Frustrated Total Internal Reflection:Evanascent Waves

Noah Johnston
Physics Department,The College of Wooster, Wooster, Ohio 44691

Abstract: This experiment investigates the phenomena of evanascent waves from frustrated total internal reflection. The experiment tries to determine what kind of function the waves in the less dense medium follows. The results show that the evanascent waves in the "gap" die off as the function $\frac{1}{T}=\gamma\left[\sinh\left(\alpha\left(\frac{x}{\lambda}\right)\right)\right]^2+\beta$. Empirically the best fit is $\frac{1}{T}=0.038\left[\sinh\left(5.58\left(\frac{x}{\lambda}\right)\right)\right]^2+0.95$ using microwaves and paraffin-air interface.

Introduction:

When light is incident on the surface of a medium with a different index of refraction (n), it is refracted or bent when it enters the new medium. The equation which describes how much the light is bent depends on the refractive indices of each medium, and the angle(θ) at which light is shown on the surface.

Snell's law[1] relates the angles of the two light paths as a function of the refractive indices.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \qquad (1)$$

When light is trying to escape a denser material, there will come an angle at which the beam is no longer refracted into the less dense medium, but will be reflected along the surface of the medium. The angle θ_2 will be ninety degrees (90°) and the incident angle theta will be θ_c:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \qquad (2)$$

This gives the critical angle θ_C for which light is totally internally reflected.

It is not appropriate to say that the light is completely reflected at angles greater than the critical angle, because evanascent waves are transmitted through the surface.[2] These waves are not in a plane wave as the wave before refraction. It is more appropriate to think of these waves as semi-circular fronts, emitting from a point on the surface. The waves die off quickly in free space, however if another dense medium is introduced close (within two wavelengths) to the surface being refracted, the evanascent waves will collect back together to make a coherent wave[2] (Fig. 1). The wave will have some fractional amplitude of the input wave, all depending on the distance between the dense mediums.

The evanascent waves act much like a barrier penetration problem from quantum mechanics. In the prisms, the Paraflin acts like a zero (0) potential on either side of the air gap with high potential ($E<V_0$). This is exactly how the barrier problem is oriented.

The frustrated total internal reflection problem proves that the wave is not totally reflected. There are evanascent waves which actually go through the surface. It is also a model system for quantum mechanical tunneling. It is not difficult to measure physical quantities which show similar results to the quantum mechanical system.

Evanascent Waves Between Two Prisms

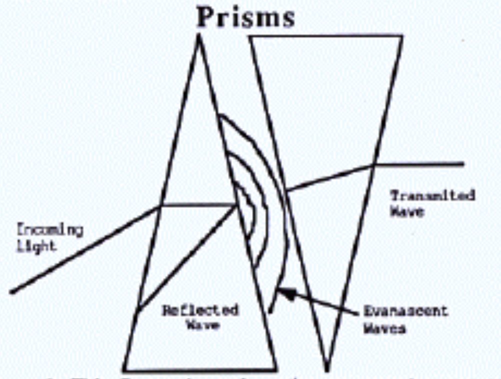

Figure 1: This figure shows how the evanascent waves are propagated between two prisms.

Theory:

The wave energy (amplitude squared) dies off as a function of distance in the gap between the two prisms. The ratio of the intensity of the transmitted wave over the input wave is the transmission coefficient, **T**. The electromagnetic wave equations state that the energies should die off as a hyperbolic sine function squared.[3]

$$\frac{1}{T} = \gamma \sinh^2(y) + \beta \qquad (3)$$

Where γ and β are a function of refractive indices and y is a linear function of x/λ.

The gap between the two prisms act as a wave suppresser. It is expected that the waves should die off as an exponential decrease. This is very similar to the way a wave is restricted in a barrier penetration problem from quantum mechanics[3]. A particle/wave is in free space traveling towards a barrier. When it hits the barrier, the amplitude of its wave is decreased, but some still can make it through the barrier.

$$T = \varepsilon e^{-\beta L} \qquad (4)$$

This shows that the amplitude of the transmitted wave should decease proportionally to an exponential function of the width of the barrier.[4]

Apparatus:

The set up for this experiment is simple. All that is needed is a "light" source, two prisms, and a detector. The "light" source used is a micro wave projector. They are electro-magnetic waves same as light, but not even close to the visible spectrum. The projector used is made by Pasco with a frequency of 10.5 GHz ($\lambda = 2.86 * 10^{-2}$ m). For prisms, two large blocks of Paraffin wax are cut to a thirty degree (30°) angle. The Detector, also manufactured by Pasco, is specifically manufactured to pick up the transmitted wavelength.

