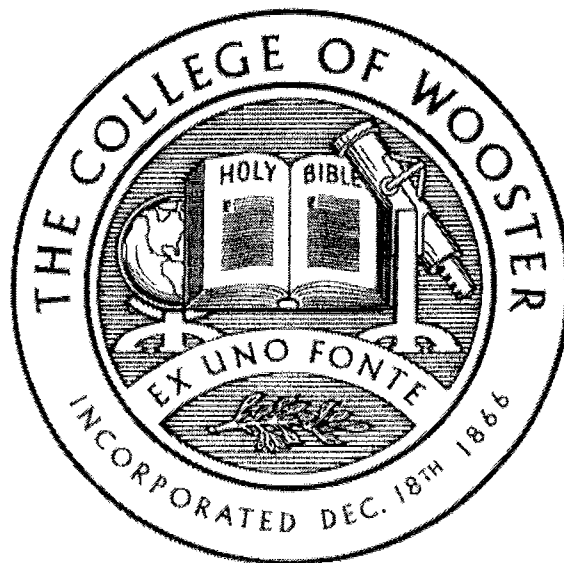


Hydraulic Jumps



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Abstract:

Hydraulic jumps are one of the most common physical phenomenon. They are seen around the world in rivers, lakes and even in the ocean. Most commonly though, they are known for being found in kitchen sinks. Hydraulic jumps rely on flow rate and water depth for their production. These effects are studied in this experiment and a basic theory of equations that could be applied are given.

Introduction:

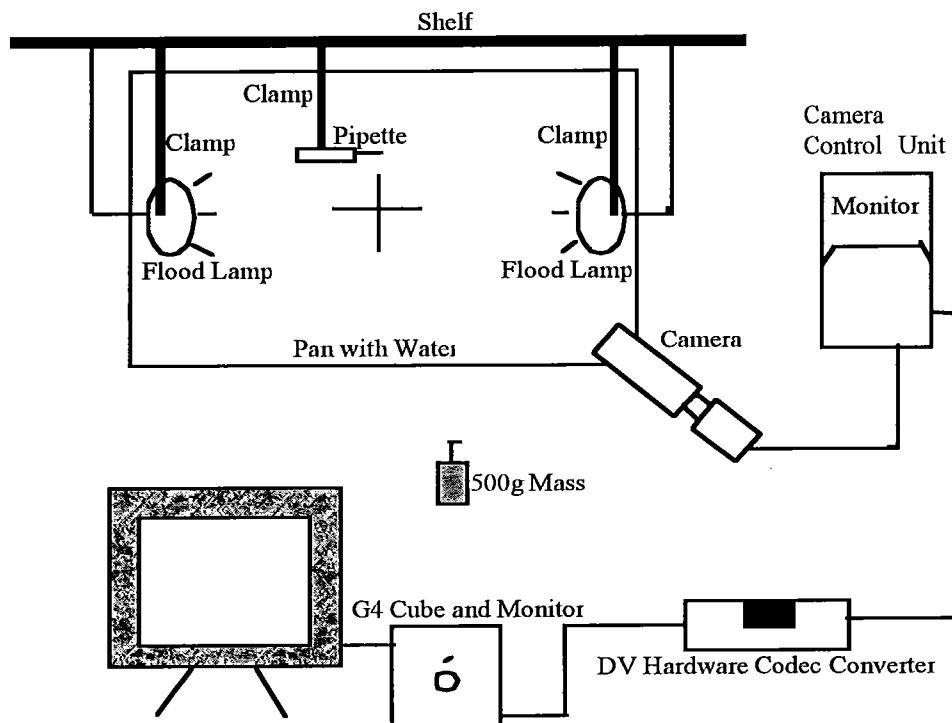
Hydraulic jumps are an incredibly common phenomena. It is likely owners of kitchen sinks have seen them quite frequently, given that that is the most common place for them to occur. Nevertheless, hydraulic jumps are usually studied because of their presence in nature. They are sometimes noticed in rivers and streams, and sometimes even in the ocean. Hydraulic jumps occur when a vertical flow of a fluid is imposed on a horizontal surface. In this experiment, the phenomenon of hydraulic jumps and the impact that the depth of a horizontal surface of water has upon them will be considered.

