Hall Coefficient of Germanium

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April 29, 1999

This experiment experimentally measures the Hall coefficient of a Germanium sample, and the number of carriers of electric current per unit volume in that sample using formulae derived within the theory of the paper. The Hall coefficient for the Germanium sample was found to be $-(1.907\pm0.071)*10^{-2}$ m^3/C, and the number of carriers was found to be $3.86*10^{20}\pm0.14*10^{20}$ $/m^3$. The Hall coefficient, and the density of free carriers for germanium has been previously found to be $-8*10^{-2}$ m^3/C,[4] and $1.0*10^{21}$ electrons/m^3 respectively[6]. These results, in particular the sign of the Hall coefficient show that conduction in Germanium is in fact performed by electrons, and not holes as in many other semi-conductors. The results of this experiment also verify previously published results, as both sets of values are of the same order.

INTRODUCTION

In 1879, Hall observed that on placing a current carrying conductor perpendicular to a magnetic field, a voltage is observed perpendicular to both the magnetic field and the current. It was puzzling that the charge carriers, which were assumed to be electrons, experienced a sideways force opposite to what was expected. This was later explained by the band theory of solids.[4]

The Hall Effect has been important in the study of the mechanism of conduction in semiconductors because both the mobility and concentration of the charge carriers may be measured, as opposed to only the conductivity with conductivity experiments.

THEORY

Assuming that electrons, with charge e are the current carriers, and they have a drift velocity, v_d in the direction of the applied electric field, they experience a force due to a transverse magnetic field, B given by:

$$F = -ev_d B \qquad \ldots 1$$

where the force, drift velocity, and the magnetic field are perpendicular to each other, and the minus sign comes from the negative charge of the electron (figure 1). The sideways force, called the Lorentz Force, causes the electrons to deflect toward one face of the conductor, due to which arises a transverse electric field, which is measured as the Hall Voltage. At thermal equilibrium, when the Lorentz Force exactly matches the force due to the electric field (the Hall Voltage), we have:

$$-ev_d B = -eE_H \qquad \ldots 2$$

where E_H is the Hall Voltage, B is the magnetic field and e is the charge of an electron.

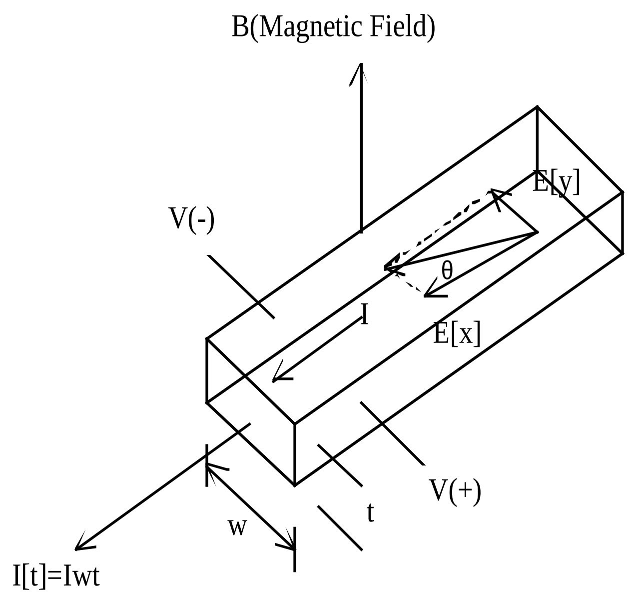


Figure 1: Voltage components for conduction by electrons in a magnetic field, where w is the width of the conductor.

To gain a good understanding of the Hall Effect, a classical particle view is beneficial. For the specific case of Germanium or any other semi-conductor, the charge carriers have a high probability of being at or around the point of injection of impurities, giving the carriers a definite location. Thus, using the classical particle treatment, the drift velocity, v_d, used in equation 1, can be used to find the current density in a semi-conductor. Assuming that the collisions that an electron undergoes are completely random, the current density, j is:

$$j = -env_d \qquad \ldots 3$$

where e is the charge of an electron, and n is the number of electrons per unit volume. The drift velocity used in equation 1 is not a very convenient parameter. Using equations 2 and 3, we can derive a convenient expression for the Hall Voltage in terms of the current density.

$$R_H = \frac{E_H}{jB} = \left(\frac{V_H}{I}\right)\frac{t}{B} \qquad \ldots 4$$

where V_H is the average Hall Voltage, or simply the Hall Voltage divided by the width across which the voltage was measured (the width of the sample). The other parameters are given sense by figure 1, and previous equations.

