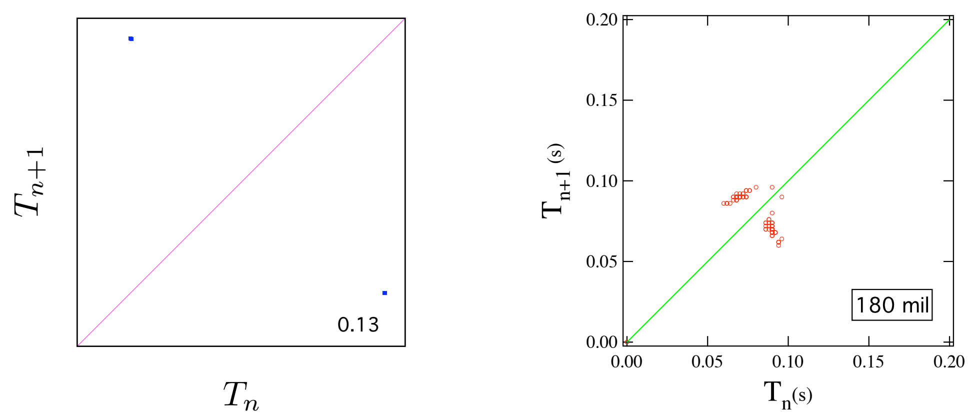
To implement the simulation of the dripping faucet, Euler-Cromer integration was used to update the position and the velocity of the drop using the acceleration calculated from the damped harmonic oscillator approximation, given in Eq. 1. Each time through the algorithm, the position was checked to determine if the drop should fall. If the distance x of the drop was less than the critical length, the mass was updated according to the flow-rate, dM/dt. If the distance x of the drop was greater than or equal to the critical length, part of the drop would fall from the spring and part would rebound to start the next drop. The falling drop had a mass proportional to the velocity v_c of the drop at the critical length. As shown in Fig. 5, the falling mass M_f is equal to $\alpha M_c v_c$, where α is a user assigned proportionality constant. The rest of the mass of the drop M is equal to $M_c - M_f$ rebounds instantaneously, as shown in Fig. 5, to the position $x = 0$ and also has its velocity v set to zero.



(a)A period-1 section plot from the computer simulation. ($k = 6.6$, $b = 3$, $g = 9.8$, $critL = 2.45$, $M_0 = 0$, $dM/dv = 0.09$).

(b)A period-1 section plot from the physical experimental.

FIG. 6: Both the computer simulation (a) and the experiment (b) showed the expected period-1 behavior with a single point.



(a)A period-2 section plot from the computer simulation. ($k = 6.6$, $b = 2.34$, $g = 9.8$, $critL = 2.5$, $M_0 = 1.2$, $dM/dv = 0.32$).

(b)A period-2 section plot from the physical experiment.

FIG. 7: Both the computer simulation (a) and the experiment (b) produced clean period-2 data, symmetric across the xy-line as expected.

V. DATA, RESULTS AND EXPLANATION

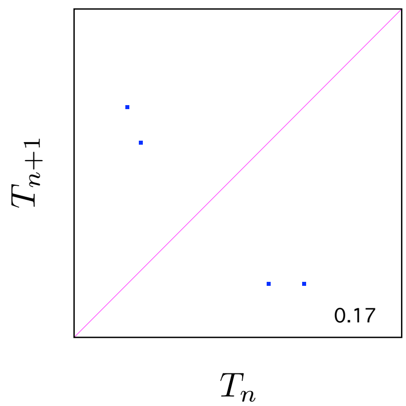
A. Period Doubling

The most uninteresting result of the dripping faucet is period-1, which is also the most common. I was able to find period-1 easily in both the simulation and the physical experiment. For period-1, T_n should always equal T_{n+1}. To help visualize this, I added the $x = y$ line with slope 1 to all of the Poincaré section plots. As shown in Fig. 6, the period-1 data falls directly on the line.