The knowledge of hydraulic jumps is relatively limited. According to "Hydraulic Jumps, Flow Separation and Wave Breaking: An Experimental Study¹," the reader is told that "It has also been studied scientifically, although relevant references and, more importantly, genuine understanding of the phenomenon, is very sparse." Scientists thus far, have divided the areas of a hydraulic jump into three regions; the supercritical region, the critical region and the subcritical region. Each of these regions defines an area of the jump and its behavior. These are regions by which the theory of this experiment was developed. The study of hydraulic jumps in continuously increasing. There is an increasing level of interest about bore occurrences around the world. A bore is an extreme hydraulic jump. They are often seen in oceans and large lakes as tidal bores. According to the Catalog of Worldwide Tidal Bore Occurrences and Characteristics, at least 67 tidal bores have occurred in at least 16 different countries². This study was written in 1988. Since then, it is a given that more bores have occurred. More knowledge of the incidences is being sought after because they are unpredictable and can be very harmful.

theory should come before this!

Experimental Setup:

Figure I:



- The experimental setup is given above. The camera is aligned at an angle which is approximately 45° to the cross in the pan. The depth of the water in the pan was varied, thus it is not listed here.

Equipment Used:

- ~ Power Mac G4 Cube and Monitor – Phys. 2785 and 2786
- ~ Olympus Encore Camera and Monitor, Olympus America, Inc. – Phys. 2781 and 2782
- ~ Two flood lamps
- ~ Fisherbrand Disposable Pipettes, 10 mL
- ~ Large metal pan for holding water
- ~ 500g mass
- ~ Four clamps used for equipment support
- ~ DV HardwareCodec Converter, Sony – Phys. 2798

Procedure:

A large pan was filled with different levels of water. For each of these levels, a pipette mounted above the pan, by means of a clamp, was used to disperse a smooth flow of water in order to create a hydraulic jump. To create the smoothest flow possible, a 500g mass was used to depress the pipette's plunger. Flood lamps were placed to either side of the pan to ensure that the camera received the proper amount of light needed for recording at 1000 frames per second. The camera was placed alongside the edge of the pan at approximately a 45-degree angle.

The Encore Olympus Camera captured the actual flows of water and/or drips first.

The camera's "shutter" menu on the control unit was adjusted as follows:

F/SEC RECORD – 1000
SHUTTER SPEED – 2X
TRIGGER POINT – 20%
F/SEC PLAY – 10.

On the "settings" menu, the brightness was adjusted to 59. This allowed for the most focused viewing setting. Each of these variables was kept consistent throughout the experiment. Items on menus which are not listed here, either could not be altered or were insignificant to the project at hand.

In order to prepare to capture the flow, the pipette needed to be filled with water. The pipette was filled in each run to the 10mL mark. To capture the flow, the "record" button on the camera was pushed, making sure that the camera was properly adjusted on the cross painted on the bottom of the pan. This ensured that the camera would be filming where the pipette water source was colliding with the pan water source. The pipette was secured directly above the cross. The 500g mass was then placed atop the plunger and released. The water then flowed from the pipette attempting to create a

hydraulic jump in the pan. Once the flow was complete, the “stop” button on the camera was pushed as quickly as possible. The footage was then played backwards in order to get to the beginning of the flow. As said earlier, the “shutter” menu was adjusted in order to play at ten frames per second in either the forward or the backwards play mode.

Once information was captured, it was moved to iMovie, where it could be analyzed. The outgoing information from the Olympus Encore and the Incoming to iMovie on the Mac G4 Cube, were converted using the Sony DV Hardware Codec Converter. Once footage had been played back on the camera, the “import” button in iMovie was pushed and the play forward button on the camera control unit was pushed. The camera sent the movie out also at ten frames per second while iMovie only had the capability to capture the movie at thirty frames per second. The movie could have been sent out at thirty frames per second, but this would not have allowed for "slow motion" play.

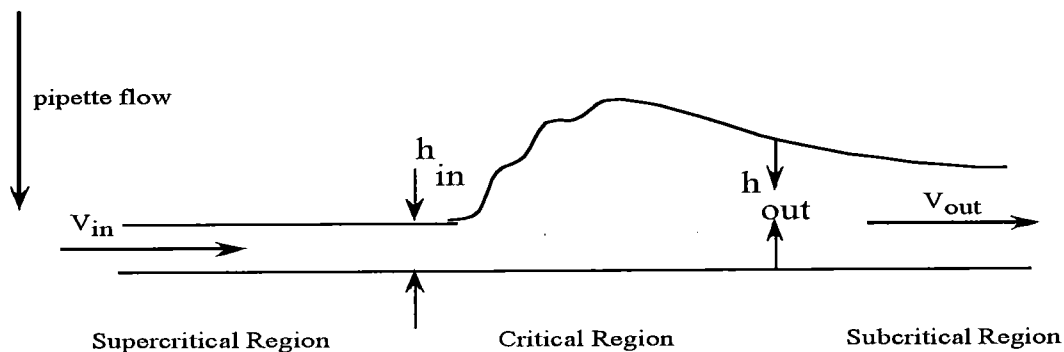
Theory:

In the introduction, the terms supercritical, critical and subcritical were introduced.

