The Effects of Moons on Saturn's Ring System

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The ring system of Saturn is a complex interaction between numerous particles, moons, and Saturn. A program was created to simulate the different aspects of the ring system and to study the effects of moons on the orbiting particles. Simply by considering both the force of gravity on each particle and collisions between particles, gaps formed at eight different resonant positions. I also studied the effects of collision elasticity and found that less elastic collisions cause the gaps to form more quickly. Furthermore, the effects of shepherding were studied by embedding an orbiting moon in the ring particles. By varying the mass ratio of Saturn to the moon, it was found that the smaller the mass ratio between Saturn and the moon, the larger the gap cleared in the ring and the larger the width of the ringlet formed in the center of the gap. It was also observed that resonances interior to the moon controlled the interior edge of gaps while resonances exterior to the moon controlled the exterior edge of the gaps.

I. INTRODUCTION

The ring system of Saturn is a dynamically rich environment with many phenomena to explore. The complex interactions between Saturn's moons and the rings are particularly interesting. To better understand the observed phenomena in ring systems, I created a computer simulation to model aspects of Saturn's ring system like resonance and shepherding. Although all of the gas giants in our solar system have rings, Saturn has the largest, most complex ring system. Also, there is more knowledge of Saturn's ring system from the wealth of information gathered by the Voyager and Cassini satellite missions. The availability of information makes Saturn's rings a a good choice for modeling.

Many of the observed phenomena in rings are created by interactions between the ring particles and orbiting moons. Although there were only 18 known moons of Saturn in 1997, that number has quickly grown to 57 and continues to rise as more discoveries are made. These moons were not observed before 1997 because 31 of these moons are less then 10 km in diameter. Even these small moons though have visible effects on the rings.

A. Cassini Division

The Cassini division in between Saturn's A and B rings is a gap caused by a 2:1 resonance with the moon Mimas. The mass ratio of Saturn to Mimas to the ring particles is about $5.6 \times 10^{26} : 3.8 \times 10^{19} : 1$ [1]. Resonance happens when a ring particle and an orbiting moon have periods that are simple fractions of one another. This causes the particle and moon to always align at the same position in their orbit and the gravitational pull from the moon will be greater at this point. Over time, the small gravitational tugs from the moon will add up causing the orbit of the ring particle to be perturbed. When the ring particle's orbit is perturbed, it will collide with other particles outside of the resonance area and its orbit will change causing a gap to form in the rings. Saturn's moon Mimas is in a 2:1 resonance with ring particles in the Cassini division. Therefore, every one time that Mimas orbits, the ring particles will complete two orbits. Since a 2:1

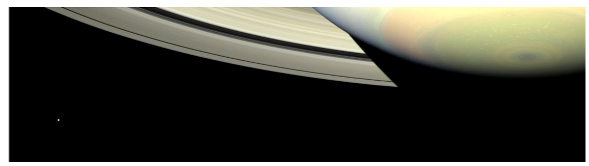

Figure 1: Cassini division and the orbiting moon Mimas [2].

resonance means that

$$T_{particle} = \frac{1}{2} T_{Moon},$$

this relationship can be used to find the 2:1 resonance position

$$\sqrt{\frac{4\pi^2}{GM_S} r_p^3} = \frac{1}{2}\sqrt{\frac{4\pi^2}{GM_S} r_M^3}$$

where M_s is the mass of Saturn, r_M is the position of the moon, and r_p is the position of resonance. This equation can be massively simplified to

$$r_p = \left(\frac{1}{2}\right)^{\frac{2}{3}} r_M.$$

Thus, the generalized resonance equation for any resonance $(m : n)$ where m and n are integers is given by

$$r_p = \left(\frac{n}{m}\right)^{\frac{2}{3}} r_M,$$

which works for both resonances inside and outside the orbit of the moon [3]. I was able to duplicate a 2:1 resonance in my model and tested how changes in the mass ratio of Saturn to moon to particle effects the gap formation.

