

Determining The Surface Tension Of Water Via Light Scattering

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(Dated: May 10, 2007)

The diffraction pattern created by the surface waves of water was analyzed and used to find an experimental value for the surface tension of water. By creating small surface waves in a dish of distilled water, an interference pattern was observed when a laser light was incident at a low angle to the surface of the water. From analysis of the interference pattern from the laser beam, the wavenumber of the surface waves was measured. From the relationship between the density of the water, the angular frequency and the wavenumber, the surface tension of water at 22°C was calculated. Three different frequencies of oscillation were used to measure and calculate the surface tension. A graph of the angular frequency squared times the density $\omega^2\rho$ versus the wavenumber cubed q^3 of the surface waves for the three frequencies used was plotted and the slope used to calculate the surface tension. This surface tension was measured to be $(75.3 \pm 1.7) \times 10^{-3}$ N/m, roughly 3.4 % greater than the accepted value at that temperature.

I. INTRODUCTION

Surface tension is a property of fluids that causes tension or contraction of the fluid molecules near the surface or plane of interaction with another material or fluid. The molecules on the interior of the liquid experience relatively equal forces in all directions from the surrounding molecules. The water molecules at the surface experience a force from fewer surrounding water molecules. The change in density at the plane of interaction however, means that there are fewer molecules of air above the surface of the water than there are water molecules below the surface of the water. These surface liquid molecules thus experience a smaller force of attraction from the gas molecules than from the interior liquid molecules and so the surface liquid molecules are drawn toward the body of the liquid [1]. Fig. 1. shows the forces of interaction of both the gas and the liquid molecules.

II. THEORY

In my experiment, controlled sinusoidal waves were created on the surface of distilled water in a dish. This sinusoidal air-water interface acted as a diffraction grating for a beam of laser light incident at a grazing angle, causing interference patterns that could be observed at a range of frequencies for the surface waves.

The relationship between the angular frequency ω and wavenumber q of surface waves in a liquid depends on the surface tension [2]. Klipstein, Radnich and Lamoreaux derive a general expression for the surface tension of a liquid

$$\sigma = \frac{\omega^2 \rho}{q^3}, \quad (1)$$

which assumed no frictional or rotational forces of the liquid. q is the wavenumber of the surface waves and $\omega = 2\pi f$ is the angular frequency of the surface waves. The surface tension σ is a measure of the amount of energy

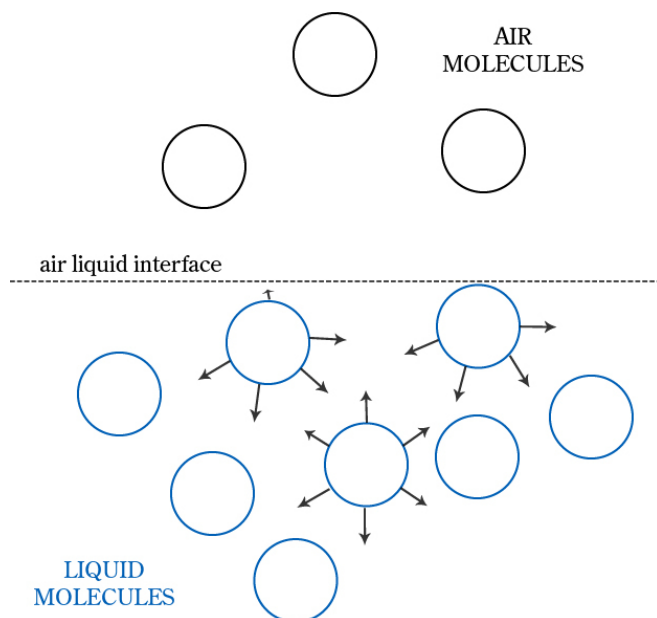


FIG. 1: A schematic of the molecular forces between the water molecules. The schematic was modeled from Isenberg [1] and shows a smaller force of attraction of water molecules near the surface where air molecules are further away than the liquid molecules.

needed to increase the area of the surface of a liquid by one square unit of distance [3]. An expression for the wavenumber of the surface waves produced is given by Weisbuch and Garbay [4] in their simple experiment

$$q = \frac{2\pi}{\lambda} \sin \left[\frac{r}{2} \right] \left(\sin \left[\theta - \frac{r}{2} \right] + \sin \left[\theta + \frac{r}{2} \right] \right), \quad (2)$$

where λ is the wavelength of the light used, θ is the angle of reflection of the incident beam, and r is the angle between the incident beam and the first maximum of the interference pattern.

