Studying the Effects of Filling a Helmholtz Resonator with Spheres

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A Helmholtz resonator, consisting of a resonating cavity with an open neck, was filled with spheres to determine the effects on the resonant frequency. A function generator and speaker were used to excite the resonator, and Fourier analysis was used to find the resonant frequency. Two sizes of marbles, glass beads, and water were used separately to fill the resonator. Frequency measurements were made at a wide range of open volumes by filling the resonator with different amounts of spheres. The volume of the spheres and the distance from the top of the resonator to the top of the spheres were measured as well. Comparisons of the resonant frequencies of spheres and water at the same height were investigated. It was found that the resonant frequency for water was much higher than the resonant frequency for spheres at the same height, indicating that the air pockets in between the spheres are having an effect on the resonant frequency. In addition, the change in frequency as a function of open volume for both the water and the spheres was studied. A peak in the resonant frequency for the spheres was observed at a certain critical open volume, approximately half of the total volume of the resonator. This effect is contrary to the theoretical dependence of frequency on open volume. The most likely explanation is that a correction is needed in the theory, as the simplest case no longer holds.

I. INTRODUCTION

Hermann von Helmholtz's study of sound was one of the most extensive and thorough in history. He studied, among other things, combinations of vibrations, beats, musical tones and their relationships, and the perception of sound by humans.[1] One of his most famous contributions to the study of acoustics, however, is his resonators. Helmholtz resonators are at their most basic a resonating cavity with an open neck.[2] The most recognizable example is an open pop bottle, which produces a tone when a stream of air is blown over the top.

The study of Helmholtz resonators is important because they are used in many applications. They are often used as sounds absorbers for noise control. The resonators are placed within walls and absorb sound at their resonating frequency. This application has been seen in air-conditioning ductwork, and automobile engines.[2, 3] These resonators are also the basis of many musical instruments, including guitars and violins.

There has been extensive research in the pursuit of improved sound absorption. Changes in the geometry of the necks of the resonators have been studied widely. Tang has investigated the effects of a tapered neck, and found that the sound absorption was enhanced.[3] Selamet and Lee looked at the effects of extending the neck into the resonating cavity. They found that the resonant frequency was lowered by this simple adjustment.[4] Selamet, Xu, Lee, and Huff lined resonators with absorbing material, and found that not only were the sound absorption qualities improved, but the resonant frequency was lowered.[5]

Something that has not been studied in detail before is the effects on the resonant frequency of filling a Helmholtz resonator with spheres. Filling the resonator with spheres differs significantly from filling the resonator with water. Water completely fills the space in the bottom of the resonator, leaving a large open resonating volume and nothing else. Spheres, on the other hand, cannot completely fill the space. Air pockets are left in between the spheres in addition to the open resonating cavity above the marbles. The resonators were filled with water to the same height as the spheres and the resonant frequency was found. In addition, the effect of the spheres on the resonant frequency was tested by finding the relationship between open volume and resonant frequency for both water and spheres. The effects of the spheres were found by comparing the measurements with water to those with spheres.

II. THEORY

A basic Helmholtz resonator consists of a resonating cavity with an open neck. Most resonators use standing waves to create the resonance. Helmholtz resonators do not work on this principle. Instead, they can be modeled as a simple mass on a spring system. The air in the neck acts as the mass, while the air pressure both inside and out of the resonator acts as the spring.[2]

The effective mass of the air in the neck is given by

$$m = \rho_0 A L', \tag{1}$$

where ρ_0 is the density of air, A is the cross sectional area of the neck, and L' is the effective length of the neck.[6] The effective length depends on the shape of the opening. In the case of the resonators used in this experiment, the effective length is given by

$$L' = L + 1.7a, \tag{2}$$

where L is the actual length of the neck, and a is the radius of the opening.[6]

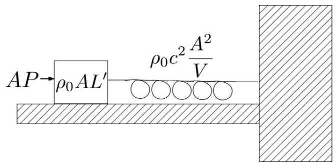


FIG. 1: A Helmholtz resonator can be thought of as a mass on a spring system. The air pressure acts as the spring, while the air inside the neck is the mass. The driving force is provided by an external sound wave.

The air pressure acts as the spring in the system. The "spring constant" of the air pressure is

$$k = \rho_0 c^2 \frac{A^2}{V}, \tag{3}$$

where c is the speed of sound, and V is the volume of the cavity.[6] A diagram of the mass on a spring system is shown in Figure 1, with the "spring" and mass labeled.