B. Shepherding

There are two observed gaps in Saturn's A ring: the Encke gap and the Keeler gap. Many of the observed gaps and tiny

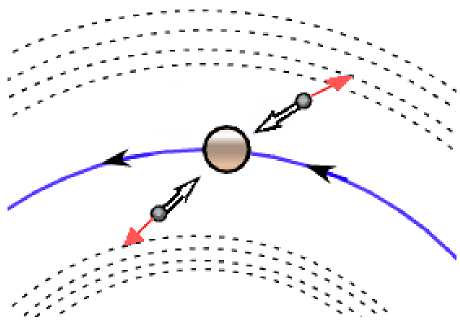

Figure 2: The attractive gravitational force from the moon results in particles being forced away from the moon's orbit. The black arrows represent the gravitational force from the moon while the red arrows indicate the net motion of the particles. This process, called shepherding, is explained below.

ringlets are caused by the forces from tiny embedded moons in the ring particles. The Encke gap is 325 km wide and created by the moon Pan [1]. Pan creates a gap by forcing particles away from its orbit, a process known as shepherding. A tiny ringlet is also observed in the center of the Encke gap. These particles follow the orbit of Pan and are in a 1:1 resonance allowing them to stay in the center of the gap. I also modeled shepherding by introducing an embedded moon into the ring system and observed the effects that changing the mass ratio of the moon and planet had on the size of the gap [1].

Shepherding is the phenomenon of moons pushing ring particles away from their orbit. As seen in figure 2, an embedded moon will attract ring particles as it orbits. However, the effective movement of the particles is away from the moon. Particles orbiting interior to the moon will be moving at a faster velocity. Therefore, the moon's gravitational force will pull the particle outward and backward. This will cause the particle to lose some of its speed and it will fall closer to the planet. As it falls closer to the planet it will pick up speed again and its new orbit will be further inward from the embedded moon.

Particles orbiting exterior to the moon will be moving at a slower velocity than the moon. Thus, the moon's force will pull the particle inward and forward. This will cause the particle to gain speed which will make its orbit bigger and it will move away from the embedded moon. As it moves further outward, the particle will start to slow down and its new orbit will be further outward relative to the moon. Therefore, even though the moon is attracting the ring particles, the combination of the planet's gravitational force and the moon's gravitational force actually causes the particles to be pushed away from the embedded moon [4].

C. Orbit of Moon

When creating the program, all of the forces had to be taken into account. The orbit of the moon could simply be found by using centripetal force and Newton's law of gravitation since it was assumed that the moon traveled in a circular orbit. Therefore, the velocity of the moon can be found by setting the centripetal force equal to the gravitational force

$$M_M \frac{v^2}{r_M} = F_M = \frac{G M_S M_M}{r_M^2},$$

where M_M is the mass of the moon, M_S is the mass of Saturn, r_M is the distance between Saturn and the moon, v is the moon's velocity, and G is the gravitational constant [5]. By solving for the velocity it can be used to obtain the moon's period

$$T_M = \sqrt{\frac{4\pi^2}{G M_S} r_M^3},$$

which is just Kepler's Third Law. Then, by finding the angular frequency of the orbit

$$\omega_M = \frac{2\pi}{T_M},$$

where $\omega_M$is the angular frequency, the x and y positions of the moon can be found over time by the equations [5]

$$x = r_M \times \cos(\omega_M t),$$

and

$$y = r_M \times \sin(\omega_M t).$$

D. Orbit of Particles

Finding the orbit of the ring particles is more complicated than the moon because forces from both Saturn and the moon must be considered. To find the force from the moon and Saturn, their distances from the particle must first be calculated. Then by setting Newton's second law equal to Newton's law of gravitation, the acceleration of the particle can be found.

$$m\overrightarrow{a} = F = -\frac{GMm}{r^2}\hat{r}.$$

Therefore, the net acceleration of the particles from both moon and Saturn's forces is [5]

$$\overrightarrow{a} = \frac{G M_M}{r'^3}\overrightarrow{r'} + -\frac{G M_s}{r_s^3}\overrightarrow{r_s}.$$

where r' is the distance from the particle to the moon and r_S is the distance from the particle to Saturn.

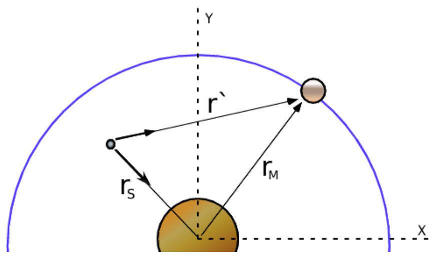

Figure 3: Both the moon and Saturn's gravitational force affect ring particles.