These refer to stages before a hydraulic jump occurs (the velocity of the flow in the pan is high), during the jump (an abrupt moment where turbulence occurs and the velocity lessens) and after the hydraulic jump (the velocity has decreased and the depth of the stream in the pan has increased), respectively. In Figure II, a depiction of the flow of water can be seen. The variables in Figure I (v_{in} , v_{out} , h_{in} , h_{out}) will be of use in equations used later in this section. v_{in} represents the velocity of the water as it hits the pan after exiting the pipette and v_{out} represents the velocity of the water during the subcritical

stage. h_{in} and h_{out} represent the depth of the water for the same stages, respectively (where the water is hitting the pan). The theory will be developed in radial (r), axial (z), and azimuthal (ϕ) coordinates. Nevertheless, the azimuthal coordinates will eventually drop out of the equation.

Figure II:



- The area labeled "supercritical region" is the area where the velocity is a high speed. Just after this region, the critical region begins. This is where an abrupt change occurs and the velocity decreases. This change is due to friction along the bottom of the pan in this experiment. The final stage has even further decreased velocity and a greater depth.

To demonstrate the change in regions, a theory which has been derived from three sources, will be used. Mainly, "The Circular Hydraulic Jump: Pursuit of analytical predictions³," but also "Air-Water Gas Transfer at Hydraulic Jump with Partially Developed Inflow⁴" and "Hydraulic Jumps, Flow Separation and Wave Breaking: An experimental study⁵." The only other important factors to recognize right now are the cross-sectional area of the pipette, a , the viscosity of the water, $\bar{\nu}$, and the radius of the jump or the area between the center of the pipette flow and the beginning of the supercritical region, R_{jump} . To begin, these variables need to be applied to give Reynolds's number².

$$Re = \frac{av_{in}}{\nu} \dots (1)$$

Why is this a vector?

~ ?

With the given information, we can make the conclusion that, presently, the only constant variables are v_{in} , R_{jump} as well as certain properties of the water such as viscosity, surface tension and gravity. From these terms, other parameters will need to be developed taking the constants' similarity into account.

$$\begin{aligned} R &= F_1(Re, Fr, We)a \\ v &= F_2(Re, Fr, We)v_{in} \end{aligned} \quad \dots (2)$$

The variables in (2) represent the Reynolds, Froude and Weber numbers which will be discussed throughout the theory section. Since the water is incompressible, the Navier-Stokes equation can be applied. D_x is the complete-derivative in x , and d_x is the partial derivative in x . Other variables are P = pressure of the flow, ρ = density, $p = \frac{P}{\rho}$ and

$\nabla \cdot \vec{u}$ which represents the viscosity terms.

$$\begin{aligned} D_t \vec{u} &= d_t \vec{u} + \vec{u} \cdot \nabla \vec{u} = \left(\frac{1}{\rho} \right) (-\nabla P) + \nu \nabla^2 \vec{u} \\ \nabla \cdot \vec{u} &= 0 \end{aligned} \quad \dots (3)$$

$$D_t u = d_t u + (u d_r + w d_z) u = -d_r p + \nu \nabla_r^2 u \quad \dots (4)$$

$$D_t w = d_t w + (u d_r + w d_z) w = -d_z p - g + \nu \nabla_z^2 w \quad \dots (5)$$

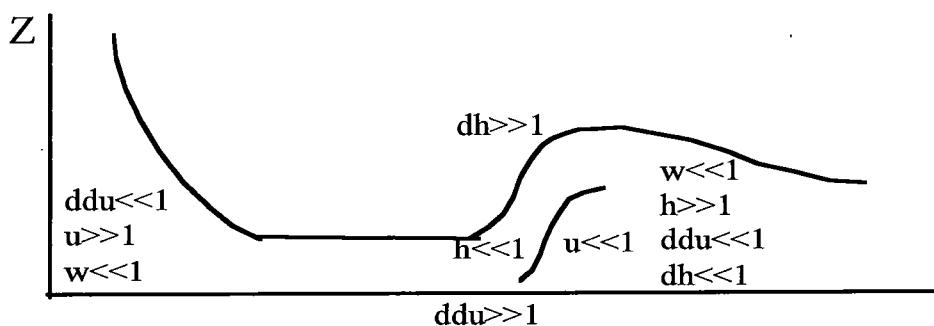
$$\begin{aligned} \nabla_z^2 &= \nabla^2 + \frac{1}{R_{jump} d_r} \\ \nabla_r^2 &= \nabla_z^2 - \frac{1}{R_{jump}^2} \end{aligned} \quad \dots (6)$$

