Angular Momentum Conservation: Astronauts At Play

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During a tumbling sequence a performer will rotate about a number of different axes. During the total rotation, the angular momentum will remain constant unless an outside torque is applied to the system. Astronauts tumbling in a zero gravity environment perform a variety of twists and somersaults. Throughout their motion, the angular momentum is constant while rotating freely. During one sequence the angular momentum was determined to be a constant 61±13 kg*m^2/sec. During a different sequence, the momentum was constant at 35.7±1.2 kg*m^2/sec.

INTRODUCTION

Biomechanics involves studying and analyzing humans in motion and the various forces acting on the body during motion. There are four biomechanical analysis techniques commonly used to analyze sporting motions[6]. The first is simply watching with the eyes and is common since no outside objects are necessary. Second, there is basic cinematographic analysis involving film or videotapes. It is useful in observing what actually happened as opposed to what supposedly happened. Next, some basic math and physics principles are applied to the film footage to see how to improve skills by changing body position or to verify certain physical principles. The fourth analysis method is biomechanic research, where sophisticated instrumentation such as high speed cameras, strobe lighting, and force detectors record the physical motions. This technique is not common for everyday analysis but better utilized by researchers wanting to improve the quality of sporting performance.

Intermediate cinematographic analysis is helpful in determining whether human motion follows basic physical principles. Divers, trampolinists, and gymnasts perform a variety of different body rotations through different axes of the body. A somersault is a body rotation through the transverse axis. A twist is a body rotation about the longitudinal axis. Somersaults are mainly performed with the body in a tight, tuck position while twists are performed with the body in full extension. The reasoning behind the respective positions is to minimize the moment of inertia and maximizing angular velocity throughout performance with conservation of angular momentum.

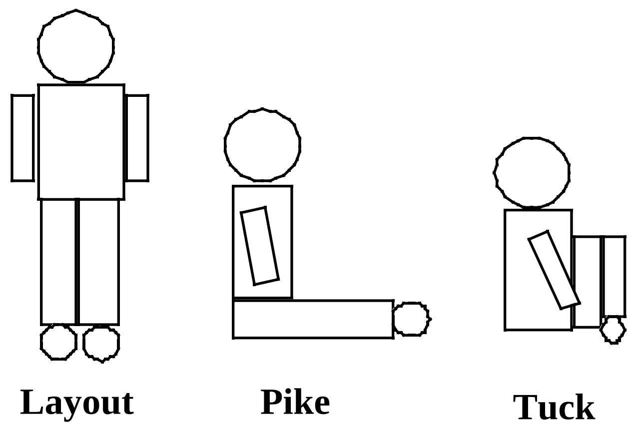

Figure 1: Common body positions held during tumbling.

A combination of somersaults can be combined with twisting to result in multiple twisting somersaults. It may seem that when performers execute twisting somersaults, then the conservation of angular momentum is violated. This is not true since the relative moments of inertia of the body adjust themselves by a repositioning of the limbs.

The NASA Skylab space station astronauts from May 1973 to February 1974 experiment with tumbling moves similar to those performed by gymnasts. The astronauts are moving in a zero gravitational field and therefore the moves have unlimited time to be executed. The principle of conservation of angular momentum is demonstrated as the astronaut tumbles, twists and rotates in space. Cinematographic analysis will be used on the Skylab footage to verify the conservation of angular momentum throughout tumbling.

THEORY

Angular momentum is always conserved unless acted on by an outside torque. If the moment of inertia of the body changes in magnitude, the angular velocities will change correspondingly. In tumbling moves, mass distribution is constantly changing as body shape is changing about the axes. It is difficult to find a precise moment of inertia for a human body since it is not a rigid system. To find the exact moment of inertia about a given axis it would be necessary to take each particle separately and multiply its mass by the square of the distance to the axis[1]. This is obviously not possible for each stage of the rotation so more general approximations must be made for the moments. To make such approximations, it is necessary to use the parallel axis theorem with the distance from each object's center to the system's center of mass.

EXPERIMENT

The segment pertaining to the astronauts tumbling on the Skylab Physics videodisk needs to be recorded as a movie on a jaz hard drive. Apple Video Player records the movie from the laserdisk using no compression for the best results. The movie can be opened in 2D Video-QT and the height of the astronaut is calibrated and set relative to a fixed origin from which all the data points can be based. The vertical height of the astronaut is 1.8 meters[3]. The origin is arbitrarily set at the midsection of the body. Sequential frames are analyzed for the position of the body and it is essential to record the frame from the actual Skylab footage which has 30 frames per second.

Two different films clips were selected to see the somersault from different perspectives. Each clip requires a different method for the estimation of the moment of inertia and the angular velocity. In one part Astronaut Bean mainly somersaults in the XY' plane while in the other Astronaut Gibson somersaults into and out of the monitor's screen.

At each successive frame mark points at the head, waist, knee and foot. The same place on the body on each frame was marked for accuracy and each frame number was recorded. The data collected by clicking on the body parts will give the coordinates of the body parts at each frame. From this, the moment of inertia and angular velocity are found.

