The Stefan-Boltzmann Law

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This experiment attempts to experimentally verify the Stefan-Boltzmann law. The resistance of a tungsten filament was calculated by measuring an applied current and voltage. The relationship between temperature and relative resistance in tungsten is well known. Thus, by varying the voltage and current and measuring the relative change in resistance, the temperature of the filament was determined. Using the previously measured voltage and current values, the power emitted by the filament could also be calculated. The emitted power of the filament was found to be proportional to the temperature raised to the 3.74 power, a 6.5% error when compared to the accepted value of 4.00.

I. INTRODUCTION

When two objects are placed in contact, heat flows from the hotter one to the colder one until the two objects reach thermal equilibrium. However, even if the two objects are not in contact and there is no physical medium to carry heat, the two objects will still eventually reach a common temperature. This occurs because objects with a nonzero temperature release and absorb energy via electromagnetic radiation. This emitted energy is commonly called blackbody or thermal radiation.

For sufficiently hot objects, a fraction of this thermal radiation is in the visible light spectrum, allowing this phenomenon to be seen with the naked eye. We commonly see this in metals, as they begin to glow when sufficiently hot. The non-visible effects of thermal radiation are easily observable in everyday practice with hot objects, such as a fire or halogen light-bulb which noticeably heat up the surrounding environment. The amount of radiation emitted is proportional to the temperature of the object, and this holds true in daily life. We frequently observe that a hot, glowing object will become brighter with an increase in temperature.

The exact relationship between the temperature of an object and the amount of radiation emitted has been explored by a number of famous physicists, including Max Planck and Wilhelm Wien. In 1879, the law that describes this relationship was first experimentally discovered by Josef Stefan[1]. Shortly later, Ludwig Boltzmann derived it theoretically.

While the the Stefan-Boltzmann law has obvious terrestrial applications, it also has a great deal of importance to astrophysicists. Measuring many of the physical properties of far away astronomical bodies directly is clearly impossible. However, by merely observing the blackbody radiation spectrum, it is possible to calculate the surface temperature and energy output of distant stars.
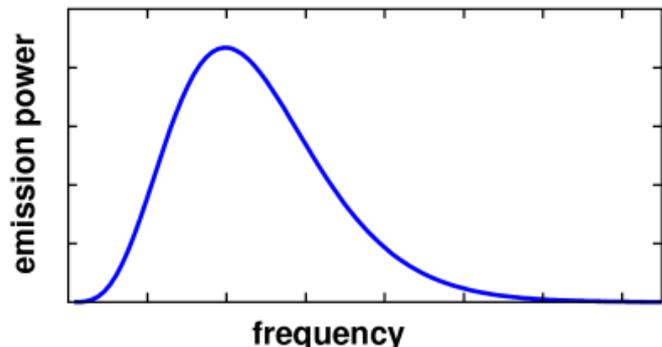

FIG. 1: The characteristic graph of the thermal radiation emitted by a hot object. Graph from reference [2].

II. THEORY

A. Stefan-Boltzmann Law

Blackbody radiation is emitted as a broad spectrum of wavelengths, as shown in figure 1. The shape of this curve is modeled by Planck's law of blackbody radiation[3], which states that for any given temperature T, the intensity I for frequency f is given by

$$I = \frac{2h}{c^3} \frac{f^3}{e^{hf/kT} - 1}. \tag{1}$$

Here h, k, and c are respectively Planck's constant, Boltzmann's constant, and the speed of light.

The wavelength with the maximum luminosity in figure 1 is given by the Wien Displacement law[4]. Formally, this law states that for some temperature T (in Kelvin), the most intense wavelength λ_{max} will be

$$\lambda_{max} \approx \frac{3 \times 10^7}{T} \text{ ÅK}^{-1}. \tag{2}$$

From this, we can make the simplification

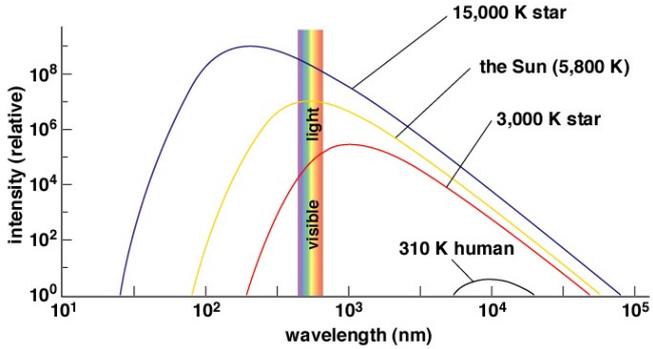$$\lambda_{max} \propto \frac{1}{T}. \tag{3}$$

FIG. 2: The thermal radiation emitted by several reference objects[5]. The hotter objects have smaller peak wavelengths and sharper curves. Note that much of the energy is not emitted as visible light.