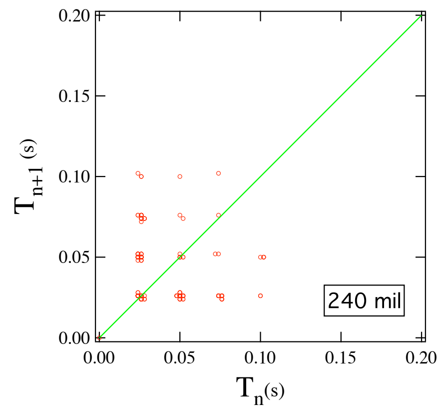
For period-2, there should be two drop times T_n, A and B. Thus, the pattern of the period ought to be ABAB repeatedly, producing a section with two data points symmetric about the xy-line. I found period-2 behavior which demonstrated this, shown in Fig. 7. I also found noisy period-2 behavior where the two drop times were repeated irregularly.

Continuing along in the period doubling regime, the simulation produced a very clean period-4. Unfortunately, for the experimental data, I never found a clean period-4. However, I did find a four by four grid which were shown to be a noisy period-4. The grid was caused by four distinct periods T_n, but they repeated in an unpredictable pattern.

For experimental period-4 it is quite difficult to determine the periodicy from the section plot and it is more informative to look at the histogram, shown in Fig. 9(a) and the plot of T_n alone, shown in Fig. 9(b). In this case the data are remarkably clear, with virtually all the T_n falling along the four lines, which indicates period-4

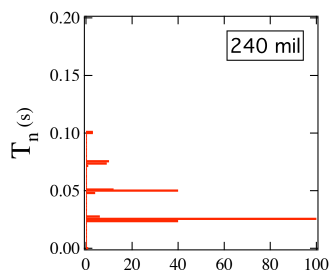


(a)A period-4 section plot from the computer simulation. ($k = 6.6$, $b = 2.34$, $g = 9.8$, $critL = 2.5$, $M_0 = 1.2$, $dM/dv = 0.32$).

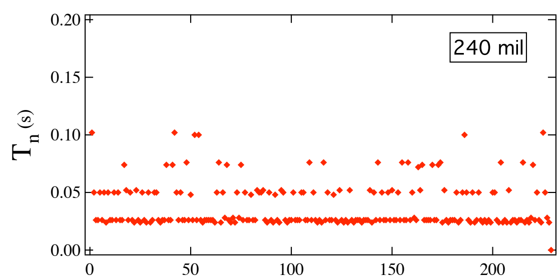


(b)A period-4 section plot from the physical experiment.

FIG. 8: The computer simulation (a) showed clean period-4 results, but the experiment (b) never showed a similar period-4. When only four different T_n were being used, the experiment produced the grid above.



(a)



(b)

FIG. 9: The histogram (a) and the T_n vs. point number (b) for the experimental period-4 data.

behavior.

I had a very hard time finding period-8, let alone a period-16, in either the experiment or the simulation. The period-8 from the simulation, shown in Fig. 10(a), has 8 points, but is not what was expected. Rather than being on a grid, like the points of period-2 or period-4, the points of period-8 seem to almost lie along a curve.

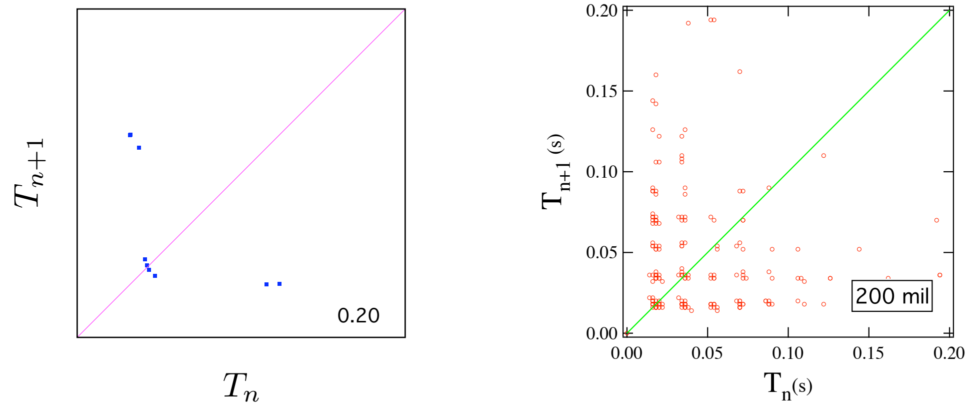
As for the experimental data, the data shown were one of my earliest data runs, and were not identified as possibly period-8 until after the period-4 shown in Fig. 8(b) was found. Unfortunately, this data was taken before I was monitoring the depth of the water in the reservoir, and is therefore not repeatable. However, as can be seen in Fig. 10(b), again the data forms a grid, although it is not quite an 8 by 8 grid.