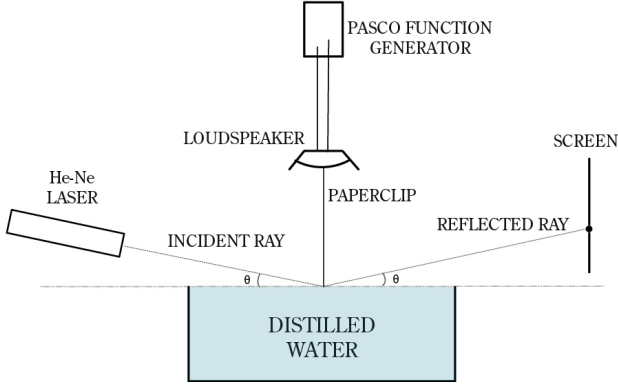


FIG. 2: A schematic of the setup of the apparatus. The container of distilled water rests on an air table with the laser beam incident near its center. The Pasco function generator and the He-Ne laser were isolated from the table.

III. EXPERIMENT

A. Set up

A paper clip attached to a loudspeaker was used to create sinusoidal surface waves in a dish of water. Incident light from an Helium-Neon laser was then aimed at a small angle, approximately 4° , grazing the surface of the water. This incident light reflected onto the wall used as a screen for the interference pattern created. The wavenumber of the water waves can be determined by measuring the maxima of an interference pattern from the diffraction of the light off the surface of the water. The air-water interface acts as a diffraction grating for the laser light. The experimental apparatus was set up as shown above in Fig. 2.

The He-Ne laser was set up on a stand behind the air table. A loudspeaker with a straightened paper clip attached was set up on a clamp stand beside the air table with the tip of the paper clip just touching the surface of the water. The circular dish was then adjusted beneath the loudspeaker so its attached paper clip was just touching the tip of the water directly above the center of the dish. The loudspeaker was attached to a Pasco function generator, which was tuned to oscillate the cone of the loudspeaker sinusoidally creating surface waves from the oscillating paper clip. The He-Ne laser was turned on with its beam skimming the surface of the water creating a single incident ray reflected on the screen. The function generator was turned on to oscillate the loudspeaker sinusoidally at a frequency of approximately 100 Hz. This frequency f was now the frequency of oscillation of the water waves in the dish. The function generator was adjusted and an approximate range of 50 Hz to 400 Hz was found to create an interference pattern at the highest amplitude of the generator. To determine the surface tension of the water, the wavenumber was measured and the frequency of oscillation recorded from the function generator as described below.

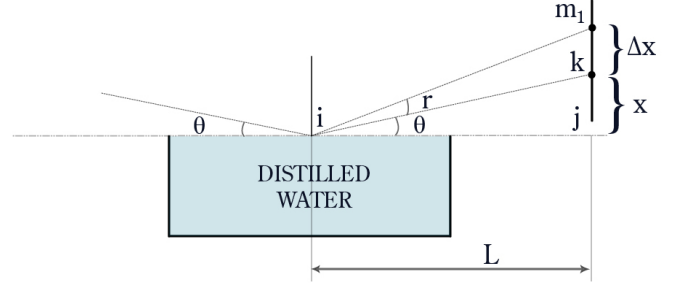


FIG. 3: A schematic of the distances and the angles of the reflected ray and the interference pattern. m_1 is the first maximum of the interference pattern, k is the reflected beam, j is the horizontal level on the screen of the surface of the water and i is the point of incidence on the surface of the water.

B. Measuring the wavenumber of the waves

From Eq. 2, the angles θ and r were measured and used to calculate the wavenumber of the surface waves. The angles θ and r are defined by

$$\theta = \tan^{-1} \left[\frac{x}{L} \right] \quad (3)$$

and

$$r = \tan^{-1} \left[\frac{\Delta x}{L} \right], \quad (4)$$

where L is the horizontal distance from the point of incidence of the laser light on the surface of the water to the screen. Δx is the distance between the reflected beam and the first maximum on the interference pattern on the screen as shown above in Fig. 3.