Procedure:

First the refractive index of the prism must be calculated. This is needed to find the angle at which the "light" wave must enter the prism in order to be totally internally reflected. After finding n, the incident angle to the outside of the prism must be calculated. This takes two applications of Snell's law.

After the projector has been set in place, the prisms are positioned touching, or with the displacement equal to zero (0). The detector should then be scanned back and forth to find a maximum reading. Next move the blocks some small Δx, and measure again. This continues until the intensity goes to zero(0).

There is a small problem with this intensity the closer it is to the prism. Therefore the detector had to be set far away from the prism, and as the prism is brought closer to it (as x gets larger) there comes a point where the reading become unreliable because the prism is too close to the detector.

Data and Analysis:

Two data runs were taken, with slightly different procedures. In the first, as the prism gets too close to the detector, the detector is moved to keep approximately the same separation from the second prism. In the second data run, the detector is started farther away. And no more data is taken once the prism is moved too close to the detector.

To look for a functional dependence of the intensity to the distance, plots are made. The X axis is the distance between the two prisms over the wavelength (2.86 cm). The Y axis is scaled to a maximum of one (1) by dividing the intensity readings by the highest reading. Then fits are tried for both the optical and the quantum mechanical results, hyperbolic sine and exponential respectively.

If the data fit the quantum mechanical theory it should look like a straight line on a semi-log plot. The semi-log plot would have the log of the scaled intensity(Log(I)) on the Y axis and the displacement over the wavelength(x/λ) on the X axis.

The two data sets have some fundamental differences. The data for the first half wavelength is very similar; however after that the two data sets diverge. Both data sets are plotted on the same axis and fit for both theories (Fig. 2). The equation to describe the way the intensity of the input wave over the transmitted wave is given by eqn. (3).[4] The quantum mechanical theory fits the data fairly well; however it is not as good of a fit as the optical theory.

The difference for the fitting coefficients of the two runs is about six orders of magnitude(not shown). The simplest explanation for the difference between data runs is that the runs were taken in slightly different ways. In the first run the detector is kept a constant distance from the prism, making the intensities drop off to a constant. The second data set is taken by moving the detector a fixed distance form the prism's initial position. Then the detector is left stationary, making the readings die off because the detector did not see the increase caused by the prism's proximity. I think the second of data is more reliable because there is not the error in approximating the distance from the detector to the prism.