$$d_r (R_{jump} u) + d_z (R_{jump} w) = 0$$

Thus far, there are factors which have not been taken into consideration. The depth of the water is going to change with the flow and the flow of the water is going to change with the depth. If one refers back to Figure II, this can once again be observed. The

supercritical region has a very small depth whereas the subcritical region has a relatively large depth. These regions are relatively well understood, while the critical regions are not. Figure 3, similar to Figure 2 found in “The Circular Hydraulic Jump: pursuit of analytical predictions³,” depicts the conditions which change dramatically during the critical region of the jump. In order to completely solve for hydraulic jump, the conditions of the jump are going to be assumed.

Figure III:



- This figure represents flow conditions which have a significant change in the critical region. This shows why it is so difficult to precisely study the occurrences of a hydraulic jump. The values in this figure relate to those in equation (4).

Next, the parameter “Q” needs to be examined. This is the flux through a cylinder of radius “r.” In these steps, the average velocity, v , is going to be recognized. The equation is as follows:

$$v = \left(\frac{1}{h}\right) \int u dz \dots (7)$$

Knowing this, the equations for “Q” may be listed.

$$Q = \int u d\phi dz$$

$$Q = 2\pi r \int u dz \dots (8)$$

$$Q = 2\pi r h v$$

Next the free surface, s , in the experiment may be examined. With the assumption that $s = 0$, the following may be concluded.

$$\begin{aligned}
 s &= h - z \\
 s &= 0 \quad \dots (9) \\
 h &= z
 \end{aligned}$$

$$\begin{aligned}
 D_t s &= d_t s + (u d_r + w d_z) s = 0 \\
 d_t h + u d_r h + w d_z (-z) &= 0 \quad \dots (10) \quad \text{at } z = h \\
 d_t h + u d_r h &= w
 \end{aligned}$$

These equations explain how the surface is created. They explain that as a fluid moves over a surface that the depth of the water will increase. It is also to be assumed that there is no stress on the system, $d_z u = 0$ at $z = h$. In addition to these conditions, one may also consider momentum flux conservation. "The Circular Hydraulic Jump: pursuit of analytical predictions³," states that "this simply comes from the fact that we may write a force law on an element of arbitrary size, and viscosity will not play a role in the net force to move the element,"(5). If there is a cylindrical element that has dimensions, which go from r_1 to r_2 , it is going to take total force to destroy it, $F_{total} = F_2 - F_1$.

$$\begin{aligned}
 F_1 &= d_t(mv) = m d_t v + v d_t m \\
 d_t m &= \rho Q \\
 d_t v &= 0 \quad \dots (11) \\
 F_{total} &= (v \rho Q)_2 - (v \rho Q)_1
 \end{aligned}$$

Thus, the force needed to destroy the element is:

$$\begin{aligned}
 F_1 &= \int P dA = 2\pi r \int \rho g (h - z) dz = 2\pi r \rho g \int_0^h (h - z) dz = 2\pi r \rho g \left[\frac{hz - z^2}{2} \right]_0^h \quad \dots (12) \\
 F_1 &= 2\pi r \rho g \left(\frac{h^2 - h^2}{2} \right) = \pi r \rho g h^2
 \end{aligned}$$

Equation (12) is a result of $P = \rho g (h - z)$. We can complete the above equations, by equating the following:

$$(v \rho Q)_2 - (v \rho Q)_1 = \pi r \rho g h_1^2 - \pi r \rho g h_2^2 \quad \dots (13)$$

Next $Q = 2\pi r \rho v h$ must be substituted into equation (13).

$$\begin{aligned} 2\pi r \rho v_2^2 h_2 - 2\pi r \rho v_1^2 h_1 &= \pi r \rho g h_1^2 - \pi r \rho g h_2^2 \\ v_2^2 h_2 - v_1^2 h_1 &= g h_1^2 - g h_2^2 \end{aligned} \quad \dots (14)$$

The equations listed in (14) complete the momentum flux conservation needed for the hydraulic jump. The volume-flux and the momentum-flux constancy may be written using $[x] \equiv x_2 - x_1$, for any positions 1 and 2.

$$\begin{aligned} [rvh] &= 0 \\ [v^2 h + gh^2] &= 0 \end{aligned} \quad \dots (15)$$

