Modeling Airplane Wings

Lauren Ault

Physics Department, The College of Wooster, Wooster, Ohio 44691

May 5, 2000

Abstract:
        An air gyroscope is used to determine the nature of the viscous force of a sphere floating on a cushion of Helium and the air resistance produced by model airplane wings. The frequency of rotation of the sphere decays exponentially because of the viscous force between the gas and the ball produces a viscous torque proportional to angular velocity. By attaching a model of an airplane wing to the gyroscope, the air resistance created by the wing is compared to the viscous torque on the ball. The drag force of the air on the wing is observed to vary as the angular velocity to a power, which is found to be $\sim 1.80 \pm 0.01$.

Introduction:

        Viscosity is the resistance that the flow of a gaseous system is subject to when it undergoes a shear stress[1]. Stress is defined in terms of forces acting across imaginary planes in a material[2]. An Ealing Air Gyroscope is a device that can be used to measure the viscous torque of Helium between a floating steel ball and an aluminum base cup. When the ball is spun in the base, the viscous friction between the gas, the ball, and the base causes the angular velocity of the ball to decrease exponentially.

        A Newtonian interpretation of viscosity considers the flow of liquid (or in this case, a gas) directly in contact with a surface. The layer of molecules in contact with the surface moves with the same velocity v as the surface. The layer in contact with the stationary surface retards the layer just above it; this layer slows the layer just above it and so on. The velocity varies linearly from 0 to v. The average velocity gradient is defined[3] as the increase in velocity divided by the distance l over which the change is made $v = \dfrac{v}{l}$. The force F required for laminar flow to slow the ball is proportional to the area A and the velocity v, and inversely proportional to the separation between the ball and the base l. The viscosity also includes[3] a coefficient of viscosity,

$$F = A\frac{v}{l}.$$

        Building a model wing to attach to the gyroscope simulates the flow of air over the wing of an airplane. The shape of an actual wing is used to create a vacuum behind the leading edge of the wing in order to increase the lift. The velocity of the air over the curved top of the wing is faster than under the wing and this allows for a greater lift over a smaller surface area[1]. The gyroscope does not allow for high enough frequencies of rotation for the wing to experience a significant lifting force; however, the air resistance created by the shape of the wing can be determined from this experiment. The drag force of the wing manifests itself as a power-law relationship in _ over a period of time.

        The wing of an airplane is designed with a curved surface from the leading to the trailing edge in order to produce a lift[4]. The air incident to the leading edge of the wing moves with a greater velocity over the top of the wing than the bottom, causing suction. The air behind the wing creates a vacuum and increases the lift[5]. The lift on the wing is also increased when the angle of incidence of the wing is increased with respect to the leading edge of the wing, as well as when the curvature of the upper part of the wing is increased.
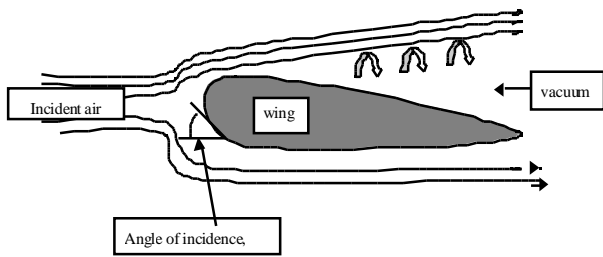
Figure 1: A model of the wing of an airplane. The air currents flow around the wing and create a vacuum behind the upper part of the wing, thus causing a lift on the entire wing. The angle of incidence at which the leading edge enters the air will also dictate the amount of lift on the wing[4].

Using three different variations on the gyroscope experiment, the relationship between the viscous torque on the rotating ball and the air drag on the wings is determined. The original set-up of the gyroscope has been modified in order to support the added weight produced by the model of the wing. The theory section proposes a power-law relationship, which is tested by the experimental data collected.