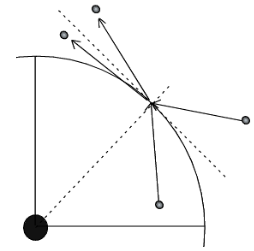

Figure 4: When particles collide, they lose some of their energy and their radial velocity decreases.

II. IMPLEMENTATION OF PROGRAM

The program simulated the rings by orbiting approximately 5000 particles around a central planet. Enough particles are needed to form a sufficient ring system; however if too many ring particles are used, it will take too long to run the program. Furthermore, collisions had to be included into the simulation because as the particles' orbits are perturbed by the orbiting moons, they will collide with other particles in the rings and their orbits will be changed.

The program begins by initializing the parameters to default settings. These parameters can then be changed by the user by using the graphical interface. Each particle is then advanced by the time step, dt, and its new position is found using fourth order Runge-Kutta integration. After the particles and the moon are advanced, the program checks to see if any particles are close enough to one another to collide. This is determined by the collision radius parameter. The new velocities of the colliding particles are calculated and the graphics are refreshed with the new positions of the particles.

A. Collisions

When the ring particles get close enough, they will collide inelastically. I defined my collisions so that the azimuthal velocity is conserved but the radial velocity is not. The elasticity of the collision is determined by the elasticity coefficient which is used to change the radial velocity of the particles. Only collisions between particles were taken into consideration and particles did not collide with Saturn or the moon.

To compute the new velocities of the colliding particles, the x and y position of the particles were first converted into r and phi coordinates by

$$v_r = v_x \left(\frac{x}{r}\right) + v_y \left(\frac{y}{r}\right)$$

and

$$v_\varphi = -v_x \left(\frac{y}{r}\right) + v_y \left(\frac{x}{r}\right),$$

where r is the distance between Saturn and the particle, v_r is the radial velocity, and v_φ is the azimuthal velocity. They are then boosted to their center of mass reference frame by finding their average velocity and subtracting this average from their original velocities. These center of mass velocities are then multiplied by the elasticity coefficient and their direction is changed by multiplying them by -1. If the elasticity coefficient is less then 1, the collisions will be inelastic. If the elasticity coefficient is equal to 1, they will collide elastically. The velocities are then taken out of the center of mass reference frame and converted back into x and y coordinates.

III. DATA AND ANALYSIS

There are many parameters and many different possibilities of phenomenon to test with the program. I tested how varying the elasticity coefficient changed the formation of the Cassini division and how changing the mass ratio of Saturn to the moon effected gap size and formation.

A. Collision elasticity

The effects of collision elasticity on the formation of a 2:1 resonance gap were tested by varying the elasticity coefficient while keeping all other parameters constant. The moon orbited at a distance of 150 with a mass of 1,000 and Saturn had a mass of 1,000,000. Over time, a gap formed in the rings at a distance if 94 to 100 from Saturn. Since the position of a 2:1 resonance is 94, the inside edge of the gap is controlled by a 2:1 resonance with the orbiting moon. However, as the elasticity coefficient was increased, it took longer for the gap to form.

To see how the coefficient effected the speed at which the gap formed, the number of particles at a distance of 94 to 98 from Saturn was averaged and plotted to see how this average number of particles in the gap area changed over time as seen in figure 5. The curves were then fit with exponentials to see how quickly the gap formed over time. For an elasticity of 0, the number of particles in the gap decayed quickly at a rate of 0.018 ± 0.001 per period of the moon while an elasticity of 0.9 caused the particles to decay at a slower rate of 0.011 ± 0.001 per period of the moon, see table I. Therefore, it will take longer for a gap to form when the collisions have a higher elasticity.