Like period-4, the possible period-8 experimental data are much easier to view as a histogram and the T_n plot. If you count the lines on the histogram, shown in Fig. 11(a), you'll find that there are 10 lines, not counting the line at $T_n = 0$, which is an artifact of the procedure. As for the plot of the T_n, it is possible to discern 6 lines before the data are scattered. Thus, these data are not conclusively period-8. However, it may be noisy period-8.

The histograms of the period-4 and period-8 data interestingly appear to follow an exponential decay. The two histograms, shown in Fig. 12 have been overlaid with an exponential fit. The period-4 histogram, shown in Fig.12(a) are very well fit by the exponential. Even the longer period values of the period-8 histogram, shown in Fig.12(b) is well fit, although the highest point of the histogram does not lie near the fit curve. Given more data at those parameters, it is possible that the period-8 data would come to fit the exponential decay as well as the period-4 data. As interesting as this behavior is, I can make no conclusions about it.

B. Chaos

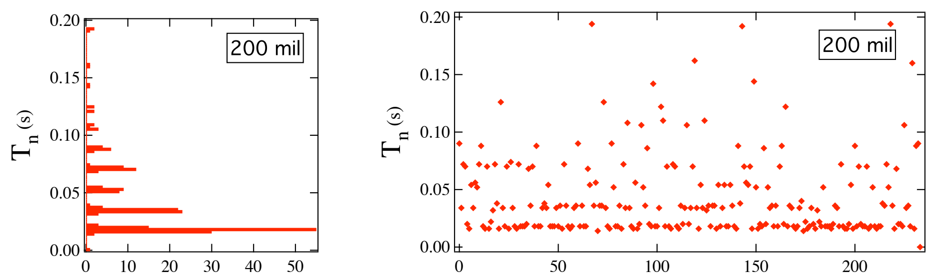
Excitingly, one of the first chaotic attractors that I found, using my simulation, was virtually identical to one Shaw found when he pioneered the study of chaos



(a)A period-8 section plot from the computer simulation. ($k = 6.6$, $b = 2.34$, $g = 9.8$, $critL = 2.5$, $M_0 = 1.2$, $dM/dv = 0.32$).

(b)A period-8 section plot from the physical experiment. These data were taken when the reservoir depth was known.

FIG. 10: Both the computer simulation (a) and the experiment (b) resulted in questionable period-8 data.



(a)

(b)

FIG. 11: The histogram (a) and the T_n (b) for the experimental period-8 data.

and the dripping faucet. Shown in Fig. 13(a), the attractor is simple, but the data is chaotic, showing extreme sensitivity to initial conditions.

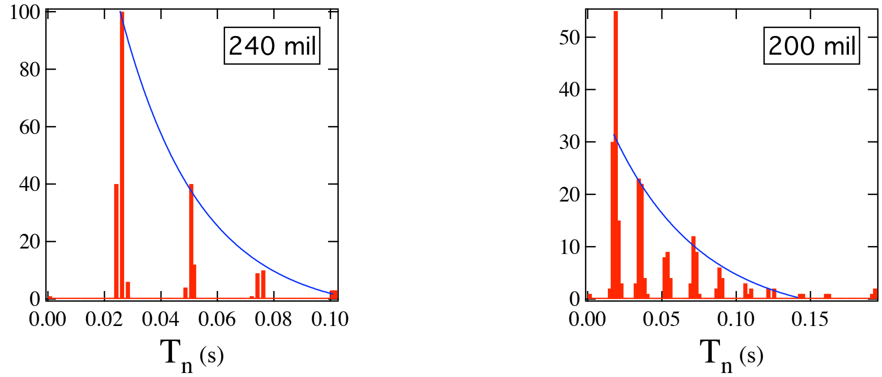
When I found chaos in the physical experiment, one of the data runs resulted in a similar attractor, shown in Fig. 13(b). Oddly, although the two attractors in Fig. 13 are of similar shape, they are inverted. The experimental attractor is almost an upside-down version the simulation attractor. It is possible that although the attractors are similar in shape, they come from significantly different parameter regimes, which caused this flipping. However, much more data would need to be acquired and analyzed before that conclusion could be made.