For the moment of inertia estimation, the body is broken into three rigid boxes. The measurements for the boxes and the body masses for each segment are approximated by Frolich[3].

	mass	height	width	depth	I
Head	47.3	80	30	18	2.65
Upper	17.3	50	30	18	0.41
Lower	11	50	30	18	0.26

Table 1: Table of mass and body dimensions(kg, cm) for a 6 foot tall male. I is in units of $kg*m^2$.

The moment of inertia in Table 1 is only the moment of inertia for the individual boxes about an axis through the center of each box. The moment needs to be found for the entire system about the center of mass. The center of mass for the entire system of three boxes must be found at each frame as it constantly changes with body repositioning. The parallel axis theorem is used to find the moment of each block about the center of mass. The total moment of inertia of the system is the sum of the three blocks about the center of mass.

Going frame by frame, the lines adjoining the body parts are also rotating and from this the angular velocity can be approximated. Finding the tangent of the angle the body rotates through and using the small angle approximation, $d\theta$ is found for each time. The time between successive frames is known and therefore angular velocity is easily calculated. Once both the moment of inertia and the angular velocity are found, the total angular momentum for each frame is calculated.

In the second somersault clip, the astronaut is tumbling into the screen. His body starts in a layout position and gradually changes from a pike to a tuck (Figure 1.) There are two tuck positions: a tight tuck and a loose tuck each with a different rotating radius.

To find the moment of inertia of the body, various shapes are compared to the different body positions (Figure 2.) A sphere is used for both tucks, a block is used for the layout and two blocks in a system are used for the pike. The pike is the only difficult moment to find and the parallel axis theorem is again necessary about the center of mass.

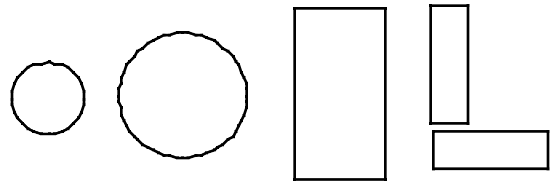

Figure 2: Various shapes associated with the body positions throughout the tumble. The total mass is assumed to be 75.6kg with the upper block as 47.3kg and the lower block as 28.3kg.

ANALYSIS AND INTERPRETATION

The astronaut performed a number of twists and somersaults in the XY' plane. The total momentum is calculated for each frame and should remain a constant value unless an outside torque is applied. At one point in the tumbling, the astronaut pushes off the wall giving himself additional momentum. Since the push or torque is applied to the system the conservation of momentum will no longer hold true. Up until the touch on the wall, the momentum should be constant.

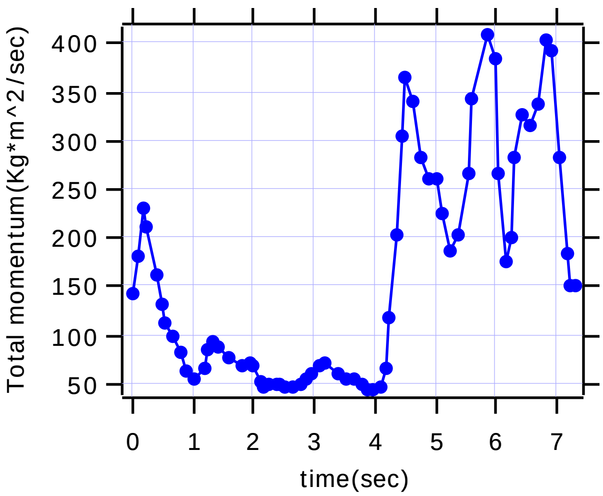

Figure 3: A plot of angular momentum versus time for a somersault in the XY' plane. The astronaut touches the wall at 4.1 seconds causing an increase in momentum. Plot smoothed once in Igor Pro.

As Figure 3 shows, from zero to 0.5 seconds, there is a spike in the curve. This is due to a different perspective angle of the camera. From 0.5 to 4 seconds, the perspective remains constant and the total momentum is roughly a constant value. Throughout this region, the astronaut is only changing his moment of inertia. The velocity will compensate for any moment changes and will keep momentum constant. The average value for the momentum is 61±13 kg*m^2/sec. The standard deviation is about a 21 percent error. The error is not unexpected due to the rough approximations made about the human body and fair resolution in the video analysis.

After the touch at 4.1 seconds, the momentum value increases. From 4 seconds to 7 seconds there is no longer a constant value for momentum. The graph looks more like a sinusoid. This is because after the push off the wall, the camera angle also changes. The astronaut is no longer in the perpendicular plane of the camera. The maxima of the sinusoid region is probably a constant momentum value of about 400 kg*m^2/sec. Each minima of the sinusoid wave is a complete revolution. The lower value is due to the perspective since not all of the momentum is observable.

The time frame from about 0.5 seconds to 4 seconds is where the angular momentum remains constant. This region is analyzed to see what the three individual blocks are doing as a function of time.

