Forced Damped Harmonic Oscillator

Ali Rauf
Physics Department, The College of Wooster, Wooster Ohio 44691

Abstract: This experiment observed resonance phenomenon for a mass on a spring exhibiting forced damped harmonic oscillations. The experiment showed excellent agreement with theoretical predictions. The resonance frequency for this forced damped harmonic oscillator, with water as the damping medium, was measured to be $\omega = 11.18 \pm 0.01$ *rads / s.*

INTRODUCTION

The simplest case of periodic motion is simple harmonic motion and it is defined as the motion of a particle whose acceleration is always directed towards a fixed point (an equilibrium position), and this acceleration is directly proportional to distance from that equilibrium position.

This type of system is unrealistic because it assumes no dissipative forces once the particle is in motion. Physical systems, however, experience various dissipative forces. My experiment uses a damped harmonic oscillator driven by a periodic driving force. The periodic driving force was provided by a motor to which a string was attached which in turn was attached to the spring that holds the vibrating mass. The appartus that was used was a Pasco Scientific Driven Harmonic Motion Analyzer (Model 9210). In such an externally driven oscillator, the amplitude of oscillations varies in response to the drive frequency. At some value of the drive frequency, the amplitude of oscillation becomes a maximum; the corresponding frequency is called the *resonance frequency*. This occurs when the driving frequency equalsthe natural frequency of the oscillating system. This experiment observed this phenomenon of resonance using the Pasco Scientific unit and water as the damping medium.

THEORY

The equation of the motion that was used to analyze the data looks like:

$$x'' + 2\beta x' + \omega^2{}_o x = F(t) \qquad (1)$$

where ω_o is the natural frequency of the spring-mass system in the absence of the driving force, F(t) is the driving force provided by the motor, and β is the damping parameter. When the system is driven at a frequency other than the natural frequency of the system, the syste oscillates at the driving frequency rather than tl natural frequency after the initial or the transie effects due to the natural frequency die out. equation 1 if the driving force is sinusoidal, tl solution of the equation consists of homogeneous solution which accounts for tl initial or transitory oscillations of the system, ar a particular solution which describes the motion be analyzed. The particular solution looks like:

$$x_p(t) = D\cos(\omega t - \delta) \qquad (2)$$

where ω is the frequency of the drive and δ is tl phase shift. Further analysis of the differenti equation 1 and its solution gives the followir relations.[1]

$$\tan\delta = \frac{2\omega\beta}{(\omega_o{}^2 - \omega)} \qquad (3)$$

and

$$D = \frac{\alpha_o}{[(\omega_o{}^2 - \omega^2)^2 + 4\omega^2\beta^2]^{1/2}} \qquad (4)$$

where α_o is a constant, $\alpha_o = \frac{F}{M}$ if the drivin force is sinusoidal and has the form $\alpha_o \sin(\omega t)$.

There are three cases of general interest as far as the damped harmonic oscillator concerned; underdamping, critical damping, and overdamping. M experiment requires the analysis of a underdamped oscillator for which β, the dampir parameter, is very small compared with ω_o. Tl system under analysis was a lightly dampe system harmonic oscillator. By this assumptio we can deduce the result[1]

$$\omega_R \approx \omega_o \qquad (5)$$

where ω_R is the resonance frequency of the oscillating system. Furthermore, we can also make the approximations about values of parameters[1] like D since at resonance $\omega_o = \omega$ and $D_{max} \approx \frac{\alpha_o}{2\omega_o \beta}$, we have

$$D = \frac{D_{max}}{\sqrt{(\Delta\omega)^2 - \beta^2}} \qquad (6)$$

Also at the half power point, $D^2 = \frac{1}{2} D^2_{max}$ which gives

$$\Delta\omega = \pm\beta \qquad (7)$$

Therefore, the experiment was conducted in a way so that I could determine D_{max} and ω_R experimentally and then compare the experimental results with the theoretical values for these parameters. This enabled me to see whether the functional form predicted by theory is consistent with the experimental data.