Other chaotic attractors were found, both with the simulation and with the experiment. For the most part, the chaotic attractors from the simulation were not similar to those from the experiment. The two shown are the most similar.

VI. CONCLUSION

I have confirmed that the dripping faucet demonstrates both period doubling behavior and chaos. Although my data do not significantly contribute to the understanding of the system, we have demonstrated that the leaky faucet exhibits a surprising wealth of complex behavior.

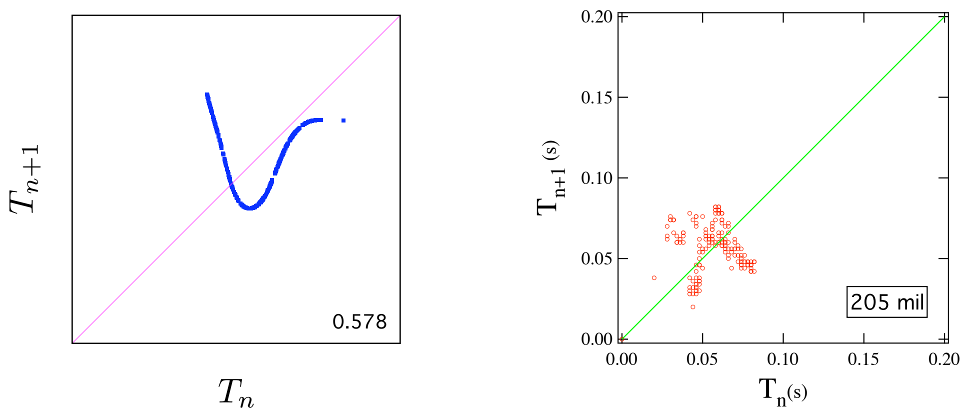
Even with such a simple system, the small amount of data that I collected demonstrates how much can be studied from the dripping faucet. Classifying the chaotic attractors of the system and finding the value of Feigenbaum's delta for the period doubling regime would be an excellent way to compare the leaky faucet to other chaotic systems. The grid pattern which was developing in the period doubling regime was unlike any data I came across in scientific journals or books. Determining if that result was a real effect or just an effect of the my setup would be a valuable way to expand upon my project. It could also be interesting to determine if the exponential fit of the period-4 and period-8 histograms was purely accidental or if there was some reason or theory behind that behavior. Lastly, we could determine the reason for the



(a)

(b)

FIG. 12: The histogram from the experimental period-4 data fit with a decaying exponential (a) and the histogram from the experimental period-8 data fit with a decaying exponential (b). No conclusions can be made about this result.



(a)A chaotic section plot from the computer simulation. ($k = 6.6$, $b = 2$, $g = 9.8$, $critL = 2.35$, $M_0 = 0$, $dM/dv = 1.67$.

(b)A chaotic section plot from the physical experiment.

FIG. 13: The chaotic attractor found by my computer simulation (a) is one of the attractors Shaw found when he worked with the dripping faucet simulation. The chaotic attractor of the experiment (b) is a similar shape, but is upside down.

inverted chaotic attractors, and whether any interesting science is behind this inversion.

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[2] S. Thornton and J. B. Marion, *Classical Dynamics of Particles and Systems*, Brooks/Cole - Thomson Learning, Belmont, CA, USA, fifth edition, 2004.

[3] A. D'Innocenzo and L. Renna, Physical Review E **55**, 6776 (1997).