With the theory already complete, if it is assumed that the jump has no thickness, $[rvh]=0$ may be rewritten as $[vh]=0$. This would then allow one to know the conditions involved directly before the hydraulic jump (v_{in}, h_{in}) and thus the conditions after the hydraulic jump (v_{out}, h_{out}). Now it is possible to seek solution on the parameters of the supercritical region which will lead to valuable assumptions about the critical and subcritical regions. This can be done by looking at equation (15) in three different ways.

$$(Fr_1)^2 = \frac{v_1^2}{gh_1} = -\frac{(\eta + 1)}{\eta^2} \quad \dots (7b)$$

$$\eta = \frac{h_1}{h_2}$$

$$R^2 h_1 h_2 (h_1 + h_2) = \frac{Q^2}{2\pi^2 g} \quad \dots (7c)$$

$$\frac{R(v_1 v_2)^2}{(v_1 + v_2)} = \frac{Qg}{4\pi} \quad \dots (7d)$$

It is said in “The Circular Hydraulic Jump: pursuit of analytical predictions³,” that is not crude to neglect factors such as drag on the bottom of the pan and surface tension. The

author says this in reference to two other scientists work, Craik and Higuera. Due to this statement, the factors will not be derived in the theory section.

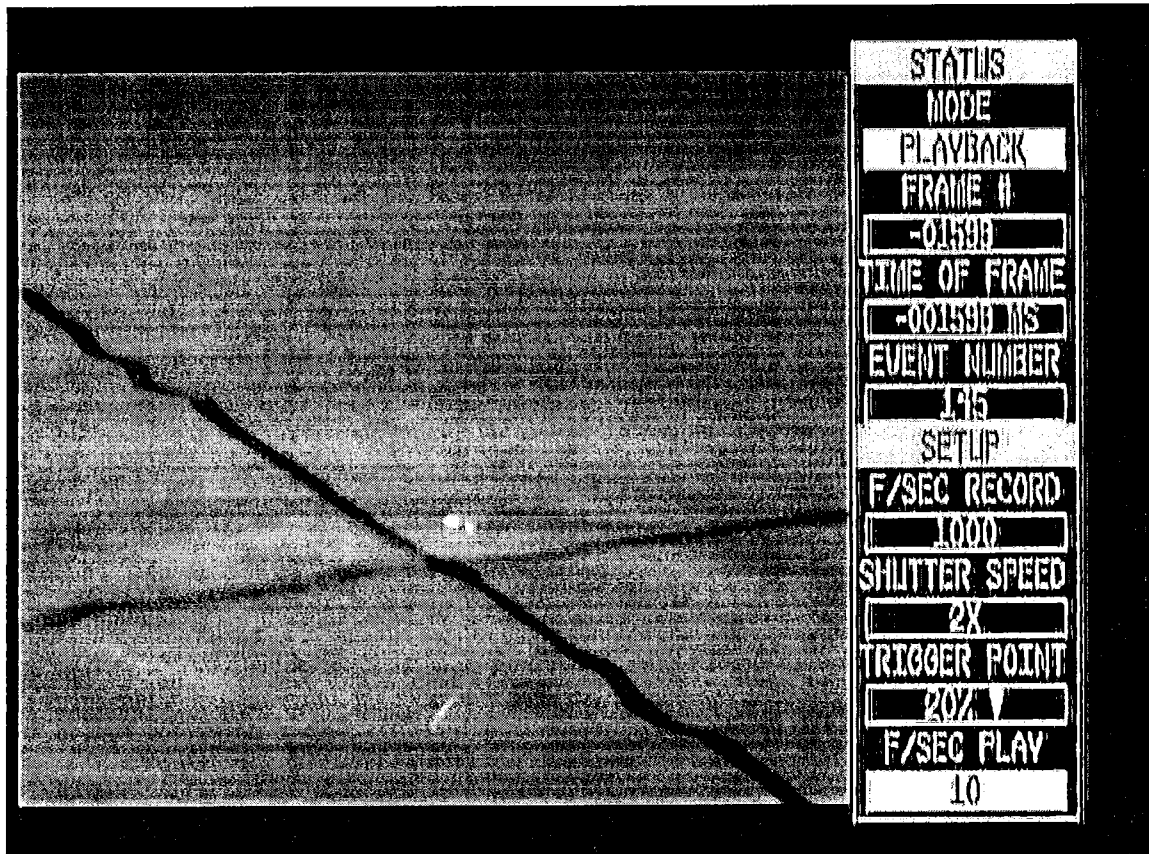
Experimental:

The hydraulic jump experiment began attempting to find a proper setup for the equipment. The original setup placed the camera above the pan, used no floodlights and water from the pipette was released by hand. This was quickly corrected when it was observed that without a greater light source the camera would in fact not be able to catch the needed footage. The camera was set to catch images at 1000 frames per second, therefore normal room lighting was not adequate. The second step then, was to add more lighting. In addition to this, the camera was moved to the side corner of the pan. It was found by observation that this gave better and more precise clips from the camera. Many of the films were taken with this setup. With more review of sources and assistance from Dr. Garg, it was thought that if as many variables as possible could be kept constant then the calculations would be simpler. Therefore it was decided to mount the pipette above the pan by means of a clamp, so that some human error would factor out. Also, instead of manually depressing the pipette plunger, a 500g mass was set atop the plunger so that the water would be dispersed at a “uniform rate.”

The data that was observed was the rate at which the water flowed from the pipette, the time taken for a given number of ripples to spread and how the depth of the water in the pan affected the size or existence of a hydraulic jump. The flow from the pipette proved to need a relatively small water depth in the pan to produce a hydraulic jump. Figure 4 gives a picture of a clip of the pipette at the deepest tested water depth. It

is obvious from this clip that no hydraulic jump was formed from the flow. It is to note that underneath the pipette flow bubbles can be observed where the pipette flow has caused turbulence. This is an indication that if the flow had been increased enough (a high enough flow rate had been achieved), that a hydraulic jump would have been witnessed.

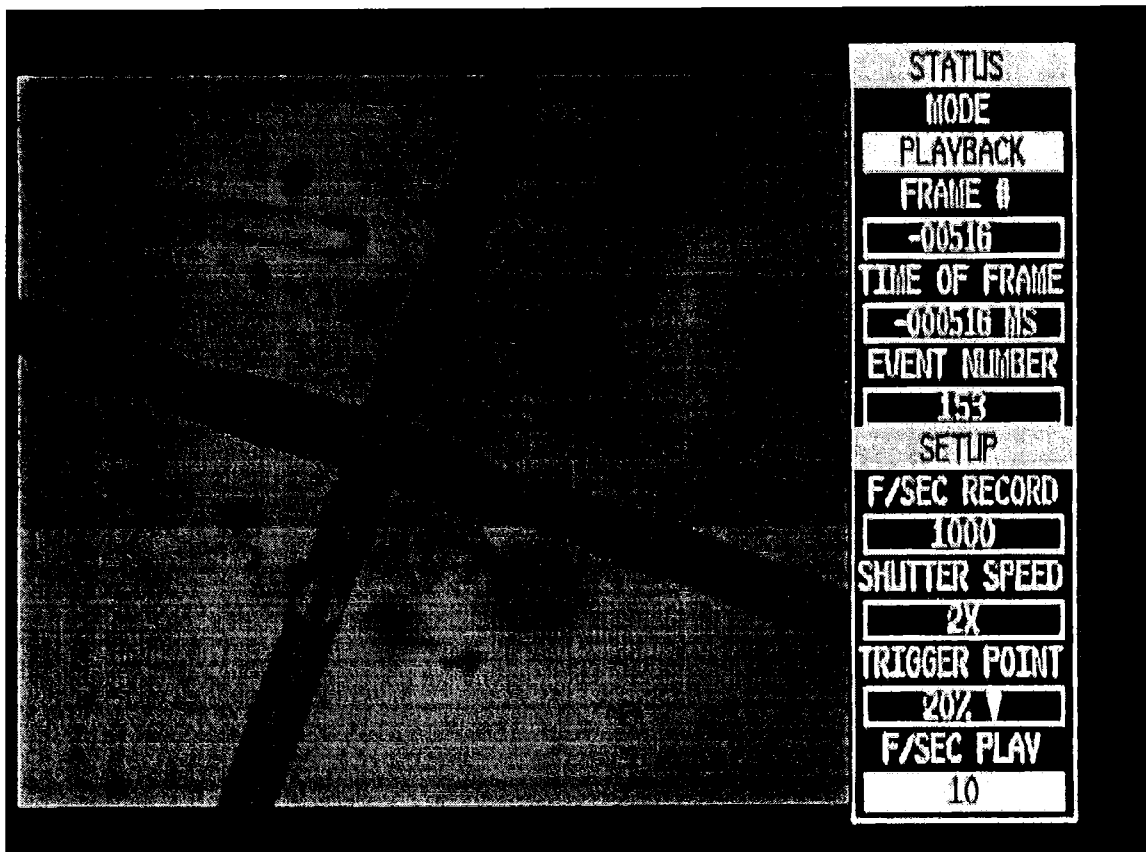
Figure IV:



- *The footage represents a pipette at the deepest pan water depth. This frame is after a smooth flow rate has been established. In this instance, there is no hydraulic jump because the depth of the water is too great. There is turbulence under the flow which represents the area where the hydraulic jump would form under appropriate conditions.*

In the next picture, figure 5, the flow of water at the depth of water in the pan may be observed from above. This picture allows the turbulent area to be more easily seen.