Theory:
The friction between the Helium gas and the ball in the gyroscope creates a torque on the ball. This torque   is dependent on the angular velocity   of the ball and results in the exponential decay of velocity over a period of time. When only the viscous torque on the ball is considered, the net   could be expected to vary as the first power of

$$\tau = -k\omega = I\frac{d\omega}{dt}.$$

(1)

The resulting expression for the angular velocity is

$$\omega = \omega_o e^{-\frac{k}{I}t}.$$

(2)

However, when an external resistance is placed on the ball, such as the model of the airplane wing, the flow is no longer laminar and hence, the resulting decay in the torque will, in general, depend on a power of

$$\tau = -k\omega^p = I\frac{d\omega}{dt}.$$

(3)

By separating the variables of (3) and integrating, the resulting time dependence for the angular velocity   and $p > 1$ becomes,

$$\frac{1}{\omega^{p-1}} = \frac{k}{I}t + \frac{1}{\omega_o^{p-1}},$$

(4)

where $_o$ is the initial angular velocity. The relationship between the angular velocity and the viscous torque is the power-law relationship in equation (3).

Procedure:
The experimental set-up of the equipment requires mounting a photo detector and the aligning a laser beam for reflection off of the 4-inch steel ball of an Ealing Air Gyroscope. A converging lens is used to collect the diverging beam and refocus it on the photodiode. Helium is pumped into the base of the gyroscope and causes the steel ball to levitate on a very thin layer of gas. The gas produces a low resistance and generally the viscous force between the gas and the ball is measured. Four thin, evenly spaced black lines are taped onto the ball with electrical tape. They cause a break in the reflected laser beam and create a pulse for the photodiode to count. The photodiode collects the pulse from the reflected laser beam and sends it to the counter through the Schmitt trigger. The Schmitt trigger sharpens the pulse from the photodiode and turns in into a TTL compatible pulse that can be read by a counter. The Hewlett Packard 5385A frequency counter has a filter to exclude high frequency noise, and is set to average the collected frequencies over 10 seconds. From the counter, the frequency values are sent to a computer via a GPIB cable where they are recorded and plotted using a LabVIEW 4.1 program. The program divides the frequency value from the counter by four (the number of breaks on the ball from the electrical tape) to get the rotational frequency of one rotation of the ball in revolutions per second and saves the elapsed time and frequency data to memory.
The model of the wing created the need for a modification of the original set-up. The weight of the wing on the side of the rod attached to the ball would cause the ball to

rotate unevenly and tip it over at low velocities. A track is needed to keep the ball and rod in an upright position for taking data. The track was constructed out of a large bolt and two nuts, one taped inside of the other. The bolt is screwed into the nuts and suspended above the rod with a lab clamp attached to a rod suspended between two lab stands. The bolt should not rest on the tip of the rod; the track is there to help keep the rod upright during the data runs that have the wing attached.

The wing is modeled after the wing of a plane and was constructed out of open call foam and packaging tape and was 26.6 cm long and 6.1 cm wide. The packaging tape was used to create a curved edge on one end that ended in a point on the opposite edge of the width (see Figure 1). The entire piece of foam was covered in tape to create a surface that would allow air to flow over, rather than through, it. The "wing" was then cut in half and re-taped with opposite edges matched together and a hole in between for the rod. Taping the opposite edges together allows for the curved edge of the wing to enter the incident air when the wing rotates.
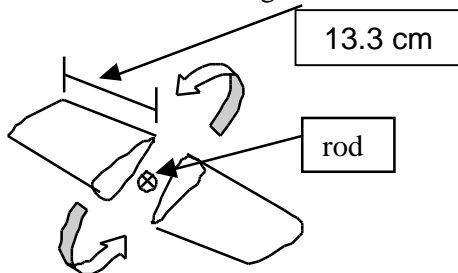
Figure 2: Top view of the wings on the rod. The two parts of the wings should be aligned as above in order for the incident air to hit the leading edge of the wing.