To illustrate this relationship between temperature and thermal radiation, the blackbody radiation spectrum for several reference objects is shown in figure 2. As expected, the hotter objects have smaller peak wavelengths.

To find the total amount of energy being radiated by an object, we merely need to find the area under the curve, which is equivalent to the integral of equation 1 over all wavelengths. This integration results in the Stefan-Boltzmann law, which states[6] that for an object of temperature T, the radiated power P will be

$$P_{rad} = \epsilon \sigma A_s T^4. \tag{4}$$

Here ϵ is the emissivity of the object, A_s is the surface area, and σ is the Stefan-Boltzmann constant. The emissivity constant depends entirely on the material of the object and is capped at 1 for an ideal blackbody. For practical purposes, the net power being radiated is more useful than the absolute radiated power. The net power of an object at temperature T in an environment of temperature T_0 is

$$P_{net} = P_{rad} - P_{absorb} \tag{5}$$

This expands to[7]

$$P_{net} = \epsilon \sigma A_s T^4 - \epsilon \sigma A_s T_0^4. \tag{6}$$

For this experiment, ϵ, σ, and A_s are constant. Additionally, T_0, the room temperature, was measured to be 295 K, while T ranged from 1500 to 2000 K. Thus T^4 will be vastly larger than T_0^4, so the environmental temperature term is negligible and can be dropped. This simplifies equation 6 to

$$P \propto T^4. \tag{7}$$

For simplicity, we take the natural log of both sides, expressing the equation as

$$\ln P \propto 4 \ln T. \tag{8}$$

B. Power and Resistance

By measuring the current in the circuit, and the voltage across the filament, it is possible to calculate the power and resistance of the filament. The rate of energy transfer P, across a potential difference V with current I is

$$P = IV. \tag{9}$$

Calculating resistance is trivial, as the resistance R of an object is defined as the potential difference V divided by the current I, or more formally

$$R \equiv \frac{V}{I}. \tag{10}$$

The measured resistance of any one piece of tungsten will depend on several physical characteristics that are object dependent, such as the length and cross sectional area. However, a change in temperature will change the tungsten's resistance, and the ratio of this new resistance to the previous resistance is object independent. That is, the relative change in resistance due to the internal temperature is the same for all tungsten objects. For example, suppose the starting reference resistance is taken at 300 K. Then for any temperature T, the dimensionless value of R_T/R_{300} will be constant for all tungsten objects.

III. EXPERIMENTAL SETUP & RESULTS

The setup consists of a power supply[8] connected to a Stefan-Boltzman lamp[9], with a voltmeter[10] and an ammeter[11] attached. An electrical schematic of the experimental setup is shown in figure 3.
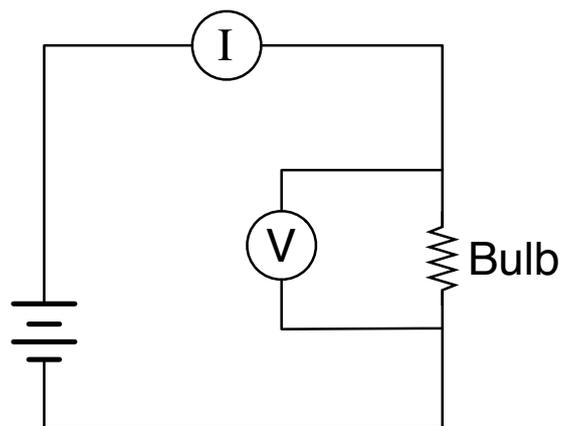

FIG. 3: The electrical schematic of the setup. The voltmeter measures the potential difference across the Stefan-Boltzmann bulb.

When current begins to pass though the filament, the filament's resistance converts some of the electrical energy into heat, raising the temperature of the bulb. As the temperature increases, so does the emitted thermal radiation. Eventually, the temperature stabilizes when the heat being generating by the resistance is in equilibrium with the energy being radiated away. It was assumed that all of the energy supplied to the bulb was emitted as radiation. That is, the energy lost to thermal conduction and convection was assumed to be negligible.

