Surface Tension Waves

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This experiment is an observation of the nature of a material's surface tension by examining the interaction of its surface with incident light. A theoretical derivation for the system predicted that the angular frequency squared goes as the third power of the wavevector. Laser light was incident, at an angle, to a sample of water. Oscillations on the surface of the water were created at distinct frequencies. The resulting surface waves reflected and diffracted the laser light. The angular separation between the central maxima and primary bright spots was captured at a distant screen. It was observed that the frequency followed a powerlaw strongly indicating the theoretical 3/2-order wavevector relationship. Further, the surface tension of water was found to be σ = (51.9 ± 9.4) dynes/cm which is less than that of pure water (σ = 72.75 dynes/cm).

INTRODUCTION

The ordinary method for determining a material's surface tension (by measuring the force required to remove a blade from a material) is only applicable to static situations. That is, it cannot be used to continuously monitor surface tension and is not feasible near the critical point.[1] A fairly reliable alternative is presented through optics. The deformation of a surface is controlled by its physical characteristics, including surface tension and density. By creating movements of known frequency on the surface of water, a series of concentric waves are established and propagate through the medium to the boundary. For sufficiently high frequency, the distance between wave-fronts is quite small. So small, in fact, that the waves may be seen as a multi-slit grating which cause Fraunhoffer-style diffraction. The information present in the diffracted pattern can be gathered at a distant screen. Following general approximations, a simplified relationship states that the driving frequency squared of surface disturbance goes as the wavevector cubed. Also, a value for the surface tension of water can be analytically deduced. For a complete theory derivation, see Klemens' *Dispersion relations for waves on liquid surfaces* (Am. J. Phys. 52, p. 451 (1984).)[2]

In this investigation, the diffraction will be observed and tested against the theoretical prediction, that $\omega = \sqrt{\left(\frac{\sigma}{\rho}\right)} q^{3/2}$. If the theory proves valid, water's surface tension will be found.

EXPERIMENT

A Uniphase model 155 SL He-Ne laser was aimed at a plane mirror that redirected the light toward a distilled water reservoir (in petri dish, at room temperature). Different incident angles were tested and 10° was selected as it allowed for the largest range of oscillation frequencies to produce an interference pattern. The location of the petri dish was fixed from the distant screen. A theoretical position for the reflected beam (given by l=153.65 cm) was calculated and the plane mirror was adjusted to ensure a precise angular alignment of θ =10°. A Pasco model PI-9587A Digital Function Generator and Amplifier was attached to a Realistic dual cone speaker system. A metal rod was attached to the speaker. The height of the petri dish was adjusted using a lab jack so that the metal rod was barely in the water (<1mm). An alcohol thermometer was used to measure the room temperature (T=20.5°C).

The function generator was dialed to the lowest frequency that gave observable diffraction (426 Hz). The diffraction information was taken at the distant screen using a pen to mark on paper the central maxima and the primary bright spots on either side of it. The amplitude setting was adjusted (increased for increasing frequency) to give the clearest diffraction pattern. Data were gathered at 50 Hz increments for each subsequent frequency, to the upper limit of 875 Hz. In a second experimental trial, additional data were gathered over the same range (550, 650, 750 Hz) for the same angle θ. Their values were in agreement with the first set and so the two runs were incorporated into a single set of data.

ANALYSIS AND INTERPRETATION

The task of this experimental investigation was to first confirm the existing theory relating frequency to wavevector, and then to determine a value for the surface tension of water. Accordingly, diffraction data were gathered. To analyze the data, they were scanned using an Epson Scanner connected to a Macintosh G4 tower. Adobe Photoshop graphic software was used to capture scans of all images (100%, 300 dpi, grayscale). See Figure 2.

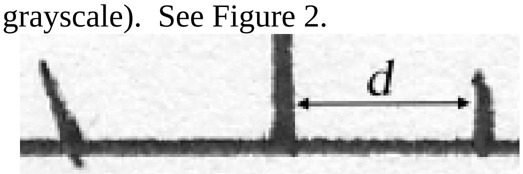

Figure 2: An example of scanned data taken at 550 Hz (enlarged).

The distance between the primary bright spots on either side of the central maxima was measured, in pixels, using the "Selection" tool. Pixel selection information was given in the "Info Window." For each data point, this double distance was recorded. The double distance (of one separation d) was halved for a mean distance, d, in pixels. The discrepancy between distances is due to the resolution with which one marks the positions of bright spots. The double measurement was taken to reduce this sort of error. To convert from pixels to centimeters, a 10 cm line was scanned. A plate scale of 1187 pixels per 10 cm, or 118.7 pixels/cm, was determined. This method of data analysis gave greater precision than ruler measurement would have allowed.