Three different variations on the apparatus were used to collect data. The first method does not use anything but the gyroscope without the wings or the track. Helium is pumped into the base of the gyroscope and ball is rotated for a starting frequency of approximately 4 Hz. The program is started and takes data of the decay of the angular frequency over a period of time. The track was added and found not to effect the viscous drag.

The final run uses the gyroscope with the track and wings in place. Again, the starting frequency is approximately the same as the previous two runs to allow for comparison to the other data points. The rod was fitted with a thick piece of tape to keep the wings from sliding down the rod into the beam of the laser.

Data and Analysis:

The data were analyzed in Igor and visualized by creating plots of the data. First, the viscous force when only the ball had a viscous force acting on it showed a linear decay (as expected) on a semi-log plot of the frequency versus the time. This data is consistent with equation (1) where the viscous force is proportional to the velocity.
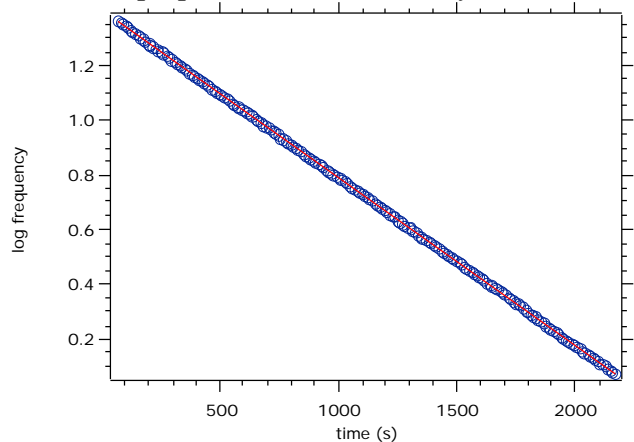
Figure 3: The plot of the decay of the frequency of the gyroscope with only the ball rotating within the base. The straight line shows the exponential decay of the angular frequency in line.

When the wing is added to the rod and the track is used to stabilize the ball, the resulting data is considered in the same way as the previous two runs. More that one set of data was taken for this part of the experiment to determine whether or not the results were consistent with one another and reproducible. Again, the data is plotted on a semi-log plot with the frequency versus time to allow for comparison to the data without the wing attached.
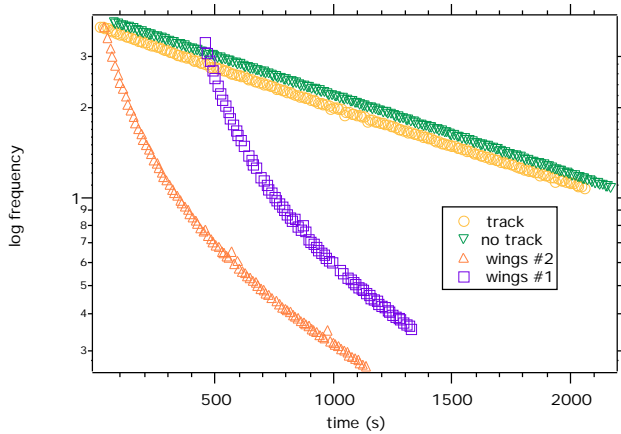
Figure 4: The decay of the frequency from each of the different methods of data collection. The run labeled "no track" is the first run without anything on the apparatus to increase the decay. The run labeled "track" is the run with only the track on. The other two runs are with the wings and the track added to the apparatus and they show similar rates of decay, but a different starting frequency.