The resistivity of the tungsten is not constant and will rise with internal temperature. As mentioned previously, this change in resistivity is universal for tungsten objects, and the ratio R_T/R_{ref} can be used to infer the temperature of the tungsten object. PASCO Scientific provides a lookup table containing the relative resistivity (R_T/R_{300}) for temperatures between 300 and 3600 K in increments of 100 K. Since the table only provides values for every 100 K, linear interpolation was used to calculate all the values in between the 100 K increments. For convenience, a simple program named "*linear*" was written that would accept any relative resistivity between two known values and calculate the interpolated temperature.

The first measurement taken was the reference resistance R_{ref}, measured at room temperature (295 K). The measured current was 0.514 A, which produced a potential difference across the filament of 0.222 V. Using equation 10, R_{ref} was calculated to be 0.432 Ω. After taking the R_{ref} measurement, the voltage across the filament was increased in increments of 0.2 V. After every increase in voltage, the current and voltage readings were allowed to stabilize, and then were recorded. The highest voltage recorded was 12.802 V, with a current of 2.825 A.

From each pair of voltage and current values, a corresponding power and temperature value was calculated. If power is related to temperature by a power law relationship, which has the form of

$$P = CT^a, \tag{11}$$

then taking the log of both sides yields

$$\ln P = a \ln T + \ln C. \tag{12}$$

Thus, if $\ln P$ versus $\ln T$ is plotted and the curve is a straight line, then the relationship between P and T is a power law and the exponent of that power law is the slope of the curve.

The experimentally determined values of $\ln P$ and $\ln T$ are plotted against each other in figure 4. In section II A, one of the assumptions made in the derivation of equation 7 was that T^4 was far larger than T_0^4. Thus, equation 7 does not hold for temperatures near 295 K. To minimize any error introduced by this approximation, only data points with an inferred temperature exceeding 1500 K were kept.

From equation 8, we know that the slope of log-log graph of power versus temperature raised to the fourth should
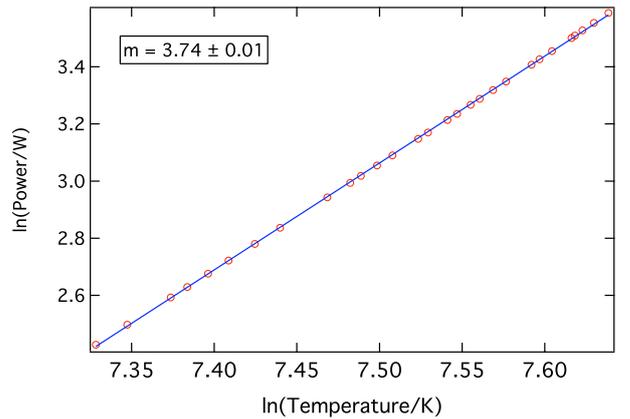

FIG. 4: The natural log of the measured power versus the natural log of the inferred temperature. The slope of the curve fit is 3.74. The uncertainty bars on this data are smaller than the size of the data points.

be exactly 4. The slope in figure 4 is only 3.74, which is a percent error of 6.5%.

IV. ERROR AND UNCERTAINTY

A. Uncertainty

The measured voltage ranged from 10 to 13 V, and the uncertainty in these measurements was ± 0.001 V, which is less than 0.01%. The current ranged from 2.5 to 2.8 A, and the uncertainty in these measurements was ± 0.001 A, which is less than 0.0025%. Propagating the uncertainty though equation 9 yields that the uncertainty in the power calculations was less than 0.04%.

The uncertainty in the resistance values was less than 0.05%, but the calculated uncertainty in the R_{ref} value was ± 0.002 Ω or 0.46%. This gives the R/R_{ref} values a maximum uncertainty value of ± 0.044 or 0.61%. The uncertainty in the R/R_{ref} values translate into an uncertainty of ± 8.0 K. Since the calculated temperatures range from 1500 to 2100 K, this is less than 0.53% uncertainty. Uncertainty that small is completely negligible in this experiment due to the much larger systematic error.

B. Error

The accepted value of the exponent in equation 4 is exactly 4 but the observed value was 3.74, a percent error of 6.5%. The uncertainty in the measurements was far too small for an error of this magnitude, suggesting a more fundamental flaw in the experiment. One possibility is that the approximation of
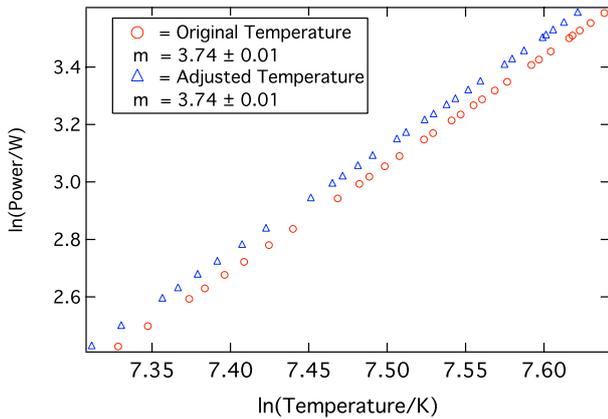
$$T^4 \approx T^4 - T_0^4 \tag{13}$$