1. Measuring angle θ

Using Eq. 3 the angle of incidence was calculated. To accurately measure the horizontal distance from the point of incidence i to the screen, the beam of a laser pointer was leveled with the surface of the water creating a horizontal line tangent to the surface of the water. The beam was adjusted so it was perpendicular to the screen and parallel to the air table. The point on the screen of the laser beam j was then marked with tape on the wall. The point of incidence i was found via careful estimation using a sheet of paper placed in the path of the beam and marking the point at which the beam and the surface of the water met. A tape measure was then used to measure the distance L from i to j . The tape measure was used to measure the vertical distance x from j to the point of reflection k of the He-Ne laser beam. The angle θ was then calculated by Eq. 3 using these values of x and L .

2. Measuring angle r

Using a carpenter's level, the tape measure was attached vertically to the screen and positioned next to the interference pattern. The distance Δx from k to the first maxima m_1 was now observed. Unavoidable vibrations in the apparatus due to vibrations in the lab environment caused the interference pattern to be slightly jumpy. Sharp digital snapshots of the pattern were taken to capture this jumpy pattern at one frame in time. From the snapshots, I was able to take more precise readings of Δx . Vector based graphing software, ADOBE ILLUSTRATOR CS2 was then used to draw perpendicular lines from the middle of the reflected beam k to the tape measure. The centers of reflected beam and the first maximum m_1 were used as the reference point for those beams and the blurred regions disregarded. This was done for the middle of the first maxima m_1 both above and below the reflected beam and an average of the distances Δx used. The angle r was then calculated via Eq. 4.

C. Determining the surface tension of water

The angular frequency of surface waves is given by the equation

$$\omega = 2\pi f \quad (5)$$

where f is the frequency of the Pasco function generator. The frequency f used was 361.5Hz. Eq. 1 was combined with Eq. 2 and Eq. 5 to get an expression for the surface tension with respect to the frequency and the angles θ and r from the interference pattern

$$\sigma = \frac{4\pi^2 f^2 \rho}{\left(\frac{2\pi}{\lambda} \sin\left[\frac{r}{2}\right] \left(\sin\left[\theta - \frac{r}{2}\right] + \sin\left[\theta + \frac{r}{2}\right]\right)\right)^3} \quad (6)$$

The wavelength λ of light from the He-Ne laser used is 632.8nm. By observing the interference pattern and measuring Δx for the the interference pattern created, the surface tension was calculated. This procedure was repeated at the frequencies 192.7Hz and 125.3Hz to acquire three values of the surface tension. Measurements for angles θ and r as well as the distance x were retaken for each frequency used to maintain accuracy after shifting the equipment. Distilled water was used in the experiment as it has a known density of 1000 kgm^{-3} .

IV. RESULTS AND DISCUSSION

The interference patterns formed from the surface waves created at different frequencies are shown below. The figures 4, 5 and 6 show the increase in distance between reflected beam and the first maxima both above and below this reflected beam. To read the distances of between the reflected beam and the first maxima, a

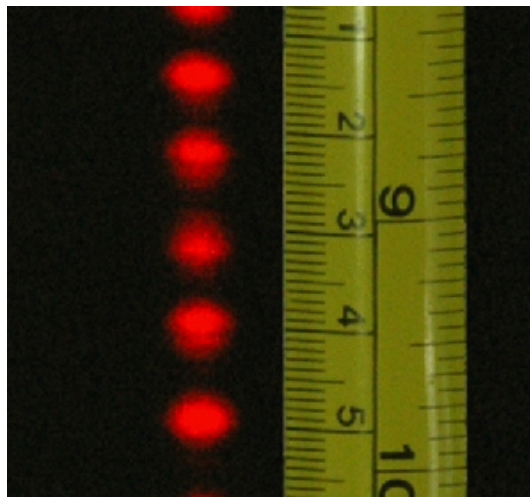


FIG. 4: Interference patterns for frequency 125.3 Hz.