The decay of the runs with the wing attached do not decay exponentially and thus, a different model for the viscous torque is necessary to describe the behavior of the trials with the wings attached. From equation (3), I propose a relationship between the viscous torque that is proportional to a power of the angular velocity . The rate of decay caused by the wing on the apparatus is modeled by plotting the natural log of $\dfrac{d}{dt}$ versus the natural log of . Igor is used to give a power-law fit of the data. The slope of the resulting fit is the power from equation (3). $\dfrac{d}{dt}$ is not really a derivative with respect to time; it is the numerical derivative, that is, the difference between consecutive data points. Due to fluctuations in the data points, some of the $\dfrac{d}{dt}$ terms are negative. Smoothing the derivative data eliminates the points that are very far off of the line from the following graph.
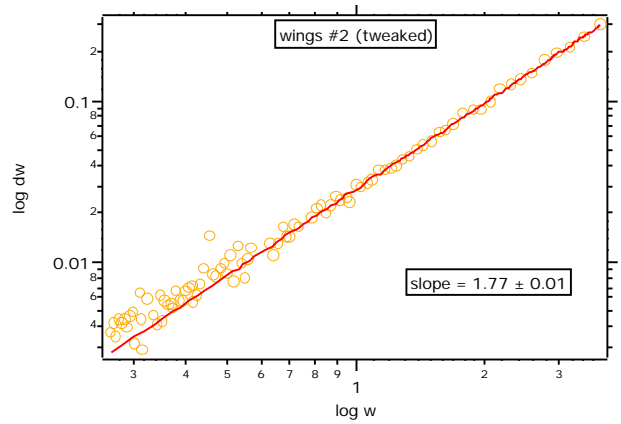
Figure 5: Plot of the numerical derivative of the frequencies versus the frequency for the second run with the wings on. A power-law fit is used to find the slope, which is the power in the equation (3)

Error in the measurement of the frequencies of rotation is caused by the average over 10 seconds. As the frequency of the ball slows, the precision decreases and this causes the increased noise toward the zero point of the plots in figure 5. The error in the measurement of the frequency is good to ± 1 bar on the ball per 10 seconds, that is, ± 0.2 radians per second.

Results:

Adding the track to the original set-up of the gyroscope did not affect the viscous force between the Helium and the ball, and the addition of the wing was possible. Without the track, the wing was too heavy for only the rod to support. The decay of the frequency of oscillation caused by the addition of the wing is much greater than the viscosity of the Helium and the ball. Therefore, the effects of the He viscosity and the extra frictional force from the track are negligible when taking the viscous drag caused by the wing into consideration.

The viscous force is not consistent with Stokes Law ($F_v = v^l$) and thus a power law was used. The resulting powers of the two sets of data that were collected with the wing are 1.81 ± 0.01 for the first trial and 1.77 ± 0.01 for the second. The average between the two is 1.79 ± 0.02 and gives a good approximation of the effect of the air drag on the wing.

Conclusions:

The decay of the frequency for only a sphere decays linearly on a semi-log plot, as expected from Stokes Law in equation (1). The addition of the track also decays linearly, but the intercept $_0$ is shifted due to the contact between the track and the tip of the rod. The addition of the wings does not agree with the model proposed in Stokes Law and creates the need for a different interpretation of the decay, which is described by equation (3). The average exponent for the decay from two different sets of data is 1.79 ± 0.02.

An extension of this project could be to measure the air drag caused by tilting the wing at different angles with respect to the incident angle of the air. The angle of incidence of the wing on the air was not varied in this experiment.

References:

[1] Encyclopedia of Science and Technology, (6th edition, McGraw-Hill Book Company, New York, 1987).

[2] "Precision Measurement and Calibration," edited by R. L. Bloss and M. J. Orloski, NBS Special Publication 300 **8**, page 452 (1972).

[3] D. C. Giancoli, Physics for Scientists and Engineers, (Prentice Hall, Englewood Cliffs, New Jersey, 1989).

[4] Lieut. G. B. Manly, Aviation from the Ground Up, (Frederick J. Drake & Co., Chicago, 1929).

[5] W. L. LePage, The ABC of Flight, (John Wiley & Sons, Inc., New York, 1928).

[6] J. Reynolds and S. Hillier, "A Demonstration of the Proportional Relationship Between Frictional Torque and Angular Velocity," The Ealing Review, 2-3, (November/ December 1977).