CONSERVATION OF ANGULAR MOMENTUM

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The conservation of angular momentum for a two disk system in which two disks, initially separated by an air cushion, are made to collide with each other, was investigated. The aim of this experiment was to determine whether the angular momentum is conserved in this system. The experiment was performed three times with different rotation rates for the disks each time. The angular momentum was found to be conserved in one of the trials when both disks were rotating at a similar rate and to have been kept reasonably constant in the other two trials. It is therefore concluded that the angular momentum is conserved in a two disk system but the effects of friction need to be carefully considered.

INTRODUCTION

Angular momentum and the principle of the conservation of angular momentum have wide applications in a wide range of phenomena, ranging from the emission of light by atoms, to spinning ice skaters, to the formation of galaxies. The conservation of linear momentum when applied to rotational motion corresponds to the conservation of angular momentum. In this experiment, a two disk system in which two disks, initially separated by an air cushion, are made to collide with each other, is investigated. The aim of this experiment is to determine whether the angular momentum is conserved in this system.

THEORY

Angular momentum is the rotational analog of linear momentum and is therefore also associated with a conservation law. Just as the linear momentum of a moving system remains constant so long as there are no forces external to the system, the angular momentum of a rotating system also remains constant in the absence of external torques. A rotating object turns through a certain angle in a given time and has an angular velocity ω in rad/s. If we consider a rotating object such as a wheel, the torque T, defined as the turning effect of a force F tangentially applied to a rotating object, given by $T = \mathbf{r} \times \mathbf{F}$ where r is the radius of the object, produces an angular acceleration given by the equation $T = I\ \alpha$ where I is the moment of inertia of the wheel about its axis of rotation and α is the angular acceleration. The moment of inertia is the ratio of the torque to the angular acceleration and given by $\frac{T}{\alpha} = \int r^2 \cdot dm = I$. The moment of inertia is thus dependent on the geometry of the object. For a solid disk rotating about an axis through its center, perpendicular to the plane of the disk, the moment of inertia is given by $I = \frac{m \cdot r^2}{2}$ where r is the radius and m the mass.

For this experiment, the calculation of the moment of inertia of the disks takes into account their particular geometry with openings in the center of radius r_i. The moment of inertia in this experiment is therefore given by $I = \frac{m(r_i^2 + r_o^2)}{2}$ where r_o is the outside radius[2]. The angular momentum of an object rotating about an axis is given by $I\omega$ and is conserved in a system with no external torques.

EXPERIMENT

In the experimental set up, two disks supported by an air cushion are initially separate, spinning about their common axis. The upper disk is then dropped onto the bottom one causing a "collision". The two disks initially slip but after a while rotate at the same rate. An optical system determines the rate of rotation of the disks and a data acquisition system connected to a computer program records the measurements. From the change in the rate of rotation, and the radii and masses of the disks, the angular momentum before and after the collision can be evaluated. The initial angular momentum and the final angular momentum can then be compared in order to determine whether the angular momentum is conserved in the experimental set up. The equation $\omega = 2\pi f$ where f is the frequency relates the angular velocity to the period, allowing the angular velocity to be determined directly from the frequency of the pulses produced by the optical system as the disks rotate.

The apparatus used in the experiment consisted of a Pasco Model 9270/9279 rotational dynamics system connected to an HP 3421A data acquisition/control unit which was directly connected to a Mac SE/30 computer running the LabView 2.2 program. Other apparatus included a Harvard trip balance, nitrogen gas, vernier calipers and meter rule.

The PASCO rotational dynamics apparatus is a precision instrument designed to facilitate quantitative study of rotational dynamics. The unit consists of two steel disks, a top disk and a bottom disk, which rotate about a spindle. Both disks have black bars on their sides and have holes through the center. Optical readers in the unit permit digital measurement of the rate of rotation of each disk in turn. The measurement is made once every 5 seconds. The unit is connected to a compressed air source, nitrogen. Each time a black bar on the circumference of a disk passes the optical readers, a pulse is produced. The LabView program records the number of pulses that pass by per second, which are displayed as a frequency in radian per second (Hz), and the time the measurement is made and saves the data as a table.

The disks are separated by a thin layer of air and rotate independently of the base and each other. A drop pin is inserted through the hole in the top disc and into a hole in the top of the spindle to keep the top disk floating on the bottom disk. When the pin is removed, the top disk drops and sits on the bottom disk. In the experiment, the unit was first checked to ensure that it was properly leveled to prevent rotation with a non-uniform velocity or acceleration. With a nitrogen pressure of about 5 psi, the LabView program was loaded and started. After a few seconds, the drop pin was slowly but not completely removed and the top disk was dropped upon the bottom disk and the data was collected till a few seconds after the collision and saved. Three runs were made, the first run with the bottom disk at rest and the top disk rotating, both the top and bottom disk simultaneously rotating at a similar rate and in the same direction for the second run and both disks rotating at a faster rate for the third run.