The applied electric field, and the transverse electric field created by the magnetic field give a resultant electric field which is not parallel to the current. The angle θ, which separates the current from the resultant electric field (figure 1), enables the derivation of a formula for the Hall Coefficient, already stated in equation 4, in terms of the charge of the carrier, and the charge density. But in fact equation 4 is a very simplified view of the system which involves many simplifying assumptions regarding the collisions of the electrons. In reality, the probability of collision depends in a complicated way on both the initial and final states.[6] When this is accounted for the equation reduces to:

$$R_H = \frac{3\pi}{8}\frac{1}{n(-e)} \qquad \ldots 5$$

Some Hall Coefficients are found to be positive, conflicting with the electron theory of conduction. The band theory of solids is essential to explain the deflection of current carriers in semi-conductors like Germanium. When the Hall Effect was first observed, the apparent opposite deflection of the electrons, which were considered the only current carriers, was puzzling. This was resolved by the band theory of solids in the early part of this century. Valence electrons occupy a complete set/band of states that spans a finite range of energies; therefore no electron may be moved to any other valence state without violating Pauli's Exclusion Principle, so valence electrons are rigid and can carry no current. In metals, the bands overlap and so electrons move freely from the upper valence bands to the lower conduction bands and thus conduct. When an electron from a filled valence band is removed, there is now a hole in the valence band for other electrons to move into, and they may now conduct electricity by moving into the hole. It appears like the hole is moving in the opposite direction of the electrons and thus, positive charges or holes are said to carry the current.

Holes may be introduced into a germanium sample by careful injection of impurities, the process called doping. This sheds light on the possibility of positive current carriers, and thus some Hall Coefficient are positive, because of conduction by positive holes in the valence bands. Equation 5 may be modified to include a positive charge instead of the –e in the denominator. Thus, the sign of the Hall Coefficient may be used to discover the nature of the charge carriers.

Experimental Setup & Procedure

To calculate the Hall Coefficient of Germanium, the power supply (Hewlett Packard 6216C) sends a current through the Ge sample, while the permanent magnet provides a strong magnetic field perpendicular to the direction of the current. A voltmeter (Keithley 197 Autoranging Microvolt DMM) is connected perpendicular to both, the direction of flow of the current and the magnetic field. This is used to make observations of the Hall Voltage, An ammeter (Tektronix CDM250 Digital Multimeter) is connected to the DC power supply to make measurements of current flowing through the germanium sample. A traveling Microscope (Gaertner Scientific Corporation), and a gauss meter (Applied Magnetics Laboratories) complete the equipment list.

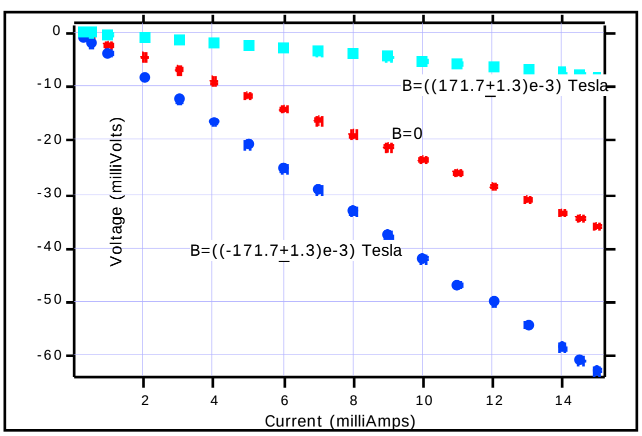
The DC power supply has control over the current passing through the sample. The current may be varied by directly changing the current, or by varying the voltage that drives the current. The instruments are very sensitive to change in either parameter, and extreme care must be taken to attain the desired level of current through the Ge sample. Once a desired current has been achieved, a transverse voltage reading is made at the voltmeter. The desired current was attained in different runs of the experiment, by alternately keeping the current and the voltage constant, and varying the other parameter.

First, the experiment was conducted without any magnetic field. Theoretically, there should be no transverse voltage as current carriers would not have deflected. Start the experiment at 15 mA, and make readings of the transverse voltage for successive decrements in the current, down to 1mA, or even lower. Then slowly increase the current back to 15 mA once again and check for reproducibility of the data. The experiment is repeated under the influence of the known magnetic field. The field is then inverted and the experiment redone yet again.

The strength of the magnetic field and the dimensions of the Ge sample are the other measurements required. To measure the strength of the magnetic field, the probe of the Gauss meter is introduced exactly in the middle of the space between the two poles, and a series of readings are made, because the field is very variable. The average value of the readings is used in the determination of the Hall Coefficient. The dimensions of the Ge sample are measured using a traveling microscope. Only the thickness of the sample is required in the determination of the Hall Coefficient. Nevertheless, measurements were made of all the dimensions of the sample. Since the Ge sample is set within a frame and should not be disturbed, its thickness is measured with a traveling microscope. The front and back end of the sample are focussed one after the other and the distance moved by the calipers recorded. Using simple trigonometric functions, the thickness is easily estimated.

Data, Error, Error Propagation & Analysis

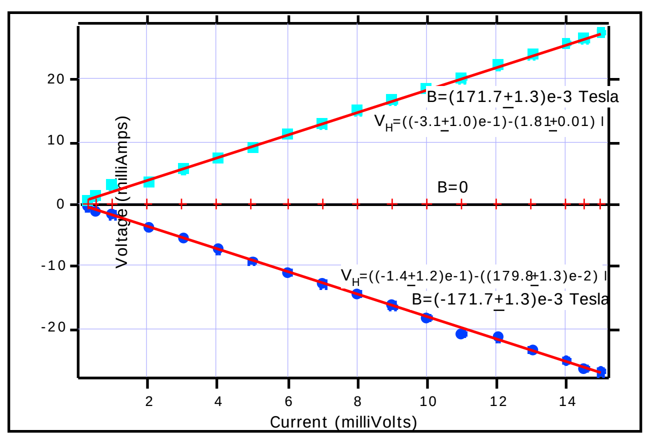
The magnetic field was measured as $(171.7\pm1.3)*10^{-3}$ Tesla, which is the SI units for magnetic fields. A conversion factor of 10^4 was used to convert between Gauss and Tesla. The dimensions of the Ge sample were measured to be $(4843.3\pm5.8)*10^{-6}$ m (length), $(483.0\pm2.6)*10^{-5}$ m (width), and $(1815.0\pm6.5)*10^{-6}$ m (thickness).



Graph 1: Transverse (Hall) Voltage plotted against the longitudinal current, uncorrected for systematic errors. The opaque squares and circles are voltages for opposite orientations of the magnetic field, while the crosses are voltages in the presence of no magnetic field.

Systematic errors, probably due to faulty calibration, causes the voltage recorded without the magnetic field to be consistently non-zero. The transverse voltage should be zero in the absence of a magnetic field because none of the charge carriers are deflected, and hence no Hall Voltage can be set up. The Hall Voltage readings with the magnetic field are thus also systematically erred by the same amount. Therefore, in graph 1, the voltage is non-zero even for a zero magnetic field. In graph 1, the error bars are constant values. The negative B field merely signifies an opposite orientation of north and south poles.

To correct for the error, the voltage readings in the absence of any magnetic field are systematically forced to zero, and adjustments are accordingly made to the Hall Voltage readings. The values of the zero magnetic field voltages are subtracted from both sets of Hall voltages. To calculate the Hall Coefficient of germanium, we look to equation 4. One method that can be employed to measure the Hall Coefficient of germanium is computing the Hall Coefficient for each of the values of current, and then using the mean and standard deviation of all those values. But there is a more elegant method. The factor V_H/I is the slope of the plot of the current and the Hall Voltage. Therefore, a line can be fitted to the values of the Hall Voltage at various values of current, and the slope of this line may be used, along with the error in fit (standard deviation), to calculate the Hall Coefficient (graph 2). The measurement of the magnetic field, and the thickness of the germanium sample is conducted as described in the procedure, and their errors are simply a standard deviation of a series of observations. These steps accounts for all the random error involved in the observations.



Graph 2: Current plotted against voltage, corrected for systematic error, such that there is no voltage reading for B=0. Straight lines are fitted to the data. The markers are consistent with graph 1.

The slope of both the lines (current plotted against Hall voltage) for the different orientations of the magnetic field, are similar. Since the values are so consistent, the average value of the two slopes (V_H/I) is used and the errors of the individual line fits also propagate as averages, since this error is greater than simply the standard deviation of the two consistent slopes. The Hall Coefficient of germanium can thus be calculated:

$$R_H = -1.907*10^{-2} \pm 0.071*10^{-2}\,\frac{m^3}{C} \qquad \ldots 6$$

On observing the sign of the Hall Coefficient of germanium, it can be concluded that the charge carriers in Ge are in fact electrons. The above value compares favorably with $-8*10^{-2}$ m^3/C, reported by Lerner et.al.[4] The negative sign may arise also imply due to the connection of the voltmeter. Keeping track of the magnetic fields therefore becomes very important.

Further, the density of the electrons is calculated using equation 5. The Hall Coefficient for germanium is known along with its associated errors, Therefore, the density of conducting electrons in Ge is:

$$n = 3.861*10^{20} \pm 0.14*10^{20}\,\text{electrons/m}^3 \ldots 7$$

The density of the current carriers is also consistent with values reported earlier. Shockely[6] reported a value of $1.0*10^{21}$ electrons/m^3, and Lerner et.al[4] reported a value of the apparent number of free electrons per atom as $1.7*10^{-9}$. To convert to the units used in this paper, (electrons/m^3) we have to find the number of atoms per unit volume for Ge. We find that there are $4.4*10^{28}$ atoms/m^3, and therefore, the density of free electrons is: $4.4*10^{28}$ atoms/m^3 * $1.7*10^{-9}$ electrons/atom = $7.48*10^{19}$ electrons/m^3.

Conclusion

The Hall Effect is important because it enables us to make measures of mobility and concentration, and gives insight into the mechanism of conductivity in semi-conductors. The Hall Coefficient and the density of carriers in Ge are well known quantities, and have been verified in this paper, given experimental limitations, with values of the same order. Also, it is established that the primary charge carriers are electrons in our sample of germanium. In some cases, conduction occurs in both the valence bands and the conduction bands. Therefore, electrons and positive holes simultaneously carry current, though in the Ge sample used in the experiment, the electrons predominate over the positive holes.

If the experiment were conducted at low temperatures, and a variable magnetic field used, the Quantum Hall Effect could have been observed. This is characterized by a step function that arises when the resistivity of the sample is plotted against the varying magnetic field.

References

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