Figure V:

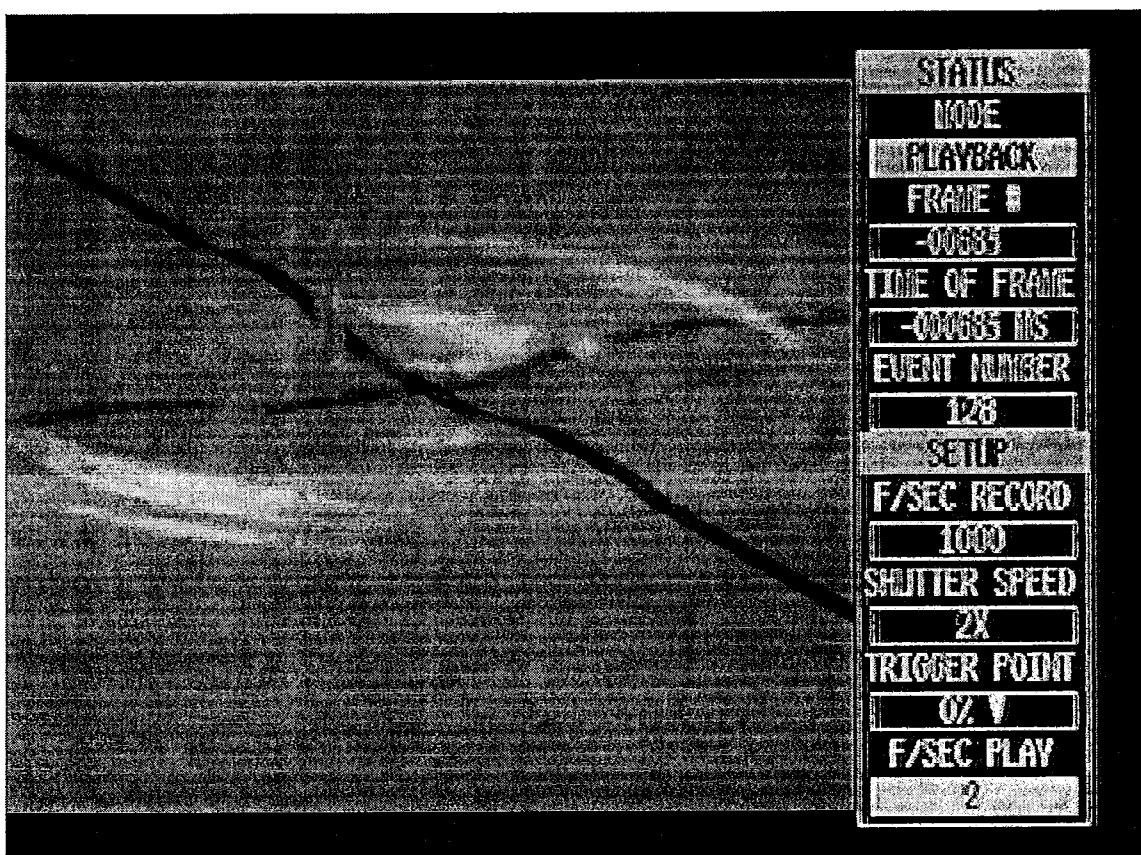


- *The picture above represents the pan water at it deepest tested depth from above. In this picture the bubbles/turbulence may be more easily seen than in the other direction.*

Once again, it is important to know that if the water had been adequately shallow, then a hydraulic jump would have formed.

Several runs were taken with the water at a very shallow depth. From these pictures, the hydraulic jump is very easily recognizable. It can be seen by looking at the smooth area that extends from the point where the pipette flow is hitting. This is the actual supercritical region. The critical region can be see immediately at the edge of the supercritical region as the point where wave formation begins. The subcritical region then, is where the waves have formed and the water depth is significantly larger than that of the supercritical region.

