The Journey Around a Sphere: The Making of an Optical Rotator

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The goal we had was to make a device to rotate modes at any arbitrary angle we choose. We built a dove prism made of mirrors. The goal of this is that we can rotate the mirror system around the center point where the laser would be directed and the laser would be rotated at twice the angle of the mirror system itself. After construction we found that the rotator was very effective. The linear fit of our data was \( \Omega = a\theta \pm b \). Our most accurate way of measuring the rotation gave us values of \( a = 2 \pm 0.2 \) and \( b = 0.2 \pm 0.6 \).

PACS numbers:

I. INTRODUCTION

The goal of the project was to design and build a device that would rotate a mode to any arbitrary angle we choose. Originally we had two designs. One was using a dove prism (Fig. 1) like device, made of mirrors, that we could rotate about the axis of the laser. The second idea was using a glass tube that would send the laser beam in a helix path via total internal reflection and therefore rotate the beam.

The design for the mirror rotator was three mirrors (Fig. 2). The first mirror would be angled at 20 degrees above horizontal. The second being below and to the right of the first and be perfectly horizontal. Finally, the third mirror would be at the same height as the first mirror and the same distance from the second mirror as the first is, but in the opposite direction and angled 20 degrees below horizontal. This mirror configuration would then be inside a cylinder that would be in a bigger cylinder so that it could rotate. The rotation would be about the axis of the mirror which would hit in the center of the cylinder and the center of the first mirror.

The design for the helix was simple (Fig. 3). It is only a glass cylinder that would have a notch in the top for the laser to go in and a notch at the bottom for the laser to exit out of. We decided to go with the mirror rotator but that will be discussed more in the next section.

II. THEORY

A. Modes

Modes are important to our experiments as they are what we are actually using in our experiments. The mathematically way to express modes comes as solutions to maxwells equations. One solution to Maxwell’s equations is:

\[
E(x, y, z, t) = E_t(\hat{r}, z) \times E_l(z, t)\hat{e}
\]  

(1)

Where \( E_t \) is the transverse aspect of the electric field and \( E_l \) is the longitudinal aspect. However, we only care about the transverse aspect, and for simplicity where \( z = 0 \). This leaves us with a solution to the equation that is:

\[
E_t(x, y, 0) = \frac{A_{nm}}{\omega_0} H_n \left( \frac{\sqrt{2} x}{\omega_0} \right) H_m \left( \frac{\sqrt{2} y}{\omega_0} \right) e^{-\frac{(x^2 + y^2)}{\delta}}
\]  

(2)

By changing the \( n \) and \( m \) values on this you get any Hermite-Gaussian mode (\( HG_{nm} \)) you desire. By simplifying this equation to the two first order modes (The zero state and one state, Fig. 4) and superimposing them and
FIG. 4: the 0 and 1 states

you get this equation.

\[ c_0 E_0(x, y) \cos(kz - \omega t) + c_1 E_1(x, y) \cos(kz - \omega t + \phi) \]  

(3)

Where \( c_0 \) and \( c_1 \) are the intensities of each superimposed mode. From now on we will use \( \cos \left( \frac{\theta}{2} \right) \) for \( c_0 \) and \( \sin \left( \frac{\theta}{2} \right) e^{i\phi} \) for \( c_1 \). To simplify the above equation it’s important to note that the zero state or \( |0\rangle = E_0(x, y) \cos(kz - \omega t) \), and that the one state or \( |1\rangle = E_1(x, y) \cos(kz - \omega t + \phi) \). By adding these changes you get our final equation;

\[ \cos \left( \frac{\theta}{2} \right) |0\rangle + \sin \left( \frac{\theta}{2} \right) e^{i\phi} |1\rangle \]  

(4)

B. Poincare Sphere and the Mirror Rotator

Hermite-Gaussian modes (which I will refer to as simply modes) propagate through space as they do this, they have both a propagation vector K and a helicity vector H. The propagation vector is determined by the direction that the mode is moving. The helicity vector is then found by \( \hat{h} = (-1)^n \hat{k} \). Where n is the number of mirror reflections the mode has experienced. We can then model the helicity vector on a Poincare sphere in order to get a better idea of what these reflections mean for the modes. Modeling them on a Poincare Sphere essentially means that we will graph the helicity vectors as unit vectors all from the same origin, in this case the center of the sphere, and use the points on the sphere to extract data from the vectors. For our purposes we see that the points form an equator line on the sphere. If we then close the area of by connecting \( \hat{h}_0 \) and \( \hat{h}_3 \) using a (unique) geodesic we can use a surface integral to find the area enclosed (Fig. 5). This integral is

\[ \int_0^\theta \left[ \int_0^\pi \sin \theta' d\theta' \right] = \Omega \]  

(5)

Finally this leads to our important conclusion that \( \Omega = 2\theta \). The importance of this enclosed area \( \Omega \) is that it is also the angle of rotation that the modes go through. In the specific case of the our mirror rotator this angle omega is twice the angle theta that our mirror rotator is rotated. This means that we can always know that \( \Omega = 2\theta \), and we don’t have to constantly go back to the pioncare sphere to find the rotation involved through our mirror rotator.