The resonator can be excited using a frequency generator and speaker tuned to the resonant frequency. This sound acts as a driving force, given by

$$F = AP, \tag{4}$$

where P is the sound pressure.[7]

The system can be described by a differential equation of the form

$$m\frac{d^2\delta}{dt^2} + k\delta = AP, \tag{5}$$

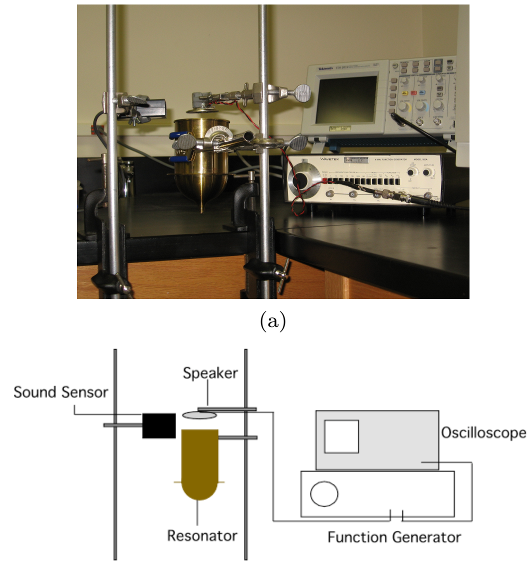
where δ is the displacement of the slug of air in the neck.[7] Solving this equation, we find that the resonant frequency is given by

$$\omega_0 = \sqrt{\frac{k}{m}}. \tag{6}$$

The frequency f is related to the angular frequency, ω_0, by a factor of 2π. By using this relationship and substituting in the expressions found in Eqs. 1 and 3, we find that the resonant frequency is

$$f = \frac{c}{2\pi}\sqrt{\frac{A}{L'V}}. \tag{7}$$

The resonant frequency is inversely proportional to the square root of the volume. This means that a larger volume would give rise to a lower frequency. As the volume decreases, the frequency should increase. It should be noted that the resonant frequency corresponds to a wavelength much longer than the dimensions of the cavity. The theoretical dependence of frequency on volume can be tested by changing the volume of the resonating cavity without altering other parameters.



(b)

FIG. 2: The setup for the experiment. A speaker is placed over the bottom opening of the resonator and connected to a function generator. The sound sensor records the data for later analysis.

III. EXPERIMENTAL

A. Procedure

A Helmholtz resonator was filled with marbles, glass beads, and water to study the effects on the resonant frequency. A function generator and speaker were used to excite the resonator, while a Pasco sound sensor was used with *DataStudio* to record the data. An oscilloscope was used to monitor the frequency of the generated signal. The Helmholtz resonator was clamped so that the ear piece was facing downwards. The speaker was positioned above the opening in the bottom of the resonator, with the sound sensor located beside the speaker. A picture and diagram of the set up are shown in Figures 2(a) and 2(b).

The resonant frequency of the empty resonator was found first. Following this initial measurement, three different types of measurements were taken. The first type was with the resonator partially or completely filled with spheres. The spheres used were two different sizes of marbles and glass beads. The smaller of the two types of marbles had a diameter of approximately 1.4 cm, while the larger had a diameter of approximately 1.6 cm. The glass beads were approximately 0.3 cm in diameter.

The second type of measurement was taken using water that came to the same height as the spheres. The height of the spheres within the resonator was also noted in some cases. The distance to the top of the first layer of marbles and the distance to the top of the second layer were measured from the flat edge of the resonator. The two distances were then averaged. The height of the spheres is not a well defined quantity, since the top surface of the spheres is not smooth, which is why this method was used. The heights were marked on the resonator both inside and out using masking tape. The resonator was filled with water until it reached one of the markings on the tape. This was done for five different heights.

The last type of measurement used water of various volumes. The volume occupied by the spheres was found using a water displacement method. A graduated cylinder was filled with a known amount of water. The spheres were then placed into the graduated cylinder. The initial volume of water was subtracted from the total volume of spheres and water to find the volume of the spheres only. The volume of water used spanned a range comparable to the volume that could be occupied by the spheres. This was done for twelve different volumes of water.

DataStudio was used to collect the data. The data were in the form of voltages as a function of time. The frequency put out by the function generator was initially at a frequency below the resonance. This frequency was gradually increased until a sharp increase in amplitude was observed. The function generator was then held at the resonant frequency for several seconds, after which point it was increased further and the amplitude died off.