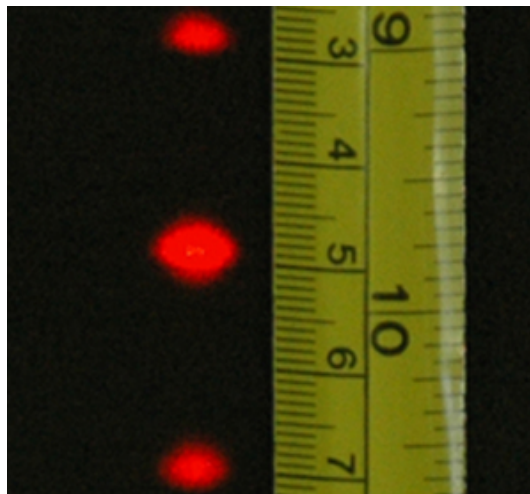


FIG. 5: Interference patterns for frequency 192.7 Hz.

digital snapshot was taken of the pattern with a tape measure running parallel to the pattern. This significantly reduced error reading the distance however the vibrations caused slightly blurry pictures which were difficult to analyze accurately. At frequency 125.3 Hz, the reflected beam was not distinct from the maximas. The frequency of oscillation was reduced to 0 Hz via the Pasco function generator and a snapshot of the reflected beam was then taken to verify its location as shown in Fig 7. The function generator was then readjusted to create the low frequency waves being analyzed before.

The frequencies of oscillation f are shown below with their corresponding values for the angle r and thus wavenumber q in Table I. The angle θ was calculated to be 4.05° for each frequency used.

A list of the frequencies and their corresponding wavenumber for the surface waves is shown below in Table I. Via Eq. 1 experimental values for the surface tension of water were calculated for the frequencies used.

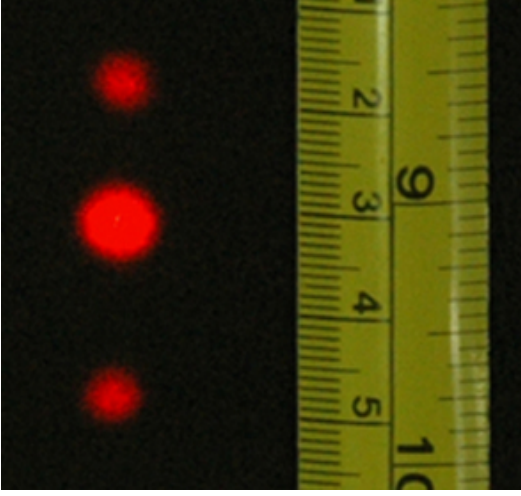


FIG. 6: Interference patterns for frequency 361.5 Hz.

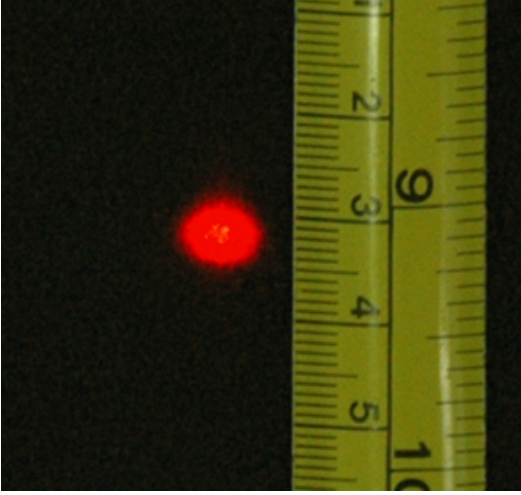


FIG. 7: Single reflected beam at $\theta = 4.05^\circ$.

The surface tension has the unit of N/m and is the force required to separate two molecules of the fluid by one unit length. The list of the accepted values of the surface tension of water at different temperatures is shown below in Table III[5]. From Table III an estimated value of surface tension at 22° was calculated to be $72.5 \times 10^{-3} N/m$.

The experiment was conducted in an optics lab isolated from the high levels of vibrations elsewhere, at a temper-

TABLE I: Table of frequency and wavenumber for the surface waves of water.

Frequency f (Hz)	Δx (m)	Angle r°	q (1/m)
361.5	0.022	0.349	4.082×10^{-3}
192.7	0.014	0.222	2.712×10^{-3}
125.3	0.010	0.159	1.943×10^{-3}

TABLE II: Table of angular frequency cubed and surface tension of water.