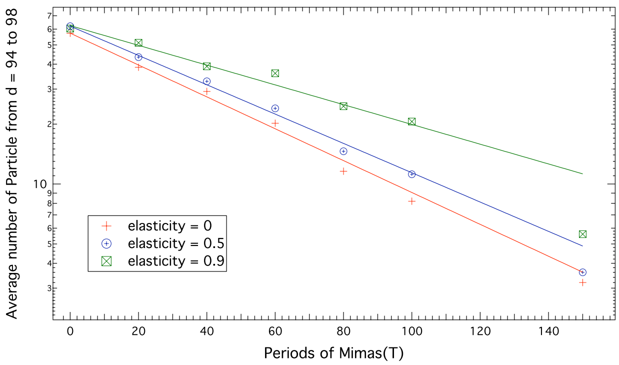

Figure 5: The number of particles at a distance from Saturn of 94 to 98 were averaged and plotted versus time for elasticities of 0, 0.5, and 0.9. The data was fit by exponentials where the equation for the lines are $e^{T/\tau}$. As the elasticity of the collisions increased, the number of particles in the gap decayed at a slower rate.

Table I: Comparison of rate of gap formation to collision elasticity. The units of τ are in Periods of the moon.

Elasticity	τ^{-1}	Chi Squared
0	0.018 ± 0.001	9.02
0.5	0.017 ± 0.001	8.38
0.9	0.011 ± 0.001	59.9

B. Embedded moons and resonances

I also ran simulations with a moon embedded in the ring particles to observe the effects of shepherding. The mass ratio of Saturn to the moon was varied to see how the mass affected observed resonances and the gap caused by shepherding. Collision elasticity was kept at 0.5 and the moon orbited Saturn at a distance of 150. The data was plotted using an image plot in Igor that graphed the radial distribution of the particles over time. As can be seen in figures 6 and 7, the embedded moon successfully clears particles away from its orbit. Furthermore, it was observed that particles at the same distance away from Saturn as the moon were not pushed away and formed a ringlet in the center of the gap.

Table II shows the edge position of the gaps and their corresponding resonance. When the mass of Saturn was increased from 100,000 to 1,000,000 and the moon's mass was held at 100, the moon cleared a smaller gap around its orbit and more resonances can be seen, as shown in figure 7. The 2:1 and 1:2 resonances took longer to appear with the larger mass ratio and were not as large.

Since the embedded moon did not clear as large a gap around itself as it did when the mass ratio was smaller, more resonances could be seen in ring particles closer to the moon that did not have the opportunity to form before. For all mass ratios, it was observed that the resonances exterior to the moon form the exterior edge of gaps while resonances interior to the moon form the interior edge of the gaps. I then ran the simulation again, this time keeping the mass of Saturn at 1,000,000 and changing the mass of the moon to only 10.

At this larger mass ratio, the moon did not have enough

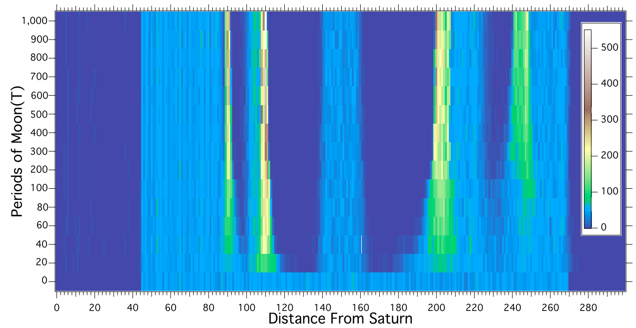

Figure 6: The effects of an embedded moon at a distance of 150 with a mass of 100 orbiting a planet of mass 100,000. The color scale corresponds to the number of particles at a given distance from Saturn where dark blue is no particles present and white is approximately 550 particles present.As time progresses, the formation of the gaps can be observed.

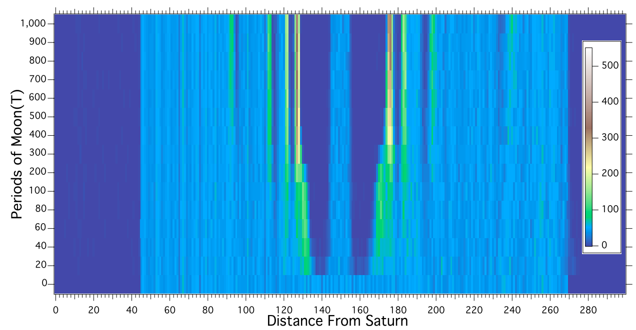

Figure 7: The effects of an embedded moon at a distance of 150 with a mass of 100 orbiting a planet of mass 1,000,000. A smaller ringlet can be seen in the center as well as gaps at 8 different resonance positions.