Figure VI:



- *The image above is of the pipette flow at a shallow depth. The hydraulic jump is very noticeable in this picture. The small size of the jump is due to the slow flow rate of the water.*

From the picture above, one may now see how depth affects the hydraulic jump. In this frame a hydraulic jump is very apparent, but in the frame of the slightly deeper water, no jump forms. Though hydraulic jumps are being observed on a small scale in this experiment, one can now see how perhaps this situation may apply in real life. If, for instance, there is a little stream slowly flowing along which suddenly hits a large rock, no jump will occur. The water will simply be caused to splash. Now, in the instance where a raging river collides with the same rock, a jump will almost surely happen. Instances such as these have been seen in California rivers frequently. In the table below, the values of the size of the hydraulic jump once a smooth flow rate has been established

compared to the depth of the water in the pan is given. Measurements of the size of the jump were taken by placing a ruler along the cross of the pan, recording a couple frames of the tool and comparing it with actual pipette flow clips.

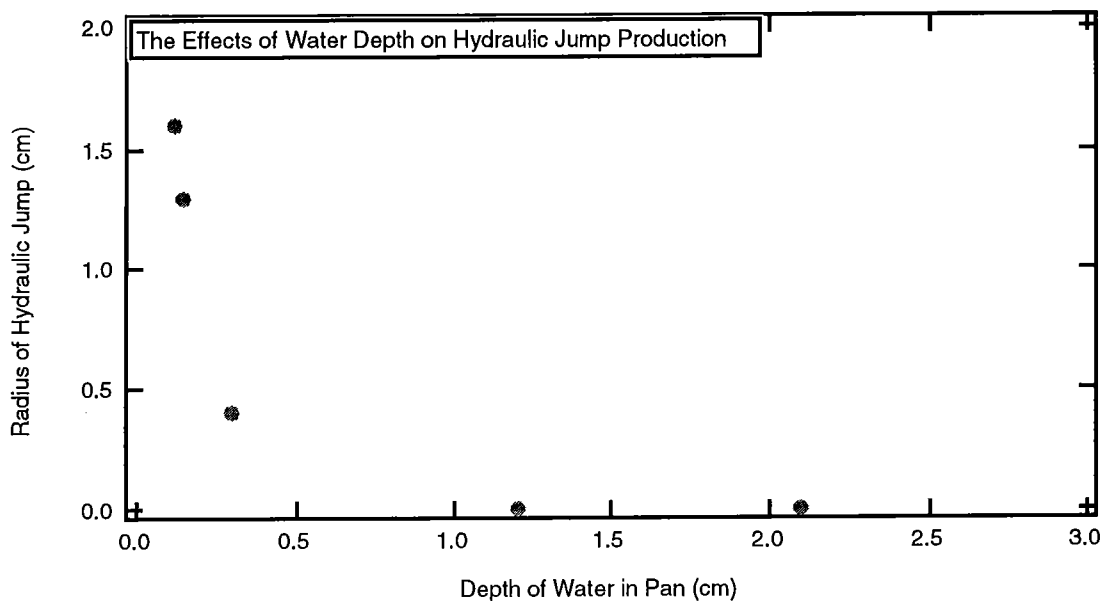
Table I:

<u>Depth</u>	0.12 cm	0.15 cm	0.30 cm	1.2 cm	2.1 cm
<u>Radius of Jump</u>	1.6 cm	1.3 cm	0.4 cm	0.0 cm	0.0 cm

These jumps and lengths were measured as accurately as the human eye would allow. They were made using a standard six-inch ruler, thus some error is inevitable. Five measurements were taken for each dimension and all errors were approximately ± 0.130 . These were found using the standard deviation tool in Excel. Measurements were taken from clips found in iMovie.

The plot in Figure 7 gives a depiction of the values in Table 1. From this chart, one can see that the decrease in jump size is completely exponential.

Figure VII:



- *This chart represents the change in radius of the hydraulic jump as compared to the depth of the water in the pan. From this chart, it can be seen that this relationship is exponential and helps to define the behavior of the pipette flows in the shallow and deep water figures.*

Another factor that can be observed when looking at a hydraulic jump, is how the speed of the water affects the size of the hydraulic jump. Though this was not properly tested for this experiment, a model can be assumed. If one were to increase the flow rate, the value for v_{in} would also be increased. Thus, the size of the supercritical region of the hydraulic jump would be increased. These values are directly related (as seen in equation (8) of the theory section).

Conclusion:

Hydraulic jumps are a phenomenon that is closer to people than thought. They occur in sinks, bathtubs, river, lakes, oceans as well as many other places. Nevertheless they are one of the least understood phenomenon today. It is important that studies continue with hydraulic jumps and their extreme, bores. They can relate to many fluid dynamics questions, which therefore relate to questions such as airfoil turbulence. In this experiment it was examined how increasing or decreasing the depth of the water in a large pan would affect the formation of a hydraulic jump at a constant flow rate. It could also be studied how the flow rate affects the jump, which was not done in this experiment.

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