Using the program Excel, tables were constructed for the data collected and exported into the program Igor. The frequency was then plotted as a function of time for each run. The disks quickly slow down due to viscous drag which gives exponential decays. The data, however, was fit to straight lines over short intervals of time in the data analysis. Lines of best fit were drawn through the sets of points on the graphs before and after the collision for each disk and extrapolated to estimate the frequency value in the time interval between the frequency value measured just before the collision and the frequency value measured just after the collision. The frequency values were then used to determine the angular momentum of each disk before the collision and the total angular momentum of the pair after the collision. This process was carried out for each graph, given in figures 1 to 3 with the extrapolated lines and a vertical line through the midpoint of the time interval in which the collision occurs for each data plot.

Since the moment of inertia is dependent upon the geometry of the objects, the mass and geometry of the disks was also measured. The inner radius and outer radius of each disk was measured as well as the thickness of the disks. From this data, the respective moments of inertia of the disks were calculated and the initial and final momentum calculated to determine whether the angular momentum was conserved. Given on the pages following the graphs of the frequency (in radian per second) as a function of the time is the data for the measurements made to determine the geometry of the disks.

RESULTS

These measured values were used to calculate the moment of inertia of the top and bottom disks. The top disk had two inner holes, one through which the spindle went through and another smaller opening at the top. In order to account for these two inner holes, the disk was treated as being comprised of two segments, an inner thinner segment with a smaller r_i and an outer thicker segment with a larger r_i. The outer radius r_o is the same for both segments and hence the moment of inertia was calculated for each segment and the sum of the moments of inertia taken as the moment of inertia of the top disk. Given in figure 4 are simple sketches of the top disk and bottom

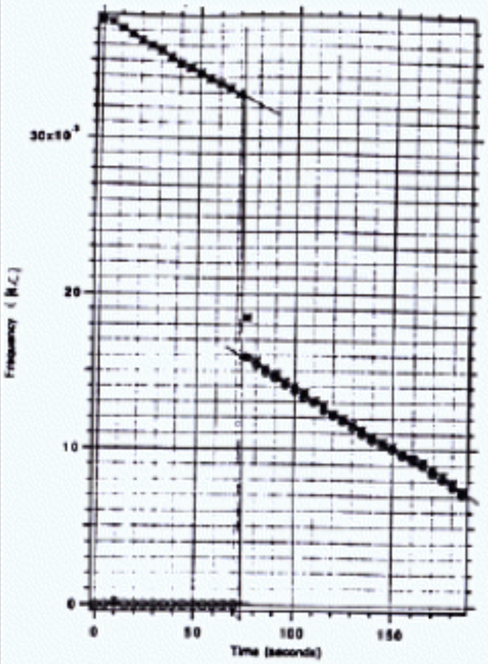

Figure 1 : Graph For the First Run
With The Bottom Disk At Rest

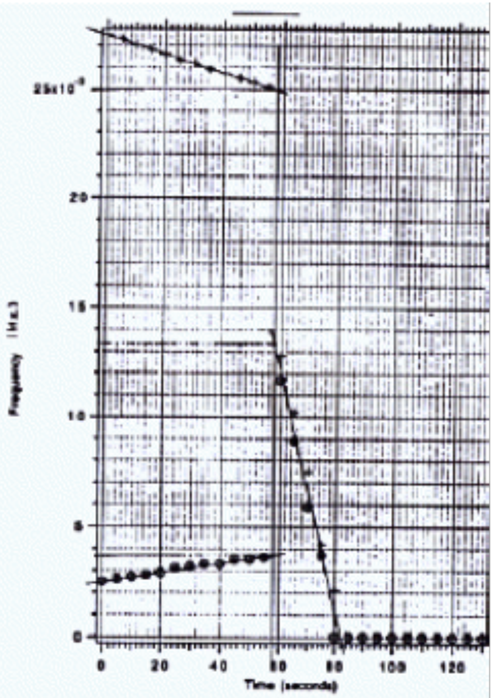

Figure 2 : Graph For Second
Run With Both Disks Rotating
At a Similar Rate In The Same
Direction.

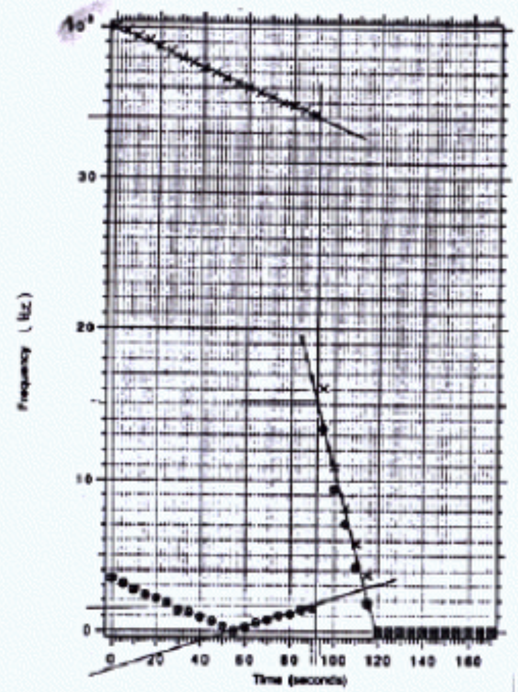

Figure 3 : Graph For Third Run with Both Disks Rotating - The Bottom Disk
Slows Down To Rest And then Resumes Rotation

disk respectively. The r_o shown is with respect to the axis of rotation of each disk.

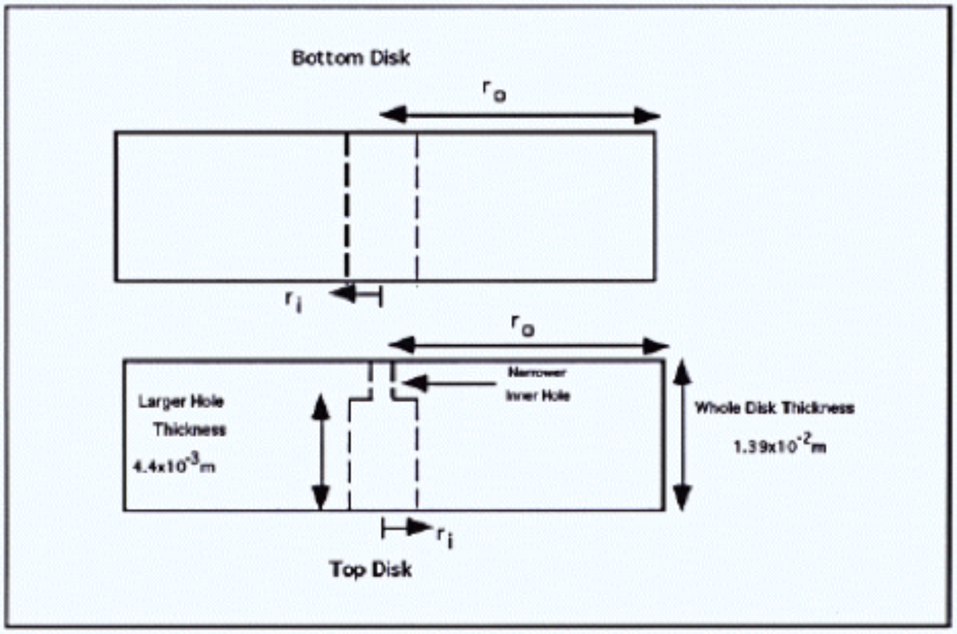

FIGURE 4: Rough sketches of the two Disks. Unlike the bottom, the top disk does not have one inner hole through it but an inner hole that drastically narrows over a small thickness at the top of the disk.

Bottom Disk	Inner radius r_i (m)	Outer radius r_o (m)	Mass (kg)
	0.00112 ± 0.00005	0.05550 ± 0.00005	1.3395 ± 0.00005
Top Disk			
Inner thickness	0.00190 ± 0.00005	0.06120 ± 0.00005	0.43590 ± 0.00005
Outer thickness	0.00790 ± 0.00005	0.05550 ± 0.00005	0.91840 ± 0.00005
Whole Top Disk.			1.35430 ± 0.00005

Table I - Geometry Of The Bottom And Top Disk As Illustrated In Fig. 4 - Used In Calculating I

Using the mass of the disk, the density was calculated to be 7783.3 kg/m^3. From this density and the calculated volumes of the segments, the masses of the inner and outer segments were determined. The moment of inertia was then calculated and the angular momentum evaluated for each run. The values of the frequencies obtained from the extrapolated graphs are given in table II. The error in each frequency value was estimated as follows : For each graph, there is a time interval between the pre and post collision frequency measurement. The midpoint of this time interval is used to estimate the frequency just before the collision for each disk and after the collision for the pair. The absolute error in this frequency value is therefore estimated as being the difference between the frequency value given by the midpoint and the value at the beginning or end of the time interval for each extrapolated line.

The given uncertainties in the momentum were determined through propagation of errors in the frequency and moment of inertia used in the calculation of the momentum. The uncertainty for each moment of inertia was determined though propagation of errors in the measurements of the radii and mass.

	Run 1	Run 2	Run 3
Top Disk (f before collision)	$32.8\times10^{-3} rad/s$ $\pm0.1\times10^{-3} rad/s$	$24.9\times10^{-3} rad/s$ $\pm6.0\times10^{-3} rad/s$	$34.0\times10^{-3} rad/s$ $\pm0.2\times10^{-3} rad/s$
Bottom Disk (f before collision)	0	$3.6\times10^{-3} rad/s$ $\pm0.02\times10^{-3} rad/s$	$1.8\times10^{-3} rad/s$ $\pm4.3\times10^{-6} rad/s$
Both Disks (f after collision)	$16.0\times10^{-3} rad/s$ $\pm0.1\times10^{-3} rad/s$	$13.4\times10^{-3} rad/s$ $\pm0.6\times10^{-3} rad/s$	$16.3\times10^{-3} rad/s$ $\pm0.1\times10^{-3} rad/s$

TABLE II: Frequency values obtained from extrapolating the data, giving the frequency value just before the collision for each disk and the value after the collision for the pair.

ANALYSIS AND INTERPRETATION

For the first run, when the bottom disk was at rest, the initial momentum was calculated to be $4.7\times10^{-4}\,kg.m^2/s \pm 2.5\times10^{-6}\,kg.m^2/s$. The final momentum was calculated to be $4.2\times10^{-4} \pm 2.8\times10^{-6}\,kg.m^2/s$.

For the second run, when both disks were rotating at a similar rate, the initial momentum was calculated to be $3.9\times10^{-4} kg.m^2/s \pm 9.0\times10^{-5} kg.m^2/s$. The final momentum was calculated to be $3.53\times10^{-4} kg.m^2/s \pm 3.0\times10^{-4} kg.m^2/s$.

For the third run, when both disks were made to rotate with the bottom disk rotating slightly faster than the top and the top's velocity decreasing, the initial momentum was calculated to be $5.05\times10^{-4} kg.m^2/s \pm 4.0\times10^{-6} kg.m^2/s$. The final momentum was calculated to be $4.28\times10^{-4} kg.m^2/s \pm 1.0\times10^{-6} kg.m^2/s$.

The aim of our experiment was to determine whether the angular momentum is conserved in a two disk system. Accordingly, it was expected that if the angular momentum is conserved, then the values of the initial and final angular momentum for the two disk system should overlap within their calculated uncertainties. For the data obtained in this experiment, such an overlap occurs in the angular momentum of the second run performed, when both disks were rotating at a similar rate, as shown in the graph in figure 2. For the other two experimental set-ups, the initial and final angular momentum do not overlap over any values within their uncertainties. Therefore, in these two cases, angular momentum is not conserved. However, the calculations for the angular momentum do not take into account the effect of friction or the rapid loss of the nitrogen air cushion. While the extrapolated data used in the calculations will incorporate this error due to friction and thus correct for it slightly, this energy loss nevertheless reduces the accuracy of the data obtained.

However, while the angular momentum values do not fall within each other's uncertainty range, the values of the initial and final angular momentum for all three experimental set ups were found to be quite close to each other. Since the conservation of angular momentum, like the conservation of momentum for linear motion, assumes a closed system, the fact that the experimental set up for this experiment isn't a closed system and is affected by friction would imply a lower angular momentum after the collision. The data obtained in this experiment clearly reflects this as the initial angular momentum is greater than the final angular momentum for all three experimental set ups. Since the angular velocity is determined through extrapolation, the accuracy of our calculation of the angular momentum before and after the collision is further reduced.

CONCLUSION

The purpose of this experiment was to investigate whether the angular momentum is conserved in a two disk system. For three different set ups of rotational motion, the angular momentum was found to be conserved in one of the systems and to have been kept reasonably constant in the other two systems. It is therefore concluded that the angular momentum is conserved in a two disk system but is not entirely conserved since the system is not entirely closed and is subject to the effects of friction i.e. frictional losses due to the air.

ACKNOWLEDGMENTS

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