The data were exported from *DataStudio* to *Igor Pro* for analysis. The Fourier transform was taken to find the resonant frequency of each run. Two plots were created from the analysis. The first was a plot of frequency versus the height of the marbles or water. This plot was used to compare the effects of water and spheres in the resonating cavity. The second plot was of the frequencies as a function of open volume. This plot was used to observe the change in relationship of resonant frequency to open volume for water, marbles, and glass beads.

B. Data and Error Analysis

The resonant frequency of the empty resonator was found first. The function generator was used to sweep through a range of frequencies. *DataStudio* created a graph of voltage versus time for each data run. Fourier analysis was used to find the frequency at resonance. All of the data points from each run were used in the Fourier analysis. A graph of magnitude versus frequency was created from the Fourier transform, with a large spike in the magnitude indicating the resonant frequency. The graph of magnitude versus frequency for the open resonator is shown in Figure 3.

Once the resonant frequency of the empty resonator was found, the resonator was filled with spheres. Marbles or beads were added until the surface was approximately level. At this point, the same process as with the open resonator was used. The voltage as a function of time was recorded, and the Fourier transform was taken of the data. The resonant frequency was marked by a large spike in the magnitude of the transform, just as for the

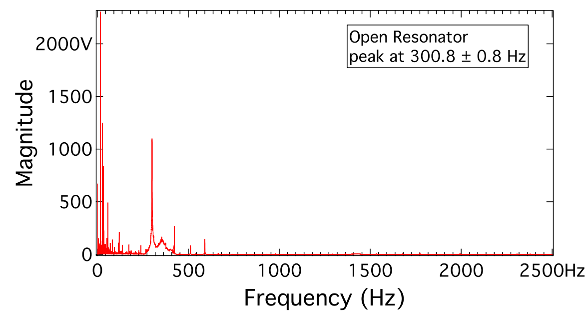


FIG. 3: The magnitude of the Fourier transform of the open resonator data. The sharp peak occurs at the resonant frequency. This peak is at 300.8 ± 0.8 Hz.

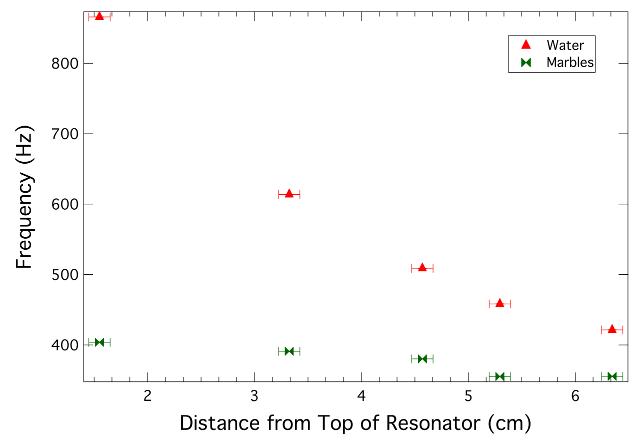


FIG. 4: The difference in resonant frequencies between water and marbles at the same height is apparent in this graph of frequency versus distance from the top of the resonator.

open resonator. This was repeated for several amounts of marbles and beads.

Once data had been collected for several amounts of spheres, water was poured into the resonator so that it matched the heights of certain amounts of the small marbles. Voltage as a function of time was collected for five heights of water. Fourier analysis was again used to find the resonant frequency. The heights are recorded as the distance from the flat edge, or top, of the resonator.

The difference in frequency is easily illustrated through a graph of frequency versus distance from the top of the resonator. This graph is shown in Figure 4. The difference between the frequency for the water and the frequency for the marbles is readily apparent.