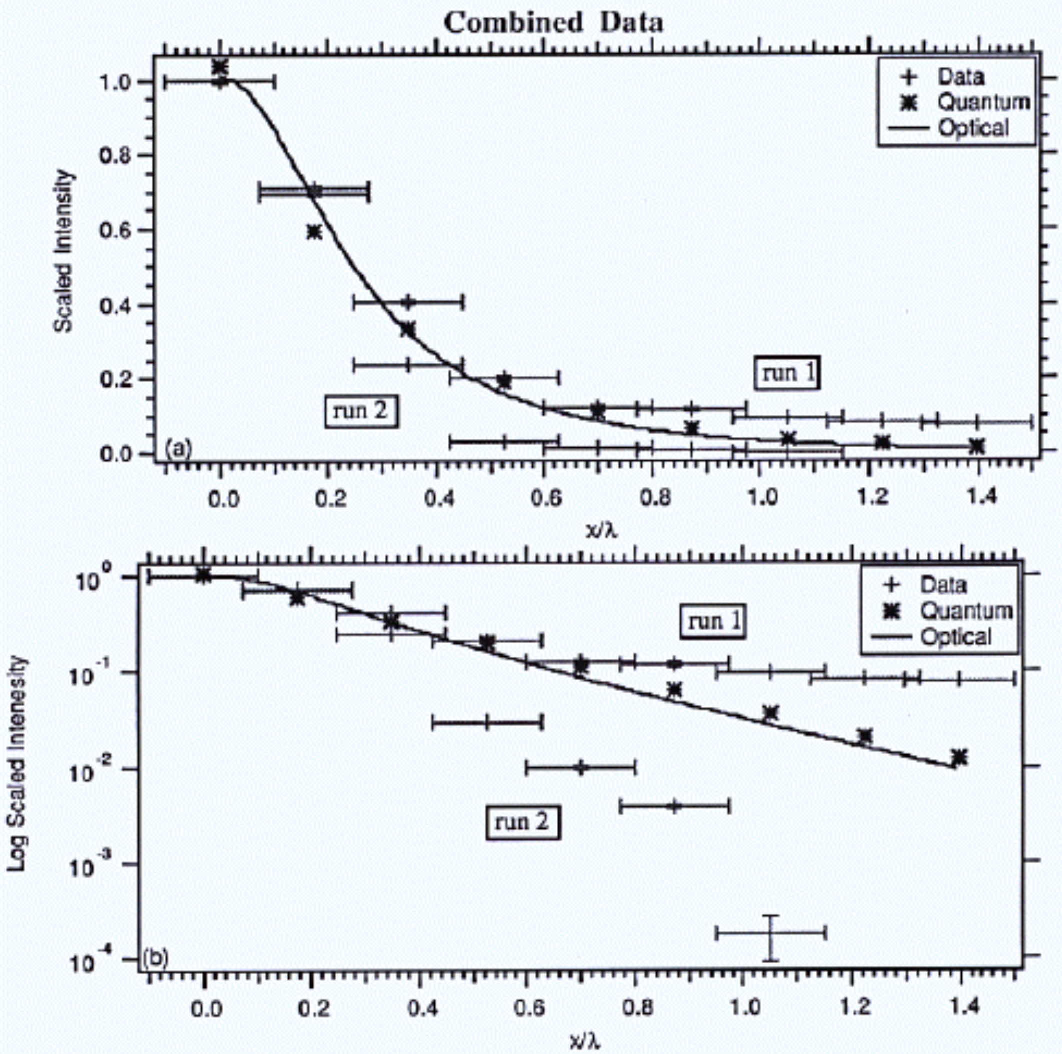

Figure 2: This is a combination of the first and second data run. For the first half wavelength, the two run are close then the first run diverges up, while the send diverges down. The fits are 1/(7.18 $\sinh^2$ (-1.48*x/λ)+0.990) and 1.04exp(-3.22x/λ). (a) intensity vs x/λ (b) log intensity vs x/λ

Therefore the data (2nd run) that fits the theory the best shows the relation of the intensity and the distance between the prisms is:

$$\frac{1}{T} = 0.038\left[\sinh\left(5.58\left(\frac{x}{\lambda}\right)\right)\right]^2 + 0.95 \qquad (5)$$

Conclusion:

The experiment is a qualitative analysis of evanascent waves. Two theories are proposed, and then the data is fit to both. The optical theory works better. The second data run almost fit the optical theory perfectly. This theory stated that the intensity should die off as a hyperbolic sine function squared is given by eqn. (3).[4] For the specific coefficients Igor was used to fit the data, giving the result in eqn. (5). The quantum mechanic theory, while not as good as the optical still fit the data. The quantum mechanical theory stated that the intensity should die off as an exponential is given by eqn. (5).[3] The best data fit (data run one) for this theory gives the coefficients:

$$T = 1.0e^{-2.6x/\lambda} \tag{6}$$

Thus, while frustrated total internal reflection does show similarities to the quantum mechanical barrier penetration problem, it is not the best theory to describe the phenomena. The optical theory derived using Maxwell's equations fits the data much more precisely.

References

[1]D. Halliday, R. Resnick, and J. Walker Fundamentals of Physics: Extended 4th edition (Wiley, New York, 1994), p 1013-1017.

[2]G. R. Fowles, Introduction to ModernOptics 2nd edition (Holt, Rinehart and Wintston, Atlanta, 1975), p 48-50.

[3]S. Zhu, A. W. Yu, D. Hawley, and R. Roy, "Frustrated Total Internal Reflection: A Demonstration and Review", Am. J. Phys. 54, p. 601-606, (1986).

[4]A. P. French, E. F. Taylor, An Introduction to Quantum Physics, (Norton, New York, 1978), p 383-389.