ω^2 (Hz ²)	q^3 (1/m ³)	σ (N/m)
5.159×10^6	6.804×10^{10}	75.83×10^{-3}
1.466×10^6	1.995×10^{10}	73.48×10^{-3}
6.198×10^5	7.330×10^9	84.55×10^{-3}

TABLE III: Table of surface tension values of water and temperature.

Temperature($^\circ$ C)	Surface Tension(N/m)
0	75.6×10^{-3}
20	72.8×10^{-3}
60	66.8×10^{-3}

ature of 22° C. An average value of $(75.3 \pm 1.7) \times 10^{-3} N/m$ was obtained from the three frequencies used. This value was 3.4 % greater than the accepted value of surface tension of water at 22° C. A straight line was used to fit the plot of $\omega^2 \rho$ versus q^3 for the three frequencies.

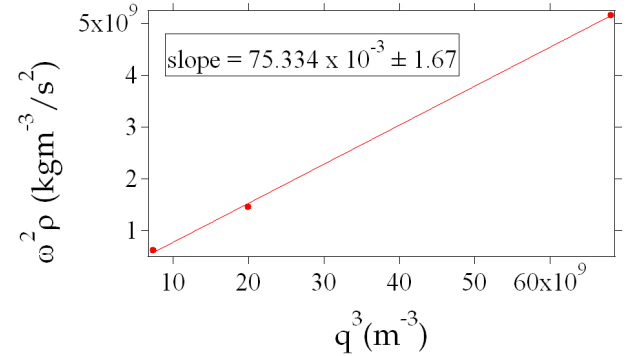


FIG. 8: Plot of the density times the angular frequencies squared versus the wavenumbers cubed.

A. Error analysis

Due to the high sensitivity to vibration of the apparatus, the frequency of oscillation of the water was interrupted and the diffraction of the incident laser light was not ideal. Vibration of the diffraction pattern on the screen indicated vibrations of the lab environment and apparatus and thus caused the reflected beams to oscillate vertically about the screen.

The greatest amount of error occurred when trying to correlate the frequency of oscillation to the angle r between the reflected beam and the first maxima. Because of the relatively small distance L between the screen and the point of incidence i , a small change in frequency f , resulted in a very small change in the separation of the

first maximum with the reflected beam Δx . With the existing difficulty of accurately reading the values for the distance Δx from the tape measure, and with the limitation in equipment available, up to a 30 Hz change in the frequency resulted in an apparent 1mm change in Δx . Since the measurements for Δx made were in the region of 0.010m to 0.022m, and the lowest possible unit of measurement was 1mm, the estimated uncertainty for Δx was ± 0.1 mm. The accuracy of this experiment's measurements could have been increased by using a screen four or five times the current distance L of the screen from the point of incidence i .

V. CONCLUSION

A sinusoidally oscillating speaker with an attached paper clip was used to create surface waves in a filled dish of distilled water. Incident laser light was reflected off the surface of the water near the center of the container and the wall used as a viewing screen. The air-water interface acted as a diffraction grating for the laser light, and interference patterns were observed and used to determine the wavenumber of the surface waves. These interference patterns were then used to find the wavenumber of the

surface waves and thus the surface tension. With the relationship between the density of the water ρ , the angular frequency ω and the wavenumber q , the surface tension $\sigma = \omega^2 \rho / q^3$ of water at 22°C was calculated. Three different frequencies of oscillation, 361.6Hz, 192.7Hz and 125.3Hz were used to measure and calculate the surface tension. A graph of the angular frequency squared times the density $\omega^2 \rho$ versus the wavenumber q of the surface waves for the three frequencies used was plotted. A slope of the linear fit of the curve determined from the derived equation for the surface tension. The surface tension was calculated to be $(75.3 \pm 1.7) \times 10^{-3}$ N/m. Despite the high value of uncertainty estimated due to the difficulty in measuring small changes in Δx , I obtained an experimental value of surface tension with only roughly 3.4 % discrepancy.

VI. ACKNOWLEDGMENT

I would like to acknowledge the help of Dr. Susan Lehman, Dr. John Lindner, Daniel Tremblay, Jon Rosch and Michael Zappitello for their assistance and guidance through my lab procedure.

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