From the geometry, the diffracted angle *r* is (for small *r*):

$$r = \tan^{-1}\left(\frac{d}{l}\right) = \frac{d}{l}$$

Computations were completed by Igor's spreadsheet calculation capabilities. The angle r measurement was used to calculate the wavevector q. A plot of ω vs. q was created. See Figure 3.

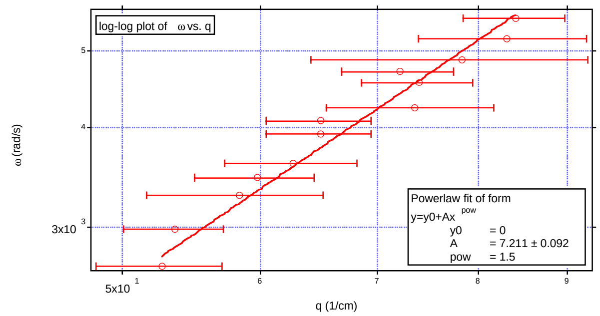

Figure 3: The log-log plot of ω vs. q.

The system does appear to be displaying the 3/2 power relationship. Noting that y0=0, the powerlaw equation takes the form:

$$y = Ax^{pow}$$

This is analogous to $\omega = \sqrt{\left(\frac{\sigma}{\rho}\right)} q^{3/2}$ with

$$\sigma = \rho A^2 = 0.99821(g/cm^3) \cdot \left(7.21 \pm 0.09(\sqrt{cm}/s)\right)^2$$

, where the value of A comes from the fit in Fig.

3.

The data for water was taken from the CRC Handbook of Chemistry and Physics.[3] When evaluated, the above relation gives the surface tension, σ=51.9 dynes/cm.

Given the physical relationship, a plot of ω^2 vs. q^3 should be a linear relationship with slope σ/ρ. See Figure 4.

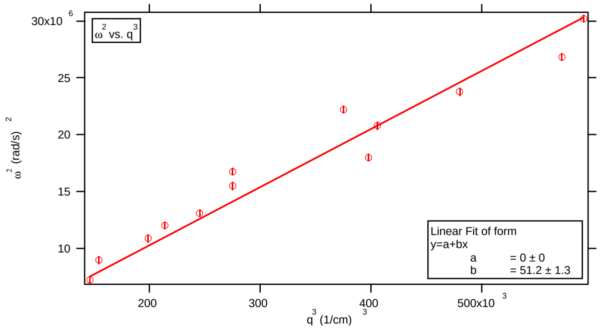

Figure 4: The linear plot of ω^2 vs. q^3.

From this slope, σ=51.1 dynes/cm. The closeness of the two surface tension values give further support to the 3/2 powerlaw relationship.

ERROR

Since the surface tension calculation is based on measured quantities, an inherent amount of uncertain error will be present in the final result. Error in measurement propagates through calculations such that the fractional errors add. Subject to:

$(\delta\theta/\theta)^2$	0.0025
$(\delta d/d)^2$	variable
$(\delta l/l)^2$	0.001446

The uncertainty in the d measurement is from the calculated discrepancy between d values, and varies for each data point. The uncertainties in l and θ are estimates and are 4 cm and 0.5°, respectively.

Allowing for uncertainty, it is clear that the data do fit a 3/2 powerlaw relationship.
Following that the maximum percent uncertainty is 18%, that level of uncertainty may be applied to the surface tension measurement. The measurement for σ becomes σ=(51.9 ± 18%) dynes/cm, or σ=(51.9 ± 9.4) dynes/cm. At 20°C (room temp was 20.5°C, approximated to 20°C for reference values), the surface tension of water is 72.75 dynes/cm. The experimentally determined value is within 70% of the accepted value (2.5 error bars).

CONCLUSION

The surface waves indeed cause a diffraction that is governed by the $\omega^2 \propto q^3$ relationship suggested by theory. For the frequency range 425-875 Hz, the surface waves produced a visible diffraction pattern. The determination of the surface tension value for water was not as accurate as desired. The deduced value may be the result of impurities in the water (for the surface tension and density would change) or other effects (the reflection of waves from the boundary of the petri dish, perhaps). By taking a second set of data which agreed with the first, the reproducibility of this experimental method was verified. The two graphical treatments of the data supported the theory and gave approximately equal values for water's surface tension.

ACKNOWLEDGMENTS

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[1] G. Weisbuch and F. Garbay, Am. J. Phys. 42, p. 355 (1979).
[2] P.G. Klemens, Am. J. Phys. 52, p. 451 (1984)
[3] D. R. Lide (ed.), CRC Handbook of Chemistry and Physics, 77th ed. (CRC Press, Boca Raton, 1974) p. **6**-8.