The volume of the spheres was also measured. By subtracting this volume from the total calculated volume, the open volume in the resonator could be calculated. The total volume of the resonator was approximated as a cylinder and a hemisphere of equal radius. This approximation yielded a volume of $502.7 \pm 0.2\,\mathrm{cm}^3$. The resonant frequency for several volumes of water was also found. This was done in order to compare the relationship between frequency and open volume for the spheres

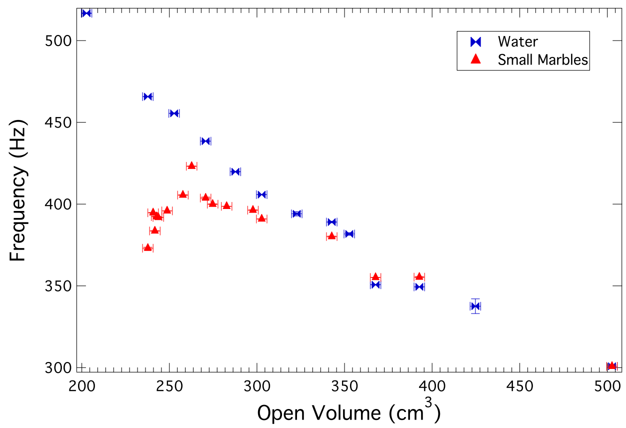


FIG. 5: A graph showing the change in frequency with open volume. There appears to be a peak in the frequencies for the marbles.

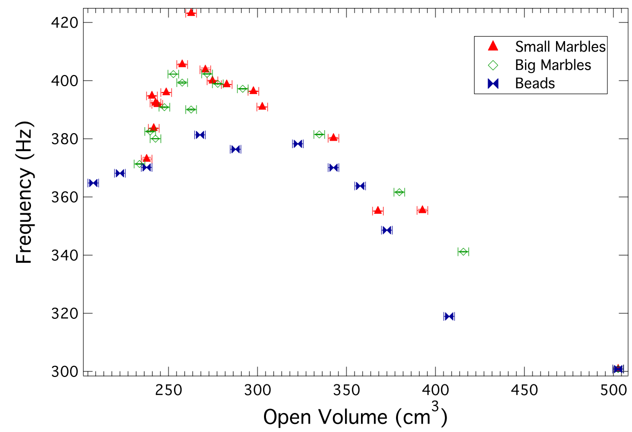


FIG. 6: A graph showing the change in frequency with open volume for the two sizes of marbles and the glass beads. The peak in frequency corresponds to an open volume of approximately half of the total volume of the resonator.

and the water.

The simplest way to view the effects of open volume on the resonant frequency is through the use of a graph. Figure 5 shows a graph showing the change in frequency as a function of open volume for the small marbles and the water. For the water, the frequency continues to increase monotonically as the volume decreases, as expected from Eq. 7. However, for the marbles, it appears that the frequency peaks at some critical volume.

Figure 6 shows the same type of graph for both sizes of marbles and the beads. The data from the two sizes of marbles overlaps and appears to peak at about the same frequency and open volume. This peak occurs at a volume of approximately 250- 260 mL, roughly half of the total volume of the resonator. There appears to be a peak in the data for the beads in about the same area as well. More data are needed for the beads to fill in large gaps in volume to determine with certainty where the peak occurs. Possible reasons for the observed decrease in frequency will be discussed later.

The error in height was taken to be 0.3 cm. The height of the marbles was not a well defined quantity. This error was thought to be reasonable given the difficulty of measuring this height. The error in the volume, except for the calculated total, was due to uncertainty in measuring with the graduated cylinder. The error in measurement was taken to be $\pm$ 3 mL. This uncertainty occurred in every volume measurement, since the same graduated cylinder was used each time. This uncertainty was small compared to the volume measurements being made.

The error in the frequency was found from the magnitude versus frequency plots for each data run. The peak was fit with a Lorentzian curve using *Igor Pro*. The width of the curve at half of the maximum value was found using the cursors on the graph. This width gave an approximate spread in the resonant frequency. The width found at half the maximum was then divided by two, and this was taken to be the error in the frequency value. The uncertainty in the frequency was typically very small, almost always less than 1 Hz with a few exceptions.

IV. RESULTS AND DISCUSSION

The data show that the spheres do not act either like water of the same volume or water of the same height within the resonator. For the equal height comparison, the resonant frequency for water is typically much greater than the resonant frequency for the spheres. This result shows that the air pockets between the spheres are contributing significantly to the resonating cavity volume in some capacity. If the air pockets were not contributing, the frequencies for water and spheres at equal heights should have been equal.

The frequencies as a function of open volume were the most surprising results. It appears from the data that there is some critical volume for the spheres. Once this volume is passed, the frequency starts to decrease with decreasing open volume. This result is contrary to the accepted theory for Helmholtz resonators. According to Eq. 7, the frequency is inversely proportional to the square root of the volume. Thus, the frequency should increase with decreasing open volume and vice versa. This theorized result was seen with the water measurements for the full range of data taken. It was also seen for a large portion of the data taken for the resonator filled with spheres. However, as can be seen in Figures 5 and 6, the theory breaks down when a large fraction of the resonator is filled with spheres.