FIG. 5: The original calculated temperature compared with the temperature calculations adjusted for possible systematic error. The slope of both curves is 3.74, illustrating that an error in the reference resistance value has no effect on the calculation of the exponent.

used in equation 7 is not valid. This seems impossible, as the smallest data point used is at 1522K. $(1522)^4$ is over 700 times bigger than $(295)^4$, which proves that the T_0^4 term is indeed negligible.

Another possible source of error lies in the R_{ref} value. When the R_{ref} value was measured in the lab, it was assumed to be the resistance of the filament at room temperature, which was 295 K. However, the lookup table provided by PASCO assumes the R_{ref} value is taken at 300 K, a 1.66% difference. Additionally, there is also another possible error in the R_{ref} value. In order to measure R_{ref}, a small amount of potential difference (0.222 V) had to be induced across the tungsten filament, perhaps this was enough to raise the temperature of the filament above 300 K. This would offset the calculated R/R_{ref} values from those in the table, providing a systematic error in the temperature values.

If the R_{ref} value was taken at 295 K instead of 300 K, and assuming an approximately linear relationship between resistance and temperature, then R_{295} differs from R_{300} by no more than 2%. Thus the value of R/R_{295} would only be about 2% bigger than R/R_{300}. If the measured values of R/R_{ref} are reduced by two percent to compensate for this possible error, then the temperatures can be recalculated.

In a similar manner to what was done with the original temperature values, the natural log of the adjusted temperature is plotted against the natural log of the power. From equation 12, we know that the slope of this line will be the exponent of the temperature. The results are shown in figure 5. The slope of the adjusted temperature and the original temperature are identical, and the only change is the y-intercept. From this, we can conclude that any calibration error in the R_{ref} value will not effect the slope of the curve in a log-log graph, and thus is not the cause of the systematic error seen in this experiment.

An assumption made in this experiment was that all of the power supplied to the bulb was emitted as radiation, and that the energy lost to thermal convection and conduction was negligible. The filament was enclosed in an airtight bulb, so the effects of possible convection currents were minimized. However, we know that the filament was much hotter than the surrounding environment, so it certainly is possible that a significant amount of heat was conducted away at the point of contact between the filament and the base of the bulb. If there was a systematic energy drain though thermal conduction, then this would depress the temperature values. These lowered temperature values could cause the discrepancy between the experimental and accepted values of the exponent in equation 7.

V. CONCLUSIONS

The relationship between the emitted power P of the thermal radiation of a blackbody object and the object's temperature T was found to be of the form

$$P = CT^a. \tag{14}$$

Here C is a constant. The value of a was found to be 3.74. The accepted value of this exponent is 4.00, so the percent error in the experimental value was 6.5%. This error is likely due to the fact that not all of the energy supplied into the filament was radiated as thermal radiation. A significant portion was probably absorbed into the surrounding environment though thermal conduction.

[1] D. Halliday, R. Resnick, J. Walker, *Fundamentals of Physics* (John Wiley and Sons, 2005)
[2] Einstein-Online, (http://www.einstein-online.info/en/spotlights/thermal_radiation/index.html) Retrieved February 21, 2007
[3] scienceworld.wolfram.com (http://scienceworld.wolfram.com/physics/PlanckLaw.html, 2007) Retrieved February 21, 2007
[4] Radiation Laws (http://csep10.phys.utk.edu/astr162/lect/light/radiation.html) Retrieved February 21, 2007
[5] (http://cse.ssl.berkeley.edu/bmendez/ay10/2002/notes/lec8.html, 2002) Retrieved February 21, 2007
[6] C. Kittel, H. Kroemer, *Thermal Physics* (W. H. Freeman and Company, 1980)

[7] S. Lehman *Physics 401 Junior Independent Study* (The College of Wooster, 2007)

[8] Hewlett Packard DC Power Supply Model 6290A

[9] Pasco Scientific Stefan-Boltsmann Lamp Model TD-8555

[10] Keithley 197 Autoranging Microvolt DMM

[11] Keithley 197 Autoranging Microvolt DMM