gravity to cause resonance gaps to form over the observed time period. The effects of shepherding could still be seen and the moon did clear a small gap around its orbit. During all three runs, when the moon shepherded particles away from its orbit, particles directly in the moon's orbit stayed forming a small ringlet in the center of the gap. The gap size formed by the moon and the ringlet width changed depending on the mass ratio of Saturn to the moon. These data are

Table II: Position of gaps formed when the moon's mass is 100 and Saturn's mass is 100,000 and 1,000,000

Resonance	Gap edge : Saturn's mass = 100,000	Gap edge : Saturn's mass = 1,000,000
2:1	92	94
3:2	112	114
4:3	—	123
5:4	—	129
4:5	—	174
3:4	—	182
2:3	198	196
1:2	238	236

Table III: Comparison of Mass ratios to gap size and ringlet width of embedded moons.

	Saturn's Mass	Moon's Mass	Gap size	Ringlet width
1	100,000	100	86	20
2	1,000,000	100	45	10
3	1,000,000	10	17	5

summarized in Table III. The smaller the mass ratio between the moon and Saturn, the larger the gap size and the larger the width of the ringlet formed in the gap.

IV. CONCLUSION

I was able to successfully create a computer program to simulate a planetary ring system. By varying the collision elasticity I studied how collisions effected the speed at which a gap formed in the particles at a 2:1 resonance position. It was determined that the more energy that is lost during collisions, the quicker the gap will form. By changing the mass ratio of Saturn to the moon, a total of eight different resonances were also observed. Furthermore, the moon was embedded in the ring particles and the effects of shepherding were succesfully simulated. The greater the mass ratio between Saturn and the moon, the smaller the gap formed by shepherding. Also, when the moon shepherded particles away from its orbit, some particles remained in the center of the gap, caught in the same orbit as the moon and formed a tiny ringlet. This occurrence is consistent with observations of the Encke Gap in Saturn's A ring. The Encke gap is cleared by the small moon Pan, however a small ringlet remains in the center of the gap [4].

When the effects of the 2:1 resonance cleared out the gap in the ring particles, particles were forced out of the gap toward the planet and the interior edge of the gap was determined by the resonance. This is consistent with observations of the Cassini division between Saturn's A and B ring. The 2:1 resonance with Mimas controls the interior edge of the Cassini division and there are a greater number of ring particles at this edge [1]. All four resonance positions interior to the orbit of the moon that were observed determined the interior edge of the gap causing particles to collect at that edge. However, resonant positions exterior to the orbit of the moon determined the exterior edge of gaps and caused particles to collect on the exterior edge. I believe that the difference in the way the gaps are formed interior to the moon as to those formed exterior to the moon is caused by the shepherding effect. The shepherding effect of moons causes particles that are interior to their orbit to be pushed further in and particles exterior to their orbit to be pushed further out. Therefore, when the gravitational tugs from the moon build up over time at the resonance positions, this causes particles interior to the moon to be slowed down and their orbit will become smaller causing them to fall into an orbit closer to the planet. Particles exterior to the moon will be speed up by the gravitational force from the moon causing the size of their orbit to increase and they will move further away from the planet. Therefore, whether it be resonance or shepherding, the attractive gravitational force of the moon in effect pushes ring particles away.

Overall, by simply including gravity from Saturn and the moon and collisions of particles, the program was able to successfully simulate both the effects of resonance and shepherding in ring systems. Tests were conducted on the effects of collision elasticity on the speed of gap formation and it was found that the less elastic the collisions were, the greater the rate at which the particles evacuated the gap. Furthermore, the effects of embedding moons into the ring particles and changing the Saturn to moon mass ratio were studied. I noticed that my simulation modeled observed aspects of Saturn's ring system like resonance gaps and shepherded gaps correctly. It was also found that the larger the mass ratio, the smaller the gap formed by shepherding and the smaller the ringlet formed in the gap. Finally, I was able to show that resonance determined the exterior edge of gaps forming outside of the moon's orbit while resonances determined the interior edge of gaps forming inside of the moon's orbit Thus, the attractive force of the moon actually pushes particles away to form gaps by resonance and shepherding.

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