The explanation for this effect is as yet unknown. There are several intriguing possibilities, however. The first possible explanation is an effective slowing of the speed of sound through acoustic wave scattering. Page *et. al* experimented with acoustic wave scattering in materials composed of spheres packed in water.[8] In the research, they found that the group velocity of the sound wave was significantly slower than the speed of sound in either the spheres or the water. This was due to the fact that the sound wave is unable to travel directly through the material, but rather bounces and scatters off of the spheres. This could be the case in the resonators. However, since the wavelength of the sound wave is substantially longer than the dimensions of the resonator in this experiment, this slowing is probably not significant enough to cause the observed effect.

A second explanation, somewhat related to the first, is that the sound is somehow interacting with the air pockets left between the spheres. It can be seen from Figure 4 that these air pockets are making a difference. If they were not affecting the resonant frequency, then the results for spheres should have been the same as those for water. The interaction between the sound waves and the air pockets would be very similar to, and perhaps indistinguishable from, the scattering described above. Again, however, this scenario is not the most likely cause, since the wavelength of the sound is much longer than the dimensions of the resonator and the air pockets.

The derivation of resonant frequency outlined previously describes the simplest case for a Helmholtz resonator with an open volume. It could be that once a certain volume is filled with spheres, or simply when the resonator has spheres inside of it, this simplest case no longer holds. The frequency found through Eq. 7 is the same for all shapes of resonators as long as a few restrictions hold. The first restriction is that the wavelength of the sound wave must be considerably longer than the dimensions of the resonator. The second is that the opening of the resonator is small.[6] The system being investigated still follows the first restriction since the wavelength of sound wave is much longer than the dimensions of the resonator. The second restriction might not hold in this system, however. As more spheres are added, the completely open cavity near the neck gets smaller. After the spheres reach a certain height, the opening might not be sufficiently small. In fact, the peak occurs when the spheres are at a distance from the top of the resonator that is approximately equal to the diameter of the opening. Once this height is attained, the resonant frequency starts to drop as though the open volume between the spheres were much larger. It is very possible that this can be accounted for through some kind of correction to Eq. 7. This is currently the most plausible explanation for the observed effect.

There are several possible ways to further investigate this effect. More data can be taken with the glass beads to fill in gaps in the current data. Spheres of different sizes could be used. This would include spheres between the size of the glass beads and the small marbles, as well as spheres larger than the large marbles. Using different sizes of spheres in the resonator at the same time would also be something to examine. Flooding the resonator with water while there are spheres inside could produce interesting effects as well. A final area of possible study would be using resonators of different sizes. This could be used to see if the observed peak occurs at the same fraction of volume for each resonator.

The results of this study have displayed an interesting effect. The change in frequency as a function of volume is contrary to theory when the resonator is filled with spheres. At this time, a definite explanation is unknown and more work is needed to investigate further.

V. CONCLUSION

The effects of filling a Helmholtz resonator with spheres were studied. This system has not been studied in detail before, so the possible effects were largely unknown. The resonator was excited using a function generator and speaker, and the data were analyzed using Fourier analysis.

In comparing the resonant frequencies of certain heights of spheres to equal heights of water, it was found that the resonant frequencies with the spheres are considerably lower than with the water. This indicates that the open air pockets between the spheres are significantly affecting the resonant frequency.

The change in resonant frequency with respect to open volume in the resonator was also investigated. It was found that for water, the resonant frequency increases as the amount of open volume in the resonator decreases. This follows the accepted theory for Helmholtz resonators. However, for the spheres, the resonant frequency had a peak, after which the frequency decreased as the open volume decreased. This result was unexpected, since the theory predicts the opposite. The peak occurred when approximately half of the total volume of the resonator was filled with spheres. It also corresponded to a point where the distance from the top of the spheres to the top of the resonator was approximately equal to the diameter of the neck.

The most likely explanation for this effect is that the simplest theory no longer holds. The accepted theory for Helmholtz resonators is for a large open cavity and a small opening. At the peak frequency, the opening is no longer small with respect to the dimensions of the cavity. A correction of some kind is needed to account for the new behavior. Other possible explanations include possible scattering effects and interactions between the sound waves and the air pockets between the spheres. Further investigation is needed to provide a definitive explanation for this effect.

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