Resonance in a Piezoelectric LTZ Ceramic

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Piezoelectricity, the interaction between mechanical and electrical systems, was studied in a ceramic containing compounds of lead, titanium, and zirconium. An AC voltage was applied to a piezoelectric ceramic, and the effect was observed. It was found that the mechanical response to an applied electric field in the ceramic is frequency dependent. Varying the frequency, a strong resonance was found at ~19.67 kHz, with a smaller resonance at ~18.6 kHz. There were other resonant frequencies observed both above and below the one studied in this experiment, and a fundamental frequency was not observed. The resonant frequency was modeled with a Lorentzian equation, the equation for a forced, damped, harmonic oscillator.

Introduction

The following experiment observes the effects on a piezoelectric LTZ ceramic when an AC voltage is applied. The frequency was varied, the mechanical stress on the material observed, and voltage readings of the phase shifts at 0° and 90° were recorded. The effect on the LTZ ceramic was resonant frequencies being produced by the mechanical stress, which in turn was produced from the AC voltage.

At certain frequencies, standing waves, some in the acoustic frequency range, are set up within the piezoelectric ceramic. In the frequency range studied in this experiment, two resonances were studied and modeled. The two models used to describe the phenomenon were 1) a Lorentzian curve focusing on the main resonance observed and 2) a curve fit containing two Lorentzian equations, taking into account both resonances observed. The two curve fits are equations derived from a driven damped harmonic oscillator. The experiment also determines if there is a model to explain where the different resonant frequencies will occur based on resonant frequencies qualitatively observed in this experiment and one quantitatively studied by Chris Ditchman.

Theory

Piezoelectricity is electric polarization of a material when a mechanical stress is applied. The converse of this phenomenon is also true; in certain materials mechanical stress is produced when an electric field is applied. This unique relationship between the electric field and the mechanical stress is directly proportional.

In order for a material to have piezoelectric properties, certain qualities must be present. The most important of these deals with the symmetry of the material. For a piezoelectric material, in this case LTZ, the crystal lattice structure cannot be centrally symmetric. If a centrally symmetric crystal were put under stress and became polarized, there would be only a reverse in polarization. Even if the entire crystal were inverted, there would be zero change in the stress of the material because it is centrally symmetric, and merely a reversal in polarization.[1]

The reason there can be a physical change in the structure of a piezoelectric material is due to ions present on the crystal structure which shift under an applied electric field, causing the physical dimensions to increase in the direction of the electric field. Conversely, when mechanical stress is applied, there is displacement of ions within the crystal lattice, causing an electric polarization.[2]

The converse piezoelectric effect (mechanical stress produced by an applied electric field) acts very much like a driven, damped harmonic oscillator. As the AC voltage is being applied to the ceramic material, it causes oscillations parallel to the axis across which the electric field is being applied. These oscillations can be modeled by looking at the equation of a weak driven, damped harmonic oscillator.

The equation of motion for a driven, damped harmonic oscillator is

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos(\omega t) \qquad (1),$$

where x is the position, β is a damping term, ω_0^2 is the ratio of the spring constant over the mass $(\frac{k}{m})$, and A cos(ωt) is the driving force.

This solution, in a force driven, damped harmonic oscillator eventually dies out in time and leaves:[3]

$$x_p(t) = D\cos(\omega t - \delta) \quad (2).$$

Solving for the coefficient D yields:

$$D = \frac{A}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2}} \quad (3)$$

For a weak harmonic oscillator at resonance equation (3) becomes:

$$D = \frac{D_{max}\beta}{\sqrt{\Delta\omega^2 + \beta^2}} \quad (4)$$

$$\text{and } D^2 = \frac{D_{max}^2\beta^2}{\Delta\omega^2 + \beta^2} \quad (5).$$

Equation (4) is the Lorentzian equation for a driven harmonic oscillator with weak damping.

The model which will be used to fit the data obtained from this experiment is equation (5) when $D^2 = \frac{D_{max}^2}{2}$, resulting in $\beta = \omega' - \omega_0$ (these values are in radians/sec. In this experiment they will be divided by 2π to give the frequency values, f, and the value of β in kHz). (For the complete development of the equation for the amplitude of a weak force driven damped harmonic oscillator see reference 3).

The following graph gives a visual presentation of what D_{max}^2 represents along with ω', ω_0, and β. Approximate values of these parameters can be obtained directly from the plot.

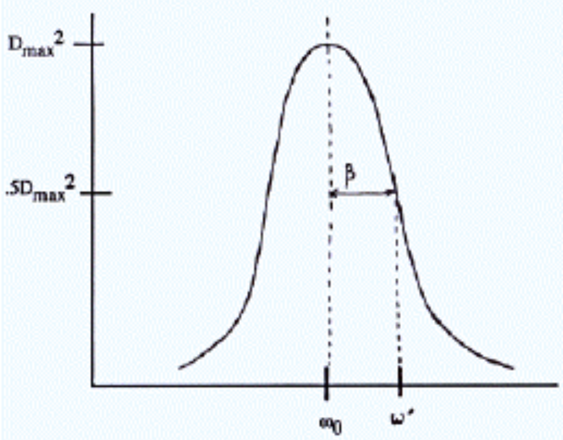

Fig. 1. The model to be used to fit data obtained in this experiment. The x-axis represents frequency and the y-axis represents the square of amplitude for the Lorentzian equation of a driven damped harmonic oscillator.

Setup/Procedure

A Wavetek sine wave generator applies an AC signal to the reference input of a SRS 510 lock-in amplifier, to the input of a Kepco BOP power supply / amplifier, and to a HP frequency counter. The output signal from the power supply/amplifier goes to the LTZ piezoelectric force driver (the "stack"). The force from the LTZ force driver is measured by a PCB force transducer in the stack. Finally, this signal is sent to the input of the lock-in amplifier. The signal coming into the lock-in amplifier is another piece of information needed for data analysis besides the frequency of the wave generator.

The force driver, or the "stack" was clamped to reduce any outside vibration effects from equipment and to make sure that the only effect on the LTZ material was from the incoming signal.

Data / Analysis

Data was collected from the frequency counter (measured in kHz), recording the in-phase (0°) and out-of-phase (90°) signals from the lock-in amplifier (measured in mV). The data was analyzed and fit to two different curves, both containing Lorentzian equations of a weakly damped, driven harmonic oscillator.

The data was analyzed using the voltage amplitude squared,

$$|A|^2 = V_0^2 + V_{90}^2 \quad (6)$$

and the phase angle between the voltage amplitude and the "in phase" (0°) voltage reading:

$$\delta = \tan^{-1}\left(\frac{V_{90}}{V_0}\right) \quad (7).$$

The uncertainty assigned to each frequency readings is ±0.0015 kHz.

Resonant frequencies were first qualitatively observed, and once a significant resonance had been found, data was taken at intervals of about 25 to 50 Hz. The data was taken between ~18.4 kHz and ~22.1 kHz, as displayed in Fig. 2. At or near resonance the in-phase and out-of-phase voltage readings were the largest, which is consistent with Eqn. (6).

Fig. 2 displays the voltage amplitude squared vs. frequency and has the following fit applied to it for the two-resonance model:

$$y = c_0 + \frac{c_1}{(x - c_2)^2 + c_3} + \frac{c_4}{(x - c_5)^2 + c_6} \quad (9),$$

taking into account both the large resonance frequency and the smaller one seen on the left part of the graph.

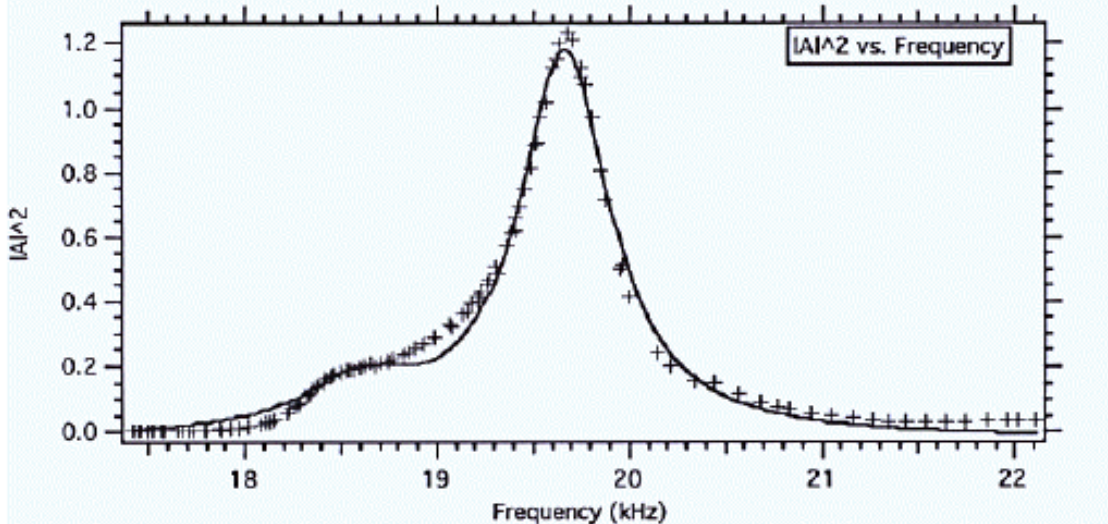

Fig. 2. Voltage amplitude squared vs. frequency with a curve fit that takes into account both of the resonance frequencies (one with a peak at about 19.7 kHz and the other at about 18.6 kHz). The data are fitted well by a two resonance model.

Coef.	Value	Standard Dev.	Theory
c_0	-0.0234	0.0074	---
c_1	0.0187	0.0057	$(D_{1max}\beta_1)^2$
c_2	18.6 (fixed)	---	$(\omega_0)_1$
c_3	0.128	0.040	β_1^2
c_4	0.0985	0.0036	$(D_{2max}\beta_2)^2$
c_5	19.6667	0.0032	$(\omega_0)_2$
c_6	0.0829	0.0033	β_2^2

Table I: These values correspond to c0, c1, c_2, c_3, c_4, c_5, and c_6 in the curve fit, along with the standard deviation from the data. It should be noted that c_2 was set to be constant, forcing the smaller of the two resonance frequencies to be 18.6 kHz.

The peak of the smaller frequency was forced to be at 18.6 kHz (c_2) so the fit would take into account this smaller frequency resonance. From the curve fit, specifically for the larger resonance, β, ω_0, and D_{max} can be determined. β is (0.288 ±0.057) kHz, f_0 is (19.6667 ±0.0032) kHz, and D_{max} is (1.090 ±0.083) mV. These values are to be compared with the fit that is to follow.

The second fit used on the same set of data but with a Lorentzian curve for one-resonance and confining the fit to the near-resonance region is:

$$y = d_0 + \frac{d_1}{(x - d_2)^2 + d_3} \quad (10).$$

Coef	Value	Deviation	Theory
d_0	.213	.041	---
d_1	.0415	.0083	$(D_{2max}\beta_2)^2$
d_2	19.666	.0065	$(\omega_0)_2$
d_3	0.0420	.0078	β_2^2

Table II: These values correspond to d_0, d_1, d_2, and d_3 in the curve fit, along with the standard deviation from the data. The standard deviation values indicate that the fit is very close to the data, supporting the theory of the piezoelectric material responding as a driven, damped harmonic oscillator.

Again, the data are well described by a driven, damped harmonic oscillator. As in the first fit, β, ω_0, and D_{max} can be determined for the larger resonance frequency. β is (0.205 ±0.088) kHz, f_0 is (19.666 ±0.0065) kHz, and D_{max} is (0.99 ±0.13) mV.

Comparing the two fits, there is some consistency between β, ω_0, and D_{max} which is what one would expect. The values for ω_0 are very consistent, the only difference in the values being the number of significant digits which can be taken (f_0=(19.6667 ±0.0032) kHz in the first fit as compared to f_0=(19.666 ±0.0065) kHz for the second fit).

Although, the values of β have a difference between them (β is (0.288 ±0.057) kHz in the first fit and β=(0.205 ±0.088) kHz for the second fit), although they are within one standard deviation of each other. The β value for the first fit is larger because in that fit two Lorentzian curves are taken into account. In any further data analysis, particularly comparing δ (angle) values to expected values from the theory, the value of β for the second curve fit (with a single Lorentzian equation) will be used.

The values for D_{max} are (1.090 ±0.083) mV for the first fit and (0.99 ±0.13) mV for the second fit. These are also within one standard deviation of each other, showing consistency between both of the curve fits.

Figure 3 is phase angle vs. frequency for the experimental data and theoretical values. The points of inflection indicate where the resonance frequencies lie. In Fig. 3, the larger of the two resonance frequencies has a point of inflection at about 19.7 kHz, consistent with Fig. 2 displaying the voltage amplitude squared. The minor resonance is difficult to pinpoint on this graph, but it falls somewhere between 18.7 kHz and 19.1 kHz.

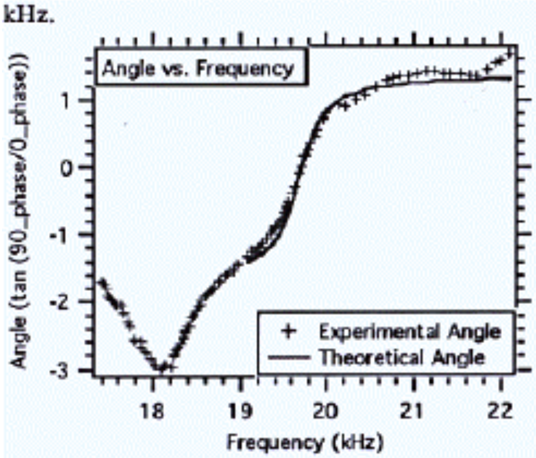

Fig. 3. Angle vs. frequency of experimental and theoretical data. The inflection points indicate where resonant frequencies occur. The major resonance has a frequency of about 19.7 kHz, the minor resonance has a frequency somewhere between 18.7 and 19.1 kHz. A scaling constant of 1.4 was used in the theoretical values to account for systematic error. The overlay further suggests the piezoelectric effect can be accurately modeled as a driven, damped harmonic operator.

The theoretical values in Fig. 3 for the angle vs. frequency are calculated between the values of 19.1 kHz and 22.2 kHz. This is the area including the main resonance frequency observed. The theoretical values were calculated from the equation:

$$\delta = \tan^{-1}\left(\frac{2\omega\beta}{\omega_0^2 - \omega^2}\right) \qquad (11),$$

In Fig. 3 the experimental data is consistent with the theoretical fit . A scaling constant of 1.4 was used to bring the theoretical values closer to the range of the data collected. The consistency of the theory with collected data continues to show that the model of a force driven, damped harmonic oscillator is accurate in describing the observed effects due to the piezoelectric material.

Conclusion

When studying piezoelectric phenomena, two resonance frequencies have been observed in the region studied. The major resonance occurring at ~19.667 kHz and a minor one occurring anywhere between 18.6 kHz to 18.8 kHz. The minor resonance observed is difficult to determine, simply because the major resonance frequency is strong in comparison.

The piezoelectric material observed in this experiment can be accurately modeled by a driven, damped harmonic oscillator. The assumption of weak damping, $\beta << \omega_0$, is an accurate one, with ω_0 being about 20 times the value of β. The Lorentzian fit(s) accurately modeled the collected data, as did the plot using Eqn. (11) to describe the observed difference in phase angle.

In trying to construct a model to describe the different resonance frequencies observed qualitatively in this experiment, and quantitatively by other experimenters, it appears there is certainly not a single fundamental frequency that can be easily explained. Some of the observed resonance frequencies may even suggest that the standing resonance waves may not be with nodes at each end of the LTZ ceramic, but even possibly have a one fixed end and an antinode at the other end. Chris Ditchman observed a strong resonance frequency at about 6.6 kHz, and a correlation between that resonance and the one observed in this experiment may be possible. The resonance he observed is about three times as small as the resonance frequency studied in this experiment.

Further experimentation is needed to have a clearer understanding of the phenomena present in LTZ ceramic material under applied AC electric fields.

[1] J.F. Nye, Physical Properties of Crystals, (Clarendon Press, Oxford, 1979), pgs. 116- 117.

[2] T.S. Hutchinson and D.C. Baird, The Physics of Engineering Solids, (John Wiley & Sons, Inc., New York, 1968) pgs.487-490.

[3] Marion Thornton, Classical Dynamics of Particles and Systems, (Saunders College Publishing, Fort Worth, 1995), pg. 125.