C. Helix

In a helix, the rotation would not be nearly as neat as in the dove mirrors. In the helix \( \Omega = 4\pi \sin^2 \left( \frac{\theta}{2} \right) \). Where \( \Theta = \arctan \left( \frac{h_x}{2\pi q} \right) \). Putting these to equations together we see that for the helix

\[ \Omega = 4\pi \sin^2 \left( \tan^{-1} \left( \frac{h_x}{2\pi q} \right) \right) \]  

(6)

Leaving the important parameters to be h and a. An important complication in this method is that the mode will have to enter and exit the helix at the same spots respectively, changing only the amount of times it goes around the tube. This means that it can only go around the helix in full loops and not at any arbitrary amount of loops. For example it could make one loop around the helix or two. However, it could not make 4.98 rotations or it would not come out at the right spot.

D. Bloch Sphere

The Bloch sphere is a way of representing all possible superpositions of the \( |0\rangle \) and \( |1\rangle \) modes Fig. 6. In the standard basis the \( |0\rangle \) state is at the top with the \( |1\rangle \) at the bottom of the sphere. At front and back of the spheres you will see the 45 degree modes, and at the left and right you will see the donut modes. You can trace out these modes mathematically by dropping down any arbitrary amount \( \theta \) on the sphere and then tracing out any arbitrary amount \( \phi \).
E. Choice

After analyzing both options using mathematica and logic. We decided that the most reasonable option was the mirror rotator. This is because the helix gives us crazy angles that we don’t wanna work with, and the mirror rotator can give us any angle we choose. The only problem with the mirror rotator is that it will be more difficult to machine accurately. However, we still believe that we can machine it accurately enough to make it effective. The final product can be viewed in Fig. 7 and in Fig. 8.

III. PROCEDURE

A. Alignment

The rotator should already be in a translator and BLAHBLAH. Align the laser through two Iris’ before adding in the rotator. Insert the base of the rotator but take out the removable middle and attempt to get the laser through the center. I found it works to cover the front of the base with paper to see where the laser is going through the rotator. Then insert the center into it. Use the first mirror to adjust the laser onto a first iris and the second mirror to adjust the laser on the second iris. It should go in the optical table to the point shown in Fig. 9.

B. Data

I used two different methods to measure the accuracy of my rotator. First, I printed out three different sized protractors. The three sizes being 5.8 cm, 11 cm, and 20.1 cm long respectively. I then would hold the pro-
TABLE I: Rotation Data with the Medium Protractor

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tractor up to the laser and measure what the angle was (while doing my best to keep it parallel to the table). It was important to have free motion with the object as I wanted to view the the laser at the tip of the protracor in order to get a better reading on the angle, but since the protracor is a half circle it moves around a large amount to stay at that edge. I decided the best protracor was the medium length one. I then took more data with this protracor. Finally in order to get more precise measurements we devised a method of getting enough play while keeping the protracor locked down. I taped the protracors to a piece of cardboard (all on the same piece). Then attached the cardboard to a rod and stand so I would have plenty of vertical play, and slide the stand onto the rail so I would also have adequate horizontal movement. I would then rotate the rotator and move the protracor accordingly in order to get accurate measurements of its angle.

IV. RESULTS & ANALYSIS

A. Physical Results

After having taken the data, I plotted it in Igor Pro to and used a best fit line to see the effectiveness of our mirror rotator. For both instances we used a linear fit, in the fashion of $\Omega = a\theta + b$. For the medium rotator we got values of $a = 2.0 \pm 0.0$ and $b = 0.1 \pm 0.3$ (Table I). However, for the rail and post method I got values of $a = 2.0 \pm 0.02 = 1$ and $b = 0.6 \pm 0.2$ (Table II).

B. Theoretical results

Beyond the scoop of the physical results, we received interesting results after we modeled what this rotator would actually do. The best way to model the rotators action is on a Bloch Sphere. This is useful because it allows both a visual and mathematically representation of what happens to a mode as it goes through my rotator. Visually you would see a change in what the mode looked like as it went to any point on the sphere. How-
ever, mathematically you would be changing \( \phi \) and \( \theta \) in Eq. 4. We found that my rotator can change both \( \theta \) and \( \phi \), only in a specific way though. My rotator can only move the mode in a vertical direction, not in a horizontal. With us a Margaret’s interferometer we can get a mode any where though. Margaret’s interferometer can move the mode any way horizontal.

V. CONCLUSION

We found that our rotator was made to very good accuracy allowing us to use it in our optical system. Our mirror rotator will be used to rotate photons states in order to detect what state they are in. For example, a zero state photon rotated 45 degrees would come equally out both ports of a Mach-Zender interferometer, just as a zero state photon rotated -45 degrees would also come out equal out both interferometers (the two ports refer to the even and odd ports which correlates to the zero and one state). However, we can correct this problem by setting my rotator to rotate these photons 45 degrees. Then one would be at 90 degrees and the other at 0 degrees. This effectively forces the photons to come completely out of one or the other port which tells as what state they were originally in. We can also use this rotator in combination with Margaret’s interferometer to access any point on the Bloch Sphere.

VI. ACKNOWLEDGMENTS

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VII. REFERENCES


VIII. APPENDICES