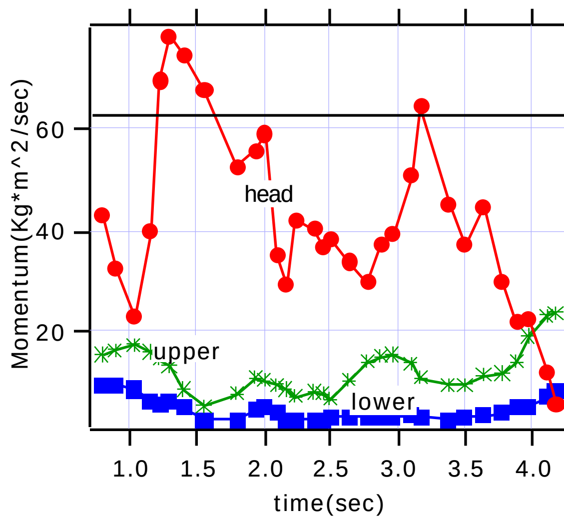

Figure 5: Plot of each block's momentum as a function of time for the segment where angular momentum is conserved. The constant momentum value is 61±13 kg*m^2/sec.

The momentum of the head block is much greater than other two blocks due to a greater concentration of mass in the upper body. The addition of the three block curves should total the constant value line at 61 kg*m^2/sec. As the curve of the head block drops, the upper or lower curves must counter by raising their curves to maintain the constant value. For example at around 2.8 seconds the head curve dips while the upper curve raises to maintain the constant value. At some points the head curve goes above the constant value line but such deviations are expected due to experimental error.

For the next somersault series, the moment of inertia estimations and angular velocities require simpler calculations. Using the estimated shapes in Figure 2, the body's moment of inertia is approximated. The various radii and lengths given in the figure are found by using the ruler in 2D Video-QT after the calibrations have been made.

Knowing the time between frames where one revolution occurs, the angular velocity is calculated. These values are used with the moment approximations to find the angular momentum. The series begins with a slower rotational speed due to the layout position. As the series progresses, the body pulls its mass closer to the turning axis and the speed accordingly increases. As the moment decreases, the velocity increases and vice versa. The ratio of the moment of inertia for the layout, pike and tuck is about 4:2:1. A plot of the total momentum versus time should result in a constant horizontal line since no outside torques are present in the system.

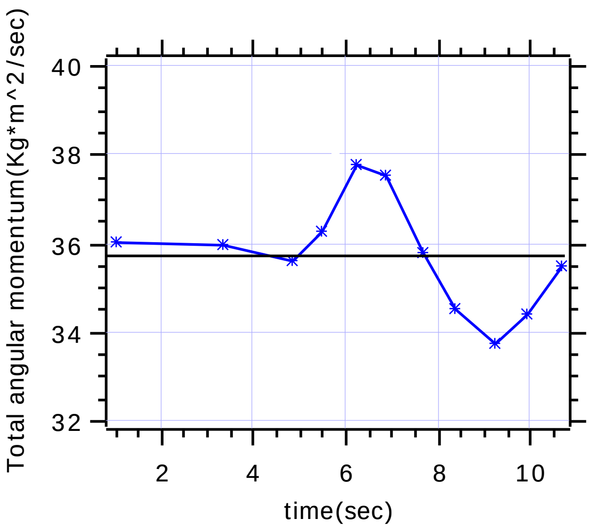

Figure 6: The plot of angular momentum versus time for a somersault.

The plot of angular momentum versus time is nearly straight and consistent with a horizontal line. The average value is 35.7±1.2 kg*m^2/sec. The standard deviation is only 3.4 percent error. This series' results are much more accurate than the somersault performed in the XY' plane. Possibly the moment of inertia estimates were more accurate since one shape instead a series of three shapes were used. The rotation about the somersaulting axis did not stray as much either. In the first clip, his motion does not remain entirely in the XY' plane and twists in and out of the screen are frequently visible and hard to measure.

CONCLUSION

When a group of leading physicists and coaches were questioned whether or not angular momentum is conserved during certain types of tumbling passes, a surprising 34 percent answered incorrectly[3]. Throughout any tumbling series of twists and somersaults, angular momentum is always conserved unless an outside torque is applied to the system.

The astronauts on the Skylab space station were filmed performing a number of gymnastic moves. Two different sequences were analyzed. The first series analyzes the astronaut in the XY' plane. His momentum remains constant at 61±13 kg*m^2/sec until he pushes off the wall. By doing this, he gives himself additional momentum. The sinusoidal motion arises after the camera angle changes. The new angle does not allow for all of the momentum to be observed at once. A constant momentum value is around 400 kg*m^2/sec.

The second series yielded more accurate results. The somersault is performed into and out of the screen. The moment of inertia estimates may have been more accurate. As the moment of inertia decreases, the angular velocity increases. The ratio of moments for the body positions is about 4:2:1, the same result Smith[7] finds. The momentum throughout this series is constant at 35.7±1.2 kg*m^2/sec. In conclusion, throughout a somersaulting series the angular momentum will be conserved unless an outside torque is applied to the system.

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