EXPERIMENT

The Pasco Scientific harmonic motion analyzer allows measurement of frequency of the drive and amplitude of the oscillating mass. One can select either the frequency display or the amplitude display on the Pasco unit.

The amplitude of the driving force can be fixed by moving a scale attached to the motor. The driver amplitude was set at 1.5±0.25 mm. The Pasco motion analyzer allows the user to vary frequency by moving a frequency control knob. This knob was used to set the driving frequency at a value and the system was allowed to oscillate for a while before recording the amplitude of the mass. The driving frequency is measured by an optical sensor (a phase set LED) mounted on an arm of the Pasco unit that holds the spring. The driver frequency is difficult to read off the Pasco unit counter because it fluctuates. The best way to determine frequency values is to average the display, with the uncertainty representing the range over which frequency values fluctuate. Measurements at different driving frequencies were made.

The raw data is shown as a plot of amplitude vs frequency in fig.1.

ANALYSIS & INTERPRETATION

I used the Lorentzian fit in Igor 1.2 to anal[y] my experimental data. The form of the Lorentzi[an] fit in Igor looks like

$$y = \frac{K_1}{(x - K_2)^2 + K_3} \qquad (8)$$

Our function was modified accordingly as

$$D^2 = \frac{\beta^2 D^2_{max}}{(\omega - \omega_o)^2 + \beta^2} \qquad (9)$$

Thus, $K_1 = \beta^2 D^2_{max}$, $K_2 = \omega_o$, and $K_3 = \beta^2$. Ig[or] gives the fitted values for K_1, K_2, K_3 as:

$K_1 = 360 \pm 20;$

$K_2 = 11.181 \pm 0.005;$

$K_3 = 0.0368 \pm 0.0024$

To analyze experimental results according to equations 8 & 9, the values for amplitu[de] squared and ω, where $\omega = 2\pi f$, and f is the driv[er] frequency were calculated. D^2 corresponds to th[e] amplitude squared term. I plotted amplitu[de] squared vs ω, the driver frequency. Fig.2 show[s] my theoretical (Lorentzian) function as well as m[y] experimental values.

From Fig.2 one can read off ω_o. We see that this value falls between 11.1 rad/s and 11 rads/s which is in excellent agreement with t[he] parameter value used in the Lorentzian fit. We ca[n] also see that D_{max} has a value between 9700 mm 9800 mm^2. Once again this agrees with m[y] experimental results. The recorded value f[or] D^2_{max} was 98.9 ± 6.0 mm.

Thus values of K_o, K_1, K_2, K_3 give the following parameter values:

$D_{max} = 98.9 \pm 6.0 mm$

$\beta = 0.192 \pm 0.046 rads/s$

$\omega_o = 11.181 \pm 0.005 rads/s$

CONCLUSION

The experimentally obtained data gave a beautiful resonance shape. Th[e] experimental data was fitted by the theoretic[al] function (Fig.2) for the forced damped harmon[ic] oscillator amazingly well. Because of th[e] excellent match between experimental and th[e] theoretical function, good agreement between th[e] theoretical and recorded experimental valu[es]

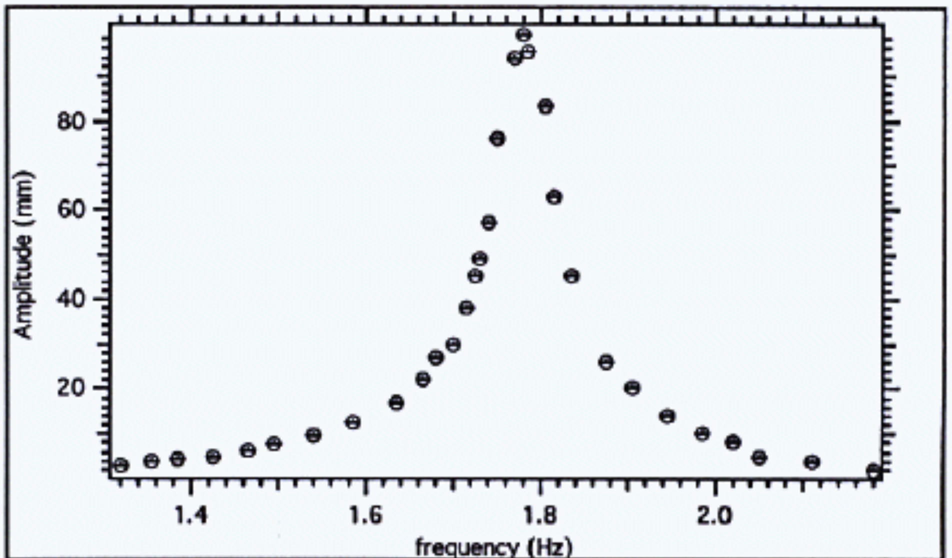

Fig.1. Plot of the raw data, in water. Amplitude (mm) of the mass-spring system vs Driving frequency (Hz). Error bars are too small to be observed.

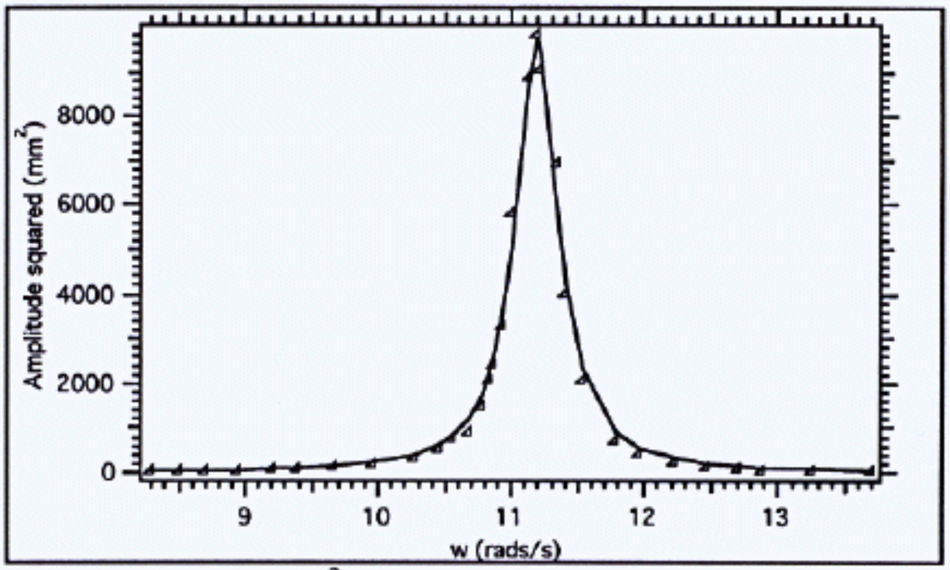

Fig.2. Amplitude squared (mm^2) vs ω (rads/s). (ω=2πf). This plot shows the experimental data and the Lorentzian fit. Markers correspond to the experimental data and the line corresponds to the fit. Error bars are not visible because they are smaller than the markers.

was obtained. The resonant frequency was found to be 11.18 ± 0.01 Hz and the maximum amplitude of the oscillator was found to be 98.9 ± 6.0 mm. A good estimate for β was found by comparing the theoretical and experimantal values. It was found to be 0.192 ± 0.046. Since this value of β is very small compared with ω_o, my assumptions regarding ω and β were justified and the results shown in the theory section are valid. Fig.2 shows that resonance occurs at ω_o. From the theory as well as the experimental values we know that $\omega_o >> \beta$ and $\omega = \sqrt{\omega_o^2 - \beta^2}$. Therefore, $\omega_o \approx \omega$ was found to be a reasonable approximation.

REFERENCES

1. J. B. Marion & S.T. Thornton, Classical Dynamics of particles and systems, 4th ed.,(Harcourt Brace College Publishing), p.126

2. D.G. Zill, A first course in differential equations, 5th ed., (Prindle, Weber & Schmidt, Boston), p.238

3. PASCO Lab Manual, p